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Λ SINGLE PARTICLE ENERGIES

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1. INTRODUCTION

We consider the binding energies of Λ hypernuclei (HN), in particular the single-particle (s.p.) energy data, which have been obtained for a wide range of HN with mass numbers $A \leq 89$ and for orbital angular momenta $\ell_\Lambda \leq 4$ [1]. We briefly review some of the relevant properties of Λ hypernuclei. These are nuclei ${}^{A}_{\Lambda}Z$ with baryon number A in which a single Λ hyperon (baryon number = 1) is bound to an ordinary nucleus ${}^{A}Z$ consisting of A - 1 nucleons = Z protons + N neutrons. The Λ hyperon is neutral, has spin 1/2, strangeness S = -1, isospin I = 0 and a mass $M_\Lambda = 1116$ MeV/c^2. Although the Λ interacts with a nucleon, its interaction is only about half as strong as that between two nucleons, and thus very roughly $V_{\Lambda N} \approx 0.5\, V_{NN}$. As a result, the two-body ΛN system is unbound, and the lightest bound HN is the three-body hypertriton ${}^{3}_{\Lambda}H$ in which the Λ is bound to a deuteron with the Λ-d separation energy being only ≈ 0.1 MeV corresponding to an exponential tail of radius ≈ 15 fm! In strong interactions the strangeness S is of course conserved, and the Λ is distinct from the nucleons. In a HN strangeness changes only in the weak decays of the Λ which can decay either via "free" pionic decay $\Lambda \rightarrow N + \pi$ or via induced decay $\Lambda + N \rightarrow N + N$ which is only possible in the presence of nucleons. Because of the small energy release the pionic decay is strongly suppressed in all but the lightest HN and the induced decay dominates. However, the weak decay lifetime $\approx 10^{-10}$s is in fact close to the lifetime of a free Λ. Since this is much longer than the strong interaction time $\approx 10^{-22}$s we can ignore the weak interactions when considering the binding of HN, just as for ordinary nuclei.

In our work we consider the Λ separation energies B_Λ defined by $-B_\Lambda = {}^{A}_{\Lambda}E - {}^{A-1}E$, where ${}^{A}_{\Lambda}E$ is the total energy of the HN and ${}^{A-1}E$ is the ground-state energy of the "core" nucleus. For orientation consider a medium to heavy HN: $A \gtrsim 20$. The Λ-nuclear interactions generate a Λ-nucleus potential which roughly follows the density distribution ρ(r) of the core nucleus, with an approximately constant value D_Λ in the interior. This well depth D_Λ is identified with the Λ binding in nuclear matter. Then for the ground s_Λ states for which the Λ is in an s state: $B_\Lambda \approx D_\Lambda - T_\Lambda$ where the Λ kinetic energy $T_\Lambda \sim A^{-2/3}$ since the radius of the Λ-nucleus potential is approximately that of the core nucleus ($R_A \sim A^{1/3}$). Figure 1 shows the experimental B_Λ vs. $A^{-2/3}$, in particular the s.p. energies obtained from

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the $\pi^+ + {}^AZ \rightarrow {}^A_\Lambda Z + K^+$ reaction [1]. Extrapolation to $A \rightarrow \infty$, i.e. $A^{-2/3} \rightarrow 0$, in particular also for the s_Λ states gives $D_\Lambda \approx 30 \pm 3$ MeV, a value which has been known for a long time.

Our aim in this and previous work [2,3] is to learn about the Λ-nuclear interactions as well as the structure of HN from the B_Λ data, making appropriate and adequate few- and many-body calculations. Here the emphasis is on the s.p. B_Λ data.

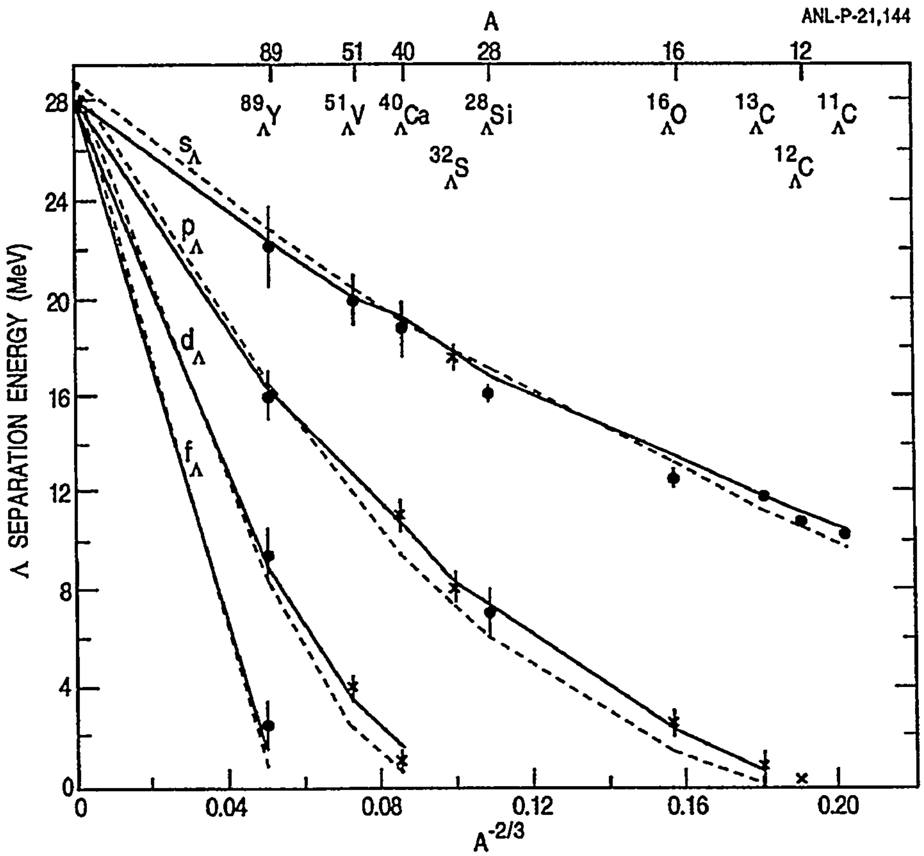

Figure 1 The experimental B_Λ are shown with errors. The curves depict the calculated B_Λ. The solid curve is for the first interaction of Table 2; the second interaction of Table 2 gives a very similar fit. The dashed curve is for a purely dispersive ΛNN potential: $V_o = 6.2$ MeV, $W = 0.016$ MeV, $Cp = 0$.

2. Λ-NUCLEAR INTERACTIONS

Our interactions are in large part phenomenological but are generally consistent with and suggested by meson-exchange models, and are such that they can be used in few- and many-body calculations.

2.1 ΛN Potential

A basic difference between the ΛN and NN potentials is that one-pion exchange (OPE) is not allowed between a Λ and a nucleon since the Λ has isospin $I = 0$ and hence there is no (strong) $\Lambda\Lambda\pi$ vertex. However, the Σ hyperon also has S = -1 but has I = 1 and there is thus a $\Lambda\Sigma\pi$ vertex. Since the Σ is only about 80 MeV heavier than the Λ, the two-pion-exchange (TPE) potential is a dominant part of the ΛN potential being dominated by the strong tensor OPE component acting twice. There will also be K,K* exchange potentials which will in particular contribute to the space-exchange and the ΛN tensor potential. The latter is of quite short range because there is no long range OPE and also quite weak because the K and K* tensor contributions are of opposite sign [4]. Also there will be short-range contributions from ω, quark-gluon exchange, etc. which we represent with a short-range Saxon-Wood repulsive potential which - somewhat arbitrarily - we take to be the same as for the NN potential [5].

We then use an Urbana-type central potential [5] with space exchange and a TPE attractive tail which is consistent with Λp scattering. For the present work we need only the spin-average potential which is

$$V_{\Lambda N}(r) = V(r) + V_x\,, \qquad V_x = -\varepsilon V(r)\left(1 - P_x\right). \tag{1}$$

P_x is the ΛN space exchange operator, V_x is the space-exchange potential with ε determining its strength relative to the direct potential which is

$$V(r) = W_0 / \left[1 + \exp\{(r - R)/a\}\right] - V_{2\pi}\,, \qquad V_{2\pi} = V_0 T_\pi^2(r)\,, \tag{2}$$

where W_0 = 2137 MeV, R = 0.5 fm, a = 0.2 fm and r is in fm. The strength of the spin-average ΛN potential consistent with Λp scattering is $V_0 = 6.15 \pm 0.05$ MeV. (In terms of the singlet and triplet strengths $V_0 = (V_s + 3V_t/4)$. $T_\pi(r)$ is the one-pion exchange tensor potential shape modified with a cut off:

$$T_\pi(r) = \left(1 + 3/x + 3/x^2\right)\left(e^{-x}/x\right)\left(1 - e^{-cr^2}\right)^2 \tag{3}$$

with x = 0.7r and c = 2.0 fm^{-2}. The space-exchange parameter, $\varepsilon \approx 0.1$-0.38, is quite poorly determined from the Λp forward-backward asymmetry. For our fits to the s.p. data we take ε to be a free parameter. This determines the odd-state potential, in particular the p-state potential to be

$$V_p = (1 - 2\varepsilon)V(r)\,. \tag{4}$$

In Table 1 we show some results for the ground state B_Λ calculated with our ΛN potential. Five-body variational Monte Carlo (VMC) calculations [6] were made for ${}^5_\Lambda$He in which a Λ is bound to an α particle [2]; for ${}^9_\Lambda$Be for which a 2α + Λ model was used implemented by appropriate VMC calculations [3], and for the well depth D_Λ for which the Fermi hypernetted chain (FHNC) method was used [2,7]. The space-exchange contribution for the s-shell HN (A $\leq$ 5) and for ${}^9_\Lambda$Be was recently obtained with the VMC method [8].

For ${}^{5}_{\Lambda}$He for all our interactions (including ΛNN potentials, see below) we obtain $E_x \approx 0.5$ MeV repulsion for $\varepsilon = 0.3$. For ${}^{9}_{\Lambda}$Be more limited calculations give $E_x \approx 1.3$ MeV.

Table 1. Experimental and calculated B_Λ (in MeV) of selected hypernuclei. The errors in the calculated B_Λ are due to the uncertainties in the strength V_0 of $V_{\Lambda N}$.

HN	Exp. B_Λ	Calculated B_Λ for ε=0	Calculated B_Λ for ε=0.3
${}^{5}_{\Lambda}$He	3.12 ± 0.02	6.1 ± 1	5.6 ± 1
${}^{9}_{\Lambda}$Be	6.71 ± 0.04	$\approx 12 \pm 2$	$\approx 10.7 \pm 2$
D_Λ	30 ± 3	74 ± 4	60 ± 4

It is clear that with only a ΛN potential fitted to Λp scattering, and even with rather large space exchange, the HN for $A \geq 5$ are strongly overbound relative to the experimental values. This is an ancient result which has been sharpened over time. Furthermore, we shall show that the s.p. data will not permit a fit with only a ΛN potential even if the requirement that this fit the scattering data is relaxed. These results imply that many-body effects are very large.

2.2 ΛNN Potential - Many-Body Effects

Many-body effects can arise for a central $V_{\Lambda N}$ through changes in the ΛN correlation function $g_{\Lambda N}$ due to the presence of other nucleons. Related, are modifications (suppression) by other nucleons of an effective interaction due to e.g. a tensor force which must act at least twice. Such tensor-force suppression of the NN force is a very important contributor to nuclear saturation. (A ΛN tensor force is suppressed much less because of its short range and weakness). For the TPE ΛN potential $V_{2\pi}$ a closely related suppression effect arises from the modifications of the propagation of the intermediate Σ or N by other nucleons (Fig. 2). Such effects have been calculated in a coupled-channel reaction-matrix approach and can give a large repulsive contribution because of the large couplings together with the small Σ–Λ mass difference [9]. We represent such suppression effects by a phenomenological (repulsive) "dispersive" ΛNN potential of the form

$$V^{D}_{\Lambda NN} = W T_\pi^2(r_{1\Lambda}) T_\pi^2(r_{2\pi}) . \tag{5}$$

where $r_{i\Lambda}$ are the Λ-nucleon separations. A strength $W \approx 0.02$ MeV gives a repulsive contribution which is roughly consistent with the suppression obtained in coupled-channel reaction matrix calculations.

The other type of three-body ΛNN force (Fig. 2) arises from TPE, appropriate to a p-wave pion interaction of the Λ with two nucleons (1 and 2), and has the form [10]

$$V^{2\pi}_{\Lambda NN} = -\left(C_p / 6\right)\left(\vec{\tau}_1 \cdot \vec{\tau}_2\right)\left\{\left[\left(\vec{\sigma}_1 \cdot \vec{\sigma}_\Lambda\right)Y\left(r_{1\Lambda}\right) + S_{1\Lambda}T\left(r_{1\Lambda}\right)\right], \left[\left(\vec{\sigma}_2 \cdot \vec{\sigma}_\Lambda\right)Y\left(r_{2\Lambda}\right) + S_2T\left(r_{2\Lambda}\right)\right]\right\} \tag{6}$$

where $\{A,B\} = AB + BA$, $Y(x) = \exp(-x)(1-\exp(-cr^2))/x$ and $T(x)$ is given by Eq. (3). S_{ij} is the tensor operator for particles i,j and $\vec{\sigma}_i$ and $\vec{\tau}_i$ are the spin and isospin Pauli operators for particle i. Theoretical estimates give $C_p \approx$ 1-2 MeV [10].

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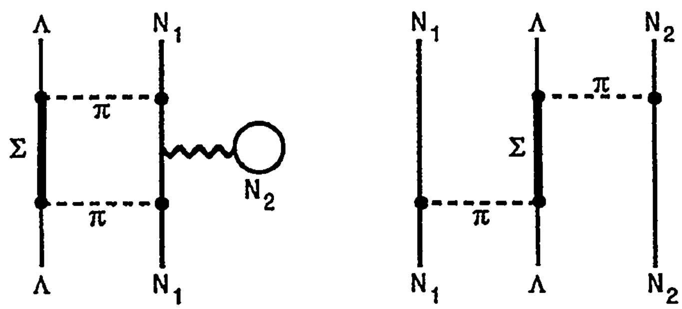

Figure 2 Diagrams for dispersive and TPE ΛNN potentials.

Thus, finally our Λ-nuclear interactions are

$$\sum_{i=1}^{A-1} V_{\Lambda N_i} + \sum_{i<j=1}^{A-1} \left(V^{D}_{\Lambda N_j N_j} + V^{2\pi}_{\Lambda N_i N_j}\right), \tag{7}$$

where the strengths of $V_{\Lambda NN}$ are chosen as described below.

3. FITS TO THE s-SHELL HN, ESPECIALLY ${}^{5}_{\Lambda}$He

3.1 The s-Shell HN

Previously we have made VMC calculations of the s-shell HN: ${}^{3}_{\Lambda}$H, ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He (both J = 0 and 1), and ${}^{5}_{\Lambda}$He. We used a Mafliet-Tjon central NN potential [11] which fits both the energies and rms radii of the core nuclei: ^{2}H, ^{3}H and ^{4}He. The calculations included ΛN, NN and ΛNN correlations: $f_{\Lambda N}$, f_{NN}, $f_{\Lambda NN} = f^{D}_{\Lambda NN} f^{2\pi}_{\Lambda NN}$. The most pertinent result for our present work is that $V^{2\pi}_{\Lambda NN}$ alone cannot provide the repulsion needed for ${}^{5}_{\Lambda}$He to compensate the overbinding obtained with only a ΛN potential, and that consequently a strongly repulsive ΛNN dispersive potential $V^{D}_{\Lambda NN}$ is required. This is because the ΛNN correlations $f^{2\pi}_{\Lambda NN}$ reduce the contribution of $V^{2\pi}_{\Lambda NN}$ from an appreciably repulsive one to one which is only slightly repulsive or even attractive, whereas the effect of correlations on the repulsive contribution of $V^{D}_{\Lambda NN}$ is much less.

3.2 $\Lambda N + \Lambda NN$ Potentials Fitted to ${}^{5}_{\Lambda}He$

For our calculations of the s.p. energies we consider three families of $\Lambda N + \Lambda NN$ potentials of the type discussed above. These are - mostly - constrained as follows:

1. The experimental value of $B_\Lambda\left({}^{5}_{\Lambda}He\right) \approx 3.1$ MeV is reproduced for an exchange parameter $\varepsilon = 0.3$. This value of ε is consistent with the s.p. data and also the Λp scattering and gives an exchange contribution ≈ 0.5 MeV for ${}^{5}_{\Lambda}He$.

2. The s-wave Λp scattering is fitted (i.e. $V_0 = 6.15 \pm 0.5$ MeV).

Our interactions are then:

I. A ΛN potential $V_{\Lambda N}$ only, with $V_0 = 5.98$ MeV. This gives the experimental $B_\Lambda\left({}^{5}_{\Lambda}He\right)$ for $\varepsilon = 0$, but gives too little scattering.

II. ΛN plus ΛNN dispersive potentials: $V_{\Lambda N} + V^{D}_{\Lambda NN}$. The strength V_0 of $V_{\Lambda N}$ covers the values allowed by ΛN scattering (6.1, 6.15, 6.2 MeV) and the strength W of $V^{D}_{\Lambda NN}$ is adjusted accordingly to give the experimental $B_\Lambda\left({}^{5}_{\Lambda}He\right)$, with more repulsion needed for more attractive $V_{\Lambda N}$ (W = 0.007, 0.011, 0.016 MeV).

III. ΛN plus dispersive plus TPE ΛNN forces: $V_{\Lambda N} + V^{D}_{\Lambda NN} + V^{2\pi}_{\Lambda NN}$ with fixed $V^{2\pi}_{\Lambda NN}$ (C_p = 2 MeV). Again V_0 (6.1, 6.16, 6.2 MeV) covers the values allowed by scattering and W is adjusted accordingly (W = 0.006, 0.01, 0.013 MeV).

4. CALCULATIONS OF THE s.p. ENERGIES

The s.p. energies B_Λ are obtained from a Schrödinger equation with a Λ-nucleus potential U_Λ and an effective mass m^*_Λ which are obtained in the local density approximation using the FHNC method [2,7]. This is used to calculate the Λ binding $D(\rho,k_\Lambda)$ for nuclear matter of density ρ and for a Λ momentum of k_Λ. Thus for $A \to \infty$:

$$-D(\rho,k_\Lambda) = \frac{\left(\Psi^{(A)}\middle|H^{(A)}\middle|\Psi^{(A)}\right)}{\left(\Psi^{(A)},\Psi^{(A)}\right)} - \frac{\left(\Psi^{(A-1)}\middle|H_N^{(A-1)}\middle|\Psi^{(A-1)}\right)}{\left(\Psi^{(A-1)},\Psi^{(A-1)}\right)}, \tag{8}$$

where $H^{(A)}$, $\Psi^{(A)}$ are the Hamiltonian and wave function of the HN and $H_N^{(A-1)}$, $\Psi^{(A-1)}$ those of the core nucleus. The (variational) FHNC wave functions are

$$\Psi^{(A)} = e^{i\vec{k}\cdot\vec{r}_\Lambda} F \Psi^{(A-1)} \text{ with } F = \left[\prod_{i=1}^{A-1} f_{\Lambda N}(r_{i\Lambda}) \prod_{i<j}^{A-1} f_{\Lambda NN}\left(\vec{r}_{i\Lambda},\vec{r}_{j\Lambda},\vec{r}_{ij}\right)\right] \tag{9}$$

and

$$\Psi^{(A-1)} = \prod_{i<1}^{A-1} f_{NN}\left(r_{ij}\right)\Phi^{(A-1)}(1,2,\cdots A-1). \tag{10}$$

$\Phi^{(A-1)}$ is the uncorrelated Fermi gas wave function for nuclear matter of density ρ. The factor F includes both ΛN and ΛNN correlations. Details of the correlation factors $f_{\Lambda N}$, f_{NN} and $f_{\Lambda NN}$ as well as of the calculational method are given in Ref. [2]. The effective mass $m^*_\Lambda(\rho)$ is obtained from a quadratic fit in k_Λ to $D(\rho,k_\Lambda)-D(\rho,k_\Lambda=0)$.

We also allow for a "fringing field" (FF) due to the finite range of the ΛN and ΛNN potentials, but do not discuss the details here, in particular since our procedure is approximate and subject to uncertainties; also the effects of a FF are relatively small and vanish for $A \to \infty$. Without a FF one has

$$U_\Lambda(r) = D\left(\rho^{(A-1)}(r), k_\Lambda = 0\right), \tag{11}$$

where the densities $\rho^{(A-1)}$ of the core nuclei are obtained from electron-scattering data. Similarly, the effective mass as a function of r is given by

$$m^*_\Lambda(r) = m^*_\Lambda\left(\rho^{A-1}(r)\right). \tag{12}$$

Finally, B_Λ is obtained as the lowest eigenvalue of the appropriate radial Schrödinger equation for an orbital angular momentum ℓ_Λ,

$$\left[\frac{-\hbar^2}{2m^*_\Lambda(r)}\frac{d^2}{dr^2} + \frac{\hbar^2\ell_\Lambda(\ell_\Lambda+1)}{r^2} + U_\Lambda(r)\right]\phi_{\ell_\Lambda} = -B_\Lambda\phi_{\ell_\Lambda}. \tag{13}$$

5. THE Λ BINDING AND EFFECTIVE MASS IN NUCLEAR MATTER

We summarize the expressions for $D(\rho)$ and $m^*_\Lambda(\rho)$ obtained with the FHNC method. We define (always for a given density ρ of nuclear matter)

$$D \equiv D(k_\Lambda = 0) = D^{\Lambda N} + D^{\Lambda NN}, \tag{14}$$

where $D^{\Lambda N}$ and $D^{\Lambda NN}$ are the ΛN and ΛNN contributions respectively. Furthermore

$$D^{\Lambda N} = D_o^{\Lambda N} + D_x^{\Lambda N}, \tag{15}$$

where the direct contribution is

$$D_o^{\Lambda N} \equiv D^{\Lambda N}(\varepsilon = 0) = \rho \int \left[g_{\Lambda N}(r)V(r) - \frac{\hbar^2}{4\mu_{\Lambda N}}\nabla^2_\Lambda \ell_n f_2(r_n)\right](d^3x) \equiv -t_o\rho. \tag{16}$$

$g_{\Lambda N}$ is the ΛN correlation function, f_2 is the ΛN correlation factor, and $\mu_{\Lambda N}$ is the ΛN reduced mass. Typically $t_o(\rho=0) \approx 400$ MeV fm^3; and $t_o(\rho)$ decreases somewhat with ρ. The exchange contribution is proportional to $\varepsilon k_F^2 \rho F_1$ where k_F is the Fermi momentum and F_1 is a form factor. Thus

$$D_x^{\Lambda N} = \varepsilon\Delta\,, \quad \Delta = -\frac{3}{10}\left(\frac{3\pi^2}{2}\right)^{2/3} b_o \rho^{5/3} F_1\,, \tag{17}$$

where

$$b_o = \frac{1}{3}\int g_{\Lambda N}\, V_{\Lambda N} r^2 d^3x\,. \tag{18}$$

The effective mass m_Λ^* is given by

$$\chi \equiv \frac{m_\Lambda}{m_\Lambda^*} - 1 = \frac{-m_\Lambda}{\hbar^2}\varepsilon b_o \rho F_2\,. \tag{19}$$

$F_1(\rho)$, $F_2(\rho)$ are form factors which represent finite range effects of $V_{\Lambda N}$ (relative to k_F^{-1}); F_1, $F_2 = 1$ for $\rho = 0$, or equivalently for a zero-range $V_{\Lambda N}$. Finite range effects are much more important for χ, i.e. m_Λ^*, than for $D_x^{\Lambda N}$. Note that exchange contributes both through $D_x^{\Lambda N}$ and therefore through the Λ-nucleus potential U_Λ as well as through m_Λ^*. For $\varepsilon > 0$, $D_x^{\Lambda N}$ is repulsive (odd state potential less attractive than even state) and $m_\Lambda^* < m_\Lambda$ thus giving a larger kinetic energy relative to that for m_Λ and therefore also an effective repulsion.

The ΛNN contribution to the Λ binding is

$$D^{\Lambda NN} = t_3 \rho^2 F_{\Lambda NN}\,, \tag{20}$$

where $F_{\Lambda NN}(\rho)$ is a form factor such that $F_{\Lambda NN} = 1$ for $\rho = 0$ and equivalently for a zero-range ΛNN potential. $F_{\Lambda NN}$ depends on various correlation functions [2] and varies by roughly a factor of two or less, depending on the specific interaction, over the density range considered ($\rho \lesssim 0.23$ fm^{-3}). The dominant ρ dependence therefore comes from the ρ^2 factor.

We define

$$D_o \equiv D(\varepsilon = 0) = D_o^{\Lambda N} + D^{\Lambda NN}\,, \tag{21}$$

which is the sum of the direct ΛN and of the ΛNN contributions, i.e. the Λ binding without the exchange contribution. The total Λ binding at ρ is then

$$D = D_o + D_x^{\Lambda N}\,. \tag{22}$$

6. FITS TO THE s.p. ENERGIES

We attempt fits to the s.p. B_Λ with our three families of interactions I-III. The exchange parameter ε is the only parameter varied for a given interaction. The well depth is given by $D_\Lambda = D(\rho_o)$ where $\rho_o = 0.165$ fm^{-3} is the density of normal nuclear matter. We recall that $D_\Lambda = B_\Lambda(A = \infty)$ for all ℓ_Λ.

Before we discuss details of our fits we emphasize the general requirements on $D(\rho)$ and m^*_Λ needed for a fit to the s.p. data. These requirements have been pointed out by Millener *et al.* Ref. [12], and will also be demonstrated in the following discussion. Thus, for a satisfactory fit to the s_Λ $(\ell_\Lambda = 0)$ data, $D(\rho)$ must have the following "saturation" properties: $D_\Lambda = D(\rho_o) \approx 30$ MeV in order to allow a satisfactory fit for large A. On the other hand for a fit for small A, $D(\rho)/\rho$ must be larger for small $\rho < \rho_o$, implying a maximum in $D(\rho)$ with ρ_{max} not very different from ρ_o. Further, to give the separation between the B_Λ for different ℓ_Λ requires quite generally that $m^*_\Lambda \approx 0.7\, m_\Lambda$ which in turn in our approach requires $\varepsilon \approx 0.3$-0.35.

<u>Fits with interaction I</u>: ΛN potential $V_{\Lambda N}$ only. This fits $B_\Lambda\left({}^5_\Lambda He\right)$ for $\varepsilon = 0$ but gives too little scattering.

1. With only a direct ΛN potential ($\varepsilon = 0$ and thus $m^*_\Lambda = m_\Lambda$): $D^{\Lambda N} = D_0^{\Lambda N} \approx 370\,\rho$ MeV, with the nonlinear dependence on ρ only a few % for $\rho \lesssim 0.23$ fm^{-3} as a consequence of the slight dependence of $g_{\Lambda N}$ and f_2 (Eq. (16)) on ρ. Thus $D_\Lambda = D_0^{\Lambda N}(\rho_o) \approx 60$ MeV. All the s.p. states and in particular the s_Λ states are then much too strongly bound, even for quite low A. If V_o were adjusted (without any justification) to give $D_\Lambda \approx 30$ MeV so as to fit the B_Λ for the heaviest HN, then conversely the B_Λ for even medium heavy HN would be much too small (and the ΛN scattering would be very much too small). Thus a direct ΛN potential cannot fit the s.p. energies.

2. With ΛN space exchange: $D^{\Lambda N} = D_0^{\Lambda N} + D_x^{\Lambda N}$. To obtain $D_{\Lambda N}(\rho_o) \approx 30$ MeV requires a large and repulsive exchange contribution $D_0^{\Lambda N} \approx -30$ MeV which is obtained for $\varepsilon \approx 0.88$. This implies a correspondingly small value of $m^*_\Lambda / m_\Lambda \approx 0.48$ at ρ_o. The Λ binding $D(\rho)$, shown in Fig. 3, then has a maximum ≈ 35 MeV at $\rho_{max} \approx 0.215$ fm^{-3}. The results for the s_Λ states are then reasonable for large A as expected, but the large Λ kinetic energy (small m^*_Λ) gives too small B_Λ for smaller A and also much too small B_Λ for the $\ell_\Lambda > 0$ states. In fact, no even tolerably adequate fit to the s.p. data can be obtained with a ΛN potential with space exchange.

If purely phenomenologically, we use $m^*_\Lambda = m_\Lambda$ together with $D_x^{\Lambda N}$ for an appropriately chosen ε, i.e. we relinquish the common origin of $D_x^{\Lambda N}$ and m^*_Λ in exchange forces, then a quite adequate fit to the s_Λ states can be obtained for $\varepsilon \approx 0.98$. However for $\ell_\Lambda > 0$ although the fits are now much better, the calculated B_Λ are somewhat too large (too small T_Λ) and the fit is of only moderate quality. There is of course no justification for taking $m^*_\Lambda = m_\Lambda$, especially since the effects of exchange on both $D_x^{\Lambda N}$ and m^*_Λ depend on quite basic many-body features and are already fully manifest in HF. Thus a central ΛN potential with and without exchange is ruled out by the s.p. data.

<u>Fits with interactions II</u>: ΛN plus dispersive ΛNN forces. We recall that for a given $V_{\Lambda N}$ the strength W of $V^D_{\Lambda NN}$ is chosen to fit $B_\Lambda\left({}^5_\Lambda He\right)$ for $\varepsilon = 0.3$, and that for more attractive $V_{\Lambda N}$ the value of W is larger since more repulsion is then required. We find that no adequate fit to the s.p. energies can be obtained for these interactions, the "fit" being worst for the smallest $V_o = 6.1$ MeV consistent with scattering, corresponding to the least

repulsive $V^D_{\Lambda NN}$. This inability to fit the s.p. energies is directly related to an insufficiently repulsive contribution $D^{\Lambda NN}(\rho_o)$ from $V^D_{\Lambda NN}$. Thus for $V_o = 6.2$ MeV, which gives the most attractive $V_{\Lambda N}$ consistent with scattering and hence to the most repulsive $V_{\Lambda NN}$, we obtain $D_{\Lambda NN}(\rho_o) \approx -26$ MeV. This together with the direct contribution $D^{\Lambda N}_o \approx 74$ MeV gives $D_o(\rho_o) \approx 48$ MeV. To obtain $D(\rho_o) \approx 30$ MeV, needed to fit B_Λ for large A, then requires a rather large exchange contribution $D^{\Lambda N}_x \approx -18$ MeV which in turn requires $\varepsilon \approx 0.44$. This implies a rather small $m^*_\Lambda(\rho_o) \approx 0.66\, m_\Lambda$ which although it allows an adequate fit to the s_Λ states gives a mediocre fit for $\ell_\Lambda > 0$ as depicted in Fig. 1. The situation is much worse for smaller V_o (< 6.2 MeV) for which $V^D_{\Lambda NN}$ is correspondingly less repulsive.

If we ignore the repulsive exchange contribution for ${}^5_\Lambda$He (i.e. if $\varepsilon = 0$) then a more repulsive $V^D_{\Lambda NN}$ is needed to fit $B_\Lambda\left({}^5_\Lambda\mathrm{He}\right)$. Now $V_o = 6.2$ MeV gives a $V^D_{\Lambda NN}$ ($W = 0.02$ MeV) just sufficiently repulsive for a satisfactory fit to the s.p. data. However, for smaller Vo a satisfactory fit is still not possible. The repulsive exchange contribution ≈ 0.5 MeV in ${}^5_\Lambda$He is thus necessary if purely dispersive ΛNN forces are to be completely ruled out. Our conclusions for a dispersive ΛNN force seem quite firm since the appropriate correlations have a relatively small effect in reducing the repulsive contribution of $V^D_{\Lambda NN}$ in ${}^5_\Lambda$He.

Table 2. The Λ binding $D(\rho_o)$ and its components (in MeV), and $m^*_\Lambda = m_\Lambda$, all at $\rho_o = 0.165$ fm^{-3} for two interactions III which fit the s.p. data for the stated values of ε.

Interaction							
V_o	W	ε	$D^{\Lambda N}_o$	$-D_{\Lambda NN}$	$-D^{\Lambda N}_x$	D	m^*_Λ / m_Λ
6.16	0.01	0.34±.015	70.5	30.5	12.3±.6	27.7±.6	0.71±.015
6.20	0.013	0.31±.015	71.9	34.5	11.3±.6	26.1±.6	0.73±.015

Fits with interactions III: ΛN plus dispersive plus TPE ΛNN forces, with $V^{2\pi}_{\Lambda NN}$ fixed. As for II, $V^D_{\Lambda NN}$ is again adjusted for any given $V_{\Lambda N}$ to give $B_\Lambda\left({}^5_\Lambda\mathrm{He}\right)$. For these interactions we obtain excellent fits to all the s.p. data for $V_o \gtrsim 6.15$ MeV ($W \gtrsim 0.1$ MeV) as shown in Fig. 1 for the 1st interaction in Table 2. This shows results for our best fits for two such interactions. All the Λ binding energies are for ρ_o. Figure 3 shows $D(\rho)$ and its components vs. ρ for the first interaction. $D(\rho)$ has the characteristic saturation features needed for a fit to the s.p. data: $D_\Lambda = D(\rho_o) \approx 27$ MeV required to fit B_Λ for large A, and a maximum ≈ 28 MeV at $\rho_{max} \approx 0.14$ fm^{-3}. Since ρ_{max} is quite close to ρ_o the uncertainties in the predicted D_Λ due to uncertainties in ρ_o are quite small (≈ 0.5 MeV) for a given interaction.

A combination $V^D_{\Lambda NN} + V^{2\pi}_{\Lambda NN}$ permits a fit to the s.p. data because the ΛNN correlations $f^{2\pi}_{\Lambda NN}$ in ${}^5_\Lambda$He strongly reduce the repulsion due to $V^{2\pi}_{\Lambda NN}$ and can even give attraction, whereas this is not so for nuclear matter, i.e. for D. Thus a sizable $V^{2\pi}_{\Lambda NN}$ which gives a small repulsive or even an attractive contribution in ${}^5_\Lambda$He can give a large repulsive contribution in nuclear matter. This together with the repulsion from $V^D_{\Lambda NN}$ (which is required for ${}^5_\Lambda$He and for which there is no such dramatic change between $A = 5$ to $A = \infty$) provides sufficient overall repulsion $D_{\Lambda NN} \approx -30$ MeV needed for the s.p. data. More generally it seems clear that what is required for our family of interactions is that the effect of correlations for $V^D_{\Lambda NN}$ does not change too much with A, whereas for $V^{2\pi}_{\Lambda NN}$ the effect of $f^{2\pi}_{\Lambda NN}$ should depend quite strongly on A in such a way as to give relatively much more attraction for small A.

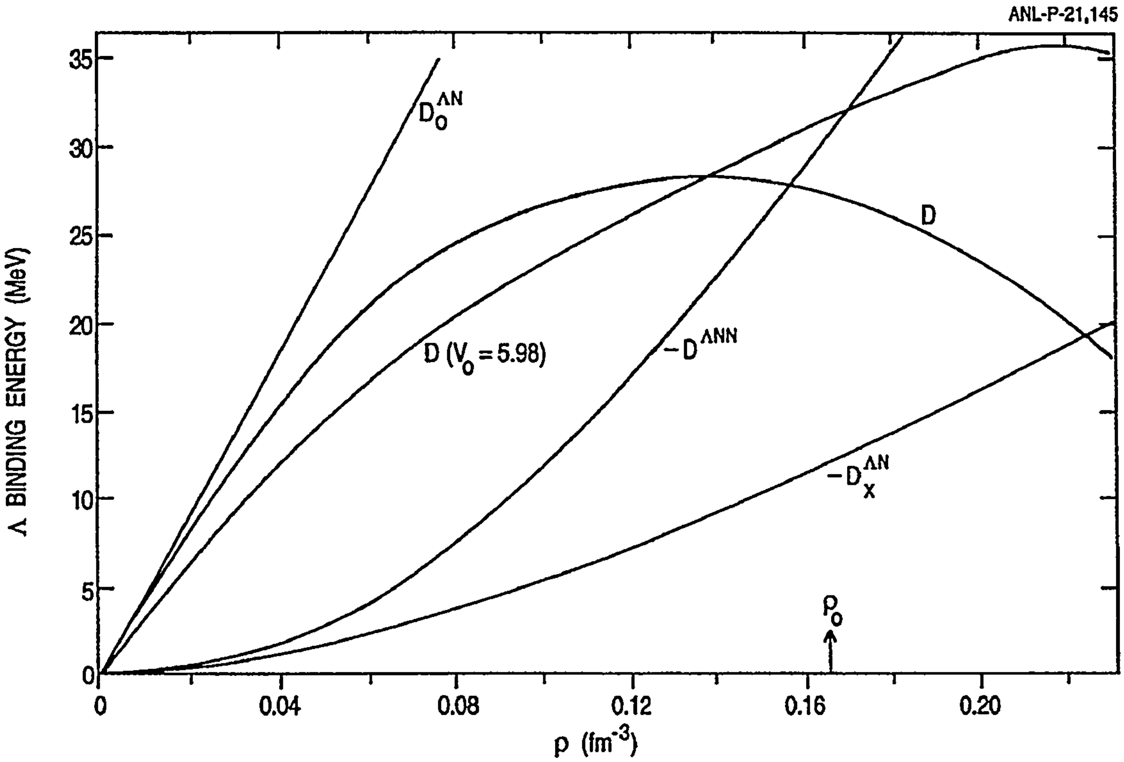

Figure 3 The Λ binding D(ρ) and its components vs. ρ for the first interaction of Table 2. Also shown is D(ρ) for interaction I (only $V_{\Lambda N}$) with $\varepsilon = 0.88$.

7. CONCLUSIONS - WHAT HAVE WE LEARNED?

1. The s.p. data require the Λ binding in nuclear matter D(ρ) to have the "saturation" features shown in Fig. 3 and which have been discussed by Millener et al. [12] and also obtained by us with our microscopic FHNC approach. With our interactions we obtain $D_\Lambda \approx 27 \pm 1$ MeV, consistent with earlier results, and required to fit B_Λ for large A. The "saturation" of D(ρ) requires a large repulsive many-body contribution, depending nonlinearly on ρ, which is identified with ΛNN forces in our approach. To account for the s.p. data for all ℓ_Λ further requires $m^*_\Lambda / m_\Lambda \approx 0.7$ (at ρ_0) which implies an exchange parameter $\varepsilon \approx 0.32 \pm .02$. This value is expected to depend only slightly (through the form factor F_2) on the details of $V_{\Lambda N}$ if this is of reasonable range and shape. The value of ε is consistent with that obtained from scattering but is much more precisely determined by the s.p. data, and implies a p-state potential (Eq. (4)): $V_p = (0.35 \pm .05)V_s$. These results depend only on the s.p. data and not on the s-shell B_Λ and the Λp scattering.

2. Λp scattering fixes the spin-average $V_{\Lambda N}$ which then determines the direct contribution to D_Λ to be $D_0^{\Lambda N}(\rho_0) \approx 70$ MeV. Many-body effects (nonlinear with ρ) are small. Even with a ΛN tensor potential such effects are quite small because of the short range and weakness of $V^T_{\Lambda N}$ [4], and probably reduce $D_0^{\Lambda N}$ by not more than a few MeV [13]. The large value of $D_0^{\Lambda N}(\rho_0)$ and the approximate proportionality of $D_0^{\Lambda N}$ with ρ

imply that a direct ΛN potential cannot fit the B_Λ data - even when a ΛN tensor force is included. With a space-exchange component, a very large $\varepsilon \approx 1$ is required to fit the s_Λ states for large A, and no reasonable fit to all the s.p. data is possible. Thus a ΛN potential with or without exchange seems excluded.

Combining the values at ρ_0 of $D_0^{\Lambda N} \approx 70$ MeV, obtained by making use of scattering, with $D \approx 27$ MeV, $D_x^{\Lambda N} \approx -12$ MeV obtained from the s.p. data gives $D^{\Lambda NN} = D - D_x^{\Lambda N} - D_0^{\Lambda N} \approx -32 \pm 2$ MeV. This large value is consistent with that required by just the s.p. data only and seems fairly well determined. Since many-body effects associated with $V_{\Lambda N}$, in particular due to a tensor force, are expected to be small, $D^{\Lambda NN}$ must be identified almost entirely with other components of the interaction such as TPE ΛNN forces and suppression of the ΛN - ΣN coupling. It is a challenge to many-body theory to account for $D^{\Lambda NN}$.

3. The results just discussed do not depend on the s-shell HN, in particular not on $B_\Lambda\left({}^5_\Lambda\mathrm{He}\right)$. If we use this to fix the strength of $V_{\Lambda NN}$ then, with an exchange contribution ≈ 0.5 MeV for ${}^5_\Lambda$He, the s.p. data excludes a purely dispersive ΛNN potential since this is insufficiently repulsive to give $D^{\Lambda NN} \approx -30$ MeV. However, a combination of dispersive and TPE forces can give an excellent fit, since the effects of correlations makes the contribution of $V^{2\pi}_{\Lambda NN}$ much less repulsive for ${}^5_\Lambda$He than for nuclear matter.

There are of course many outstanding questions although we believe that the results discussed above in 1 and 2 as well as the elimination of purely dispersive ΛNN forces discussed in 3 are reasonably secure. However, the detailed nature of the ΛNN forces, in particular the relative strengths of $V^{D}_{\Lambda NN}$ and $V^{2\pi}_{\Lambda NN}$ is largely uncertain. Further elucidation requires, in particular, complete microscopic calculations of the s-shell and heavier HN which include 3-body tensor correlations for $V^{2\pi}_{\Lambda NN}$. Such calculations are in progress in particular for ${}^5_\Lambda$He and ${}^{17}_\Lambda$O [Ref. 14] and will greatly help to clarify the character of the ΛNN forces. For heavier hypernuclei completely microscopic calculations are not yet feasible. However there are possible improvements in the local density approximation as used by us. Amongst these is the question of core distortion by the Λ. This is addressed by Professor Usmani in his talk where the will show that - for spherical nuclei - this distortion is very small even for quite light HN because the "forcing" action of the Λ on the core is much reduced because of the saturation features of $D(\rho)$. Other questions relate to an adequate inclusion of the (nuclear matter) rearrangement energy and to the adequacy of the local density approximation, in particular to a better treatment of the fringing field.

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