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A Geometric Weighted Elliptic Grid ReGeneration Method For 3D Unstructured ALE Hydrodynamics

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A Geometric Weighted Elliptic Grid ReGeneration Method for 3D Unstructured ALE Hydrodynamics

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The theory for an elliptic mesh generator is developed for use in the advection step of a 3D ALE algorithm. This mesh generator is derived by a variational principle for an unstructured 3D grid using finite elements. An arbitrary weight function is introduced which is based on the geometric properties of the existing mesh. These geometric weights should allow for a smooth grid that retains the general shape of the original mesh.

1. INTRODUCTION

In hopes of combining the best attributes of running Lagrangian and Eularian, ALE calculations have been developed over the past 15 years. An ALE (arbitrary Lagrangian-Eulerian) formulation of hydrodynamics consists of two steps, first the complete Lagrangian calculation and then a modified Eulerian, or advection step. When going from the Lagrangian step to the advection step, the mesh is regenerated, or relaxed, to alleviate any tangled or irregular zones created by the Lagrangian calculation.This new mesh needs to be smooth, with no bow-ties or distorted elements, and no drastic changes in zone size.

For the 2D arena, grid generation has been studied extensively for structured, finite difference meshes. The more popular techniques include the Winslow-Crowley[1] method and the Thompson-Thames-Mastin[2] method, both based on Laplace's equation. More recent work includes using the variational principle[3,4] to derive grid generation algorithms. The variational approach uses the concept of a reference grid which defines the desired properties of the grid on the physical space. The new mesh is then generated by minimizing the functional integrals. Some theoretical work[5] has been done in using the equipotential grid generation algorithm for 3D structured and unstructured meshes. The more robust algorithms result in elliptic grid generators which have several unique properties such as a high degree of differentiability of interior nodes and an independence of interior nodes to boundary nodes.

In this paper, we study the problem of generating a mesh at each time step for 3D unstructured ALE hydrodynamics. We want our algorithm to generate a smooth mesh, that retains the general shape of the original grid, i.e. coarsely zoned regions remain coarse, finely zoned regions remain fine. Our approach is to formulate variational integrals, apply a finite element technique and solve the resulting equation iteratively. Since geometric control is desired, a weighting term will be added to the initial integral.

2. DESCRIPTION OF THE METHOD

Our goal is to derive an elliptic mesh generator with the ability to adapt itself to some geometric feature of the existing mesh. Since we will be dealing with an unstructured grid, it is convenient to derive this equation from variational principles. We wish to minimize the integral

$$I = \frac{1}{2}\int W\,|\nabla f|^2\,dx^3. \qquad (1)$$

where $f = f(x, y, z)$ and $W = W(x, y, z)$, the geometric weighting term. Applying the variational principle gives

$$-\nabla\bullet\ W\nabla f = 0, \qquad (2)$$

which is a "weighted" Laplace's equation. The geometric weighting term will give us control over the placing of interior lines.

3. NUMERICAL SOLUTION OF THE METHOD

An unstructured grid has no defined mapping between a generalized index coordinate (i, j, k) and the physical space (x, y, z), so the traditional finite difference formalism makes no sense for the problem we are trying to solve. The ALE algorithm, however, uses a finite element approach to describe its unstructured grids. It is reasonable, then, to apply this technique to our functional (1) derived from variational principles.

We begin by approximating our function $f(x, y, z)$ as the expansion of some basis function

$$f(x, y, z) = \sum_{\alpha} f_{\alpha} N_{a}(x, y, z) \qquad (3)$$

where f_{α} is the value of f at grid node α and $N_{\alpha}(x, y, z)$ is the basis function associated with grid node α and

$$N_{\alpha} = \begin{Bmatrix} 1 & \text{at } \alpha \\ 0 & \text{otherwise} \end{Bmatrix}. \qquad (4)$$

Substituting (3) into (1) yields

$$I = \frac{1}{2}\int W\left[\nabla\left(\sum_{\alpha} f_{\alpha} N_{\alpha}\right)\right]\left[\nabla\left(\sum_{\beta} f_{\beta} N_{\beta}\right)\right] dx^3$$

$$= \frac{1}{2}\sum_{\alpha,\beta} f_{\alpha} f_{\beta}\int W\, \nabla N_{\alpha} \bullet \nabla N_{\beta}\, dx^3$$

$$= \frac{1}{2}\sum_{\alpha,} M_{\alpha\beta} f_{\alpha} f_{\beta} \qquad (5)$$

where

$$M_{\alpha\beta} = \int W\, \nabla N_{\alpha} \bullet \nabla N_{\beta}\, dx^3. \qquad (6)$$

To minimize the functional I, we require that

$$\frac{\partial I}{\partial f_{\alpha}} = \frac{1}{2}\sum_{\beta} M_{\alpha\beta} f_{\beta} = 0 = M_{\alpha\alpha} f_{\alpha} + \sum_{\beta \neq \alpha} M_{\alpha\beta} f_{\beta} \qquad (7)$$

or

$$f_{\alpha} = -\frac{1}{M_{\alpha\alpha}} \sum_{\beta \neq \alpha} M_{\alpha\beta} f_{\beta}. \qquad (8)$$

The matrix equation given by (8) is then solved iteratively.

A coordinate transformation from physical space (x, y, z) to logical space (ξ, η, ζ) is then applied to (8), yielding

$$M_{\alpha\beta} = \int w(\xi) \sum_{j,k} \tilde{g}_{jk} \frac{\partial N_{\alpha} \partial N_{\beta}}{\partial \xi_j\, \partial \xi_k} J\, d\xi^3 \qquad (9)$$

where $w(\xi) = \frac{W(\xi)}{J(\xi)}$, $\tilde{g}_{jk} = \sum_{i} \frac{\partial \xi_j \partial \xi_k}{\partial x_i \partial x_i}$, and J is the determinant of the Jacobian. The basis function is defined as $N_{\alpha}(\xi, \eta, \zeta) = H(\xi)\, G(\eta)\, F(\zeta)$, with element α defined as in Fig. 1.

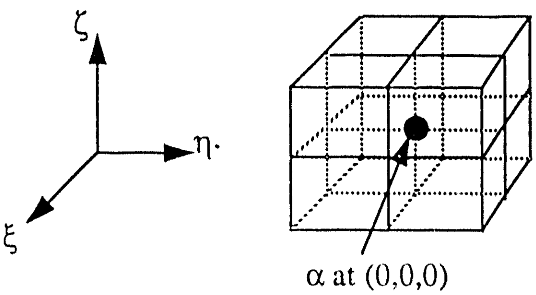

Fig. 1

4. DETERMINATION OF THE GEOMETRIC WEIGHT

The geometric weight, W, is defined in such a manner as to control the amount of smearing or spreading of grid surfaces. This will be done by analyzing the actual grid spacing between nodes in the x-

, y- and z-directions. For

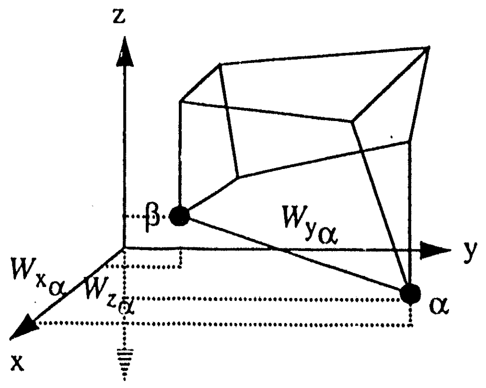

Fig. 2

example, when analyzing node α in Fig. 2, the geometric weighting term between α and β is

$$W_{x_\alpha} = \frac{1}{|x(\alpha) - x(\beta)|} \tag{10}$$

$$W_{y_\alpha} = \frac{1}{|y(\alpha) - y(\beta)|} \tag{11}$$

$$W_{z_\alpha} = \frac{1}{|z(\alpha) - z(\beta)|} \quad . \tag{12}$$

Nodes that are close to each other will be more heavily weighted, causing some movement to spread the zones apart, whereas zones that are far from each other have small weights, causing very little motion.

5. PRELIMINARY 2D STRUCTURED MESH FINDINGS

To determine the feasibility of using a geometric weight for controlling the movement of grid lines, we solved the elliptic equation (2) on a 2D structured mesh. Discretizing the equation was done using the finite volume method where Green's Identity yields the boundary integral

$$\int_{\Gamma_{ij}} \left[\lambda_{1_{ij}} \frac{\partial \xi}{\partial x} \cos(\mathbf{n}, x) + \lambda_{1_{ij}} \frac{\partial \xi}{\partial y} \cos(\mathbf{n}, y)\right] d\Gamma_{ij}, \tag{13}$$

$$\int_{\Gamma_{ij}} \left[\lambda_{2_{ij}} \frac{\partial \eta}{\partial x} \cos(\mathbf{n}, x) + \lambda_{2_{ij}} \frac{\partial \eta}{\partial y} \cos(\mathbf{n}, y)\right] d\Gamma_{ij} \tag{14}$$

with

$$\lambda_{1_{ij}} = \frac{1}{|x_i - x_{i-1}|} \text{ and } \lambda_{2_{ij}} = \frac{1}{|y_j - y_{j-1}|} . \tag{15}$$

This integral was approximated by the midpoint rule and the resulting symmetric, positive definite matrix was solved with the preconditioned conjugate gradient method. The preliminary findings were good, as seen in the following figures:

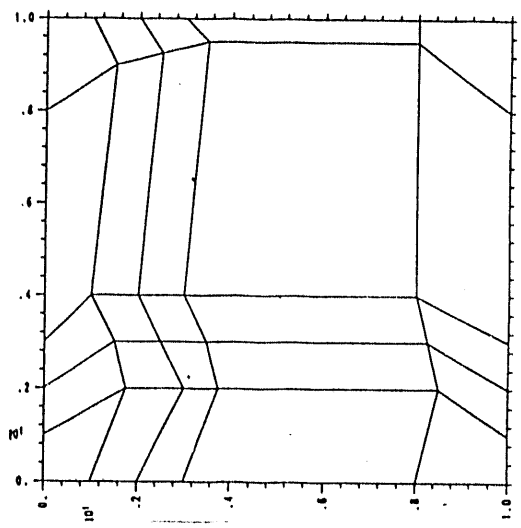

Fig. 3 Original Physical Mesh

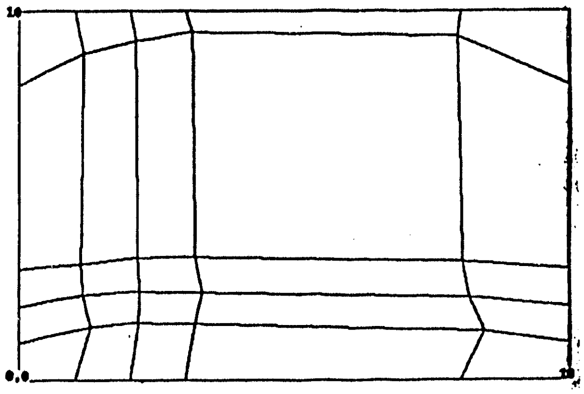

Fig. 4 Geometric "Weighted" Mesh Regenerator

6. CONCLUSIONS

Elliptic mesh generators have proven fairly successful when used on 2D structured finite difference grids. Limited use of a variational algorithm with zonal weights, such as region or pressure weighting, for 2D unstructured problems has been used with good results, and this algorithm is currently being applied to a 3D unstructured mesh. Our 3D unstructured algorithm, derived from the variational principle, which uses geometric weights as apposed to zonal weights for controlling the mesh regeneration, should allow for a smooth grid, that retains the general shape of the original mesh.

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