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Missile Sizing for Ascent-Phase Intercept*

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Abstract

A computer code has been developed to determine the size of a ground-launched, multistage missile which can intercept a theater ballistic missile before it leaves the atmosphere. Typical final conditions for the inteceptor are 450 km range, 60 km altitude, and 80 sec flight time. Given the payload mass (35 kg), which includes a kinetic kill vehicle, and achievable values for the stage mass fractions (0.85), the stage specific impulses (290 sec), and the vehicle density (60 lb/ft³), the launch mass is minimized with respect to the stage payload mass ratios, the stage burn times, and the missile angle of attack history subject to limits on the angle of attack (10 deg), the dynamic pressure (60,000 psf), and the maneuver load (200,000 psf deg). For a conical body, the minimum launch mass is approximately 1900 kg. The missile has three stages, and the payload coasts for 57 sec. A trade study has been performed by varying the flight time, the range, and the dynamic pressure limits.

With the results of a sizing study for a 70 lb payload and $q_{max} = 35,000$ psf, a more detailed design has been carried out to determine heat shield mass, tabular aerodynamics, and altitude

dependent thrust. The resulting missile has approximately 100 km less range than the sizing program predicted primarily because of the additional mass required for heat protection. On the other hand, launching the same missile from an aircraft increases its range by approximately 100 km.

Sizing the interceptor for air launch with the same final conditions as the ground-launched missile reduces its launch mass to approximately 1000 kg.

1. Introduction

A serious threat in today's world comes from short-range ballistic missiles. It is desirable to contemplate the design of an intercept missile which can destroy the ballistic missile as early as possible in the ascent phase. The primary question to be answered is how big does such an interceptor have to be.

The approach followed here is similar to that of Ref. 1. After a missile shape is selected and achievable values are prescribed for the stage mass fractions, the stage specific impulses, and the vehicle mass density, the launch mass is minimized with respect to the stage payload mass ratios, the stage burn times, and the angle of attack history. Also, inequality constraints are imposed on the angle of attack, the dynamic pressure, and the maneuver load. Heat shield mass is determined a posteriori, and its effect is included by reducing the standoff range. While ground-launched missiles are the primary concern of this paper, air-launched missiles are discussed briefly.

Conceptual design considerations are discussed in Section 2. In Section 3, a physical model

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for the interceptor is presented. After the performance index and constraints are discussed in Section 4, the optimization problem is stated in Section 5. Section 6 contains numerical results, while discussion and conclusions appear in Section 7.

British engineering units are used throughout the paper and the sizing code. However, because of the intended use of this interceptor, the specifications and the results are in metric units, the only exception being the dynamic pressure.

2. Conceptual Design

It is desired to size a missile which can intercept a short-range (400-600 km) ballistic missile in its ascent phase before it reaches exo-atmospheric altitudes. For simplicity, the interceptor and the ballistic missile are assumed to operate in the same vertical plane. If the intercept altitude is assumed to be 60 km, current theater ballistic missiles reach that altitude in 85-100 sec. Allowing 20 sec for launch detection, trajectory estimation, and downloading guidance information leaves 65-80 sec of flight time for the interceptor. Based on the range of the target missile, a standoff range on the order of 450 km is desired for the safety of the launch platform. Since launch mass is proportional to standoff range and flight time, typical values for the final distance, altitude, and time of the interceptor are taken to be 450 km, 60 km, and 80 sec, respectively.

Because of its high average flight speed (5 km/sec), the shape of the interceptor is chosen to be a cone of half angle 4.5 deg. This shape gives good structural rigidity and aerodynamic stability. A cylindrical shape would have much less structural strength and would require fins for aerodynamic stability. Next, the missile is assumed to have four stages. If the mission requires fewer stages, the sizing code reduces the number of stages. Finally, the payload of the interceptor contains a kinetic kill vehicle and is assumed to have a total mass of 35 kg. To give the kinetic kill vehicle time to achieve the inter-

cept, the payload is allowed to coast for 5 sec before hitting the target.

3. Physical Model

Missile sizing is essentially a mass and aerodynamics problem in that statistical formulas are used to eliminate any need to consider the structure and the engines at this stage of the design process. Hence, it is assumed that the mass fraction of each stage, the specific impulse of each stage, and the vehicle density are known. Ranges of achievable values can be obtained from existing missiles.

Recall that the launch mass is being minimized with respect to the stage payload mass ratios, the stage burn times, and the angle of attack history. Hence, at the beginning of an optimization iteration, values of these quantities are known, either guessed or computed.

The equations of motion used to calculate the trajectory of the missile are those for flight in a vertical plane over a nonrotating spherical earth, that is,

$$\begin{aligned}\dot{x} &= r_s V \cos \gamma / (r_s + h) \\ \dot{h} &= V \sin \gamma \\ \dot{V} &= (T \cos \alpha - D - mg \sin \gamma) / m \\ \dot{\gamma} &= (T \sin \alpha + L - mg \cos \gamma) / mV \\ &\quad + V \cos \gamma / (r_s + h).\end{aligned}\quad (1)$$

Here, x is the horizontal distance, h is the altitude, V is the velocity, γ is the flight path angle, T is the thrust, D is the drag, L is the lift, m is the mass, α is the angle of attack, r_s is the radius of the earth, and g is the acceleration of gravity. In this paper, thrust, drag, lift, and mass satisfy the functional relations $T = \text{Const}$, $D = D(h, V, \alpha)$, $L = L(h, V, \alpha)$, and $m = m(t)$, so that the angle of attack is the only control.

For a spherical earth,

$$g = g_s [r_s / (r_s + h)]^2 \quad (2)$$

where $g_s = 32.174 \text{ ft/sec}^2$ and $r_s = 20,925,646 \text{ ft}$. The atmosphere is assumed to be exponential

so that the density satisfies the relation

$$\rho = \rho_s \exp(-h/\eta) \quad (3)$$

where $\rho_s = .0023769$ slug/ft³ and $\eta = 23,800$ ft. The speed of sound is assumed constant at $a=1000$ ft/sec.

Given the payload mass (35kg) and the payload mass ratio σ of each stage, the payload mass of each stage is given by

$$\begin{aligned} m_{P,4} &= m_{\text{payload}} \\ m_{P,k-1} &= m_{P,k}/\sigma_k, \quad k = 4, \dots, 1 \end{aligned} \quad (4)$$

where $m_{P,0}$ is the launch mass. Next, the fuel mass and the structural mass of each stage are obtained from the relations

$$\begin{aligned} m_{F,k} &= \varepsilon_k(m_{P,k-1} - m_{P,k}) \\ m_{S,k} &= [(1 - \varepsilon_k)/\varepsilon_k]m_{F,k} \end{aligned} \quad (5)$$

where the mass fraction of each stage

$$\varepsilon_k = m_{F,k}/(m_{S,k} + m_{F,k}) \quad (6)$$

is known.

Given the burn time of each stage, the constant mass flow rate is given by

$$\beta_k = m_{F,k}/t_{B,k} \quad (7)$$

so that, from the definition of specific impulse, the constant thrust of each stage becomes

$$T_k = g_s I_{sp,k} \beta_k \quad (8)$$

where the specific impulse of each stage is known. A lower limit is imposed on the burn time to limit the propellant mass flow rate.

At this point, the mass during each stage is given by

$$m = m_{0,k} - \beta_k(t - t_{0,k}), \quad t_{0,k} \leq t \leq t_{f,k} \quad (9)$$

where the initial mass of each stage is the final mass of the previous stage. Also, the initial time of each stage is given by

$$t_{0,1} = 0, \quad t_{0,k} = \sum_{i=1}^{k-1} t_{B,i}, \quad k = 2, 3, 4, \quad (10)$$

and the final time of each stage is

$$t_{f,k} = t_{0,k} + t_{B,k} \quad (11)$$

Since the payload coasts for $t_C = 5$ sec before hitting the target, the final time of the engagement is

$$t_f = t_{f,4} + t_C \quad (12)$$

To compute the aerodynamics, the angle of attack history is represented by a piecewise linear function of time with three equally spaced nodes in each stage. Theoretically, the angle of attack history can have jumps at the stage times, but it is not practical to have a rapidly changing angle of attack near the stage points. Hence, the angle of attack is assumed to be continuous, meaning that the last node of one stage is the first node of the next. A total of nine nodes is needed for four stages.

The drag and the lift are computed from the relations

$$D = \frac{1}{2} C_D \rho S V^2, \quad L = \frac{1}{2} C_L \rho S V^2 \quad (13)$$

where C_D is the drag coefficient, C_L is the lift coefficient, and S is the reference base area. In turn, the drag and lift coefficients are obtained from the axial and normal force coefficients as follows:

$$\begin{aligned} C_D &= C_A \cos \alpha + C_N \sin \alpha \\ C_L &= -C_A \sin \alpha + C_N \cos \alpha \end{aligned} \quad (14)$$

Initially, C_A was modeled as a constant, and C_N , as a function of α only (see Ref. 1). This model indicated that the optimal missile only had three stages. As a result, the angle of attack of the fourth stage is prescribed to be zero because it is just additional payload coast; only seven nodes are now needed to represent the angle of attack history.

To improve the aerodynamics, the methodology of Ref. 2 has been used to generate the axial and normal force coefficients of a conical body of half angle $\delta = 4.5$ deg as functions of

angle of attack, Mach number ($M = V/a$), and Reynolds number. The resulting functions are approximated by

$$\begin{aligned} C_A &= C_{A_0} + 4.0E - 7 h + 7.0E - 4 \alpha \\ C_N &= 0.035 \alpha \end{aligned} \quad (15)$$

with h in ft and α in deg. The quantity $C_{A_0}(M)$ is obtained by linearly interpolating the following Mach number table:

$$\begin{aligned} M &= 0.4, 0.9, 1.2, 2.0, 3.0, 5.0, \\ &\quad 10., 15., 30. \\ C_{A_0} &= .16, .14, .26, .20, .145, .08, \\ &\quad .04, .03, .025 \end{aligned} \quad (16)$$

With this model, the drag and lift coefficients satisfy the functional relations $C_D = C_D(h, V, \alpha)$ and $C_L = C_L(h, V, \alpha)$.

Finally, the base area of each stage is obtained by setting density d times stage volume equal to stage weight. The results are

$$S_k = \pi R_k^2, \quad R_k = \left[\frac{3 \tan \delta}{\pi} \frac{m_{P,k-1} g_s}{d} \right]^{1/3} \quad (17)$$

where δ is the cone half angle.

The initial conditions used for this study are the following:

$$\begin{aligned} t_0 &= 0 \text{ sec}, \quad x_0 = 0 \text{ ft}, \quad h_0 = 0 \text{ ft}, \\ V_0 &= 100 \text{ ft/sec}, \quad \gamma_0 = 85 \text{ deg}, \end{aligned} \quad (18)$$

where the initial velocity is due to a rail launch. The more accurate aerodynamic model consistently drives the launch angle toward 90 deg. Hence, the launch angle has been fixed at 85 deg to reduce the number of optimization parameters.

Finally, the achievable design values are chosen to be

$$\epsilon_k = 0.85, \quad I_{sp,k} = 290 \text{ sec}, \quad d = 60 \text{ lb/ft}^3. \quad (19)$$

These values are optimistic based on current technology; they have been chosen to reflect technology available in 10-15 years.

4. Performance Index and Constraints

The optimization problem is to minimize the launch mass so the performance index is given by

$$F = (m_{P,0}/.0685218)/10,000 \quad (20)$$

where the first scale factor converts launch mass from slugs to kilograms. Scaling the problem makes the optimization algorithm work better.

Next, the prescribed final conditions are on the range (450 km), the intercept altitude (60 km), and the flight time (80 sec). They are used in the scaled forms

$$\begin{aligned} C_1 &= X_f/(450 \times 3280.84) - 1 = 0 \\ C_2 &= h_f/(60 \times 3280.84) - 1 = 0 \\ C_3 &= t_f/80 - 1 = 0 \end{aligned} \quad (21)$$

where X_f and h_f are in ft.

In general, a stage payload mass ratio varies between 0 and 1. The value 0 means no payload, whereas the value 1 means no stage. To prevent the optimization code from generating values outside this range, inequality constraints are imposed on the stage payload ratios as follows:

$$\begin{aligned} C_{3+k} &= \sigma_k - 0.1 \geq 0, \quad k = 1, 4 \\ C_{7+k} &= 1.0 - \sigma_k \geq 0, \quad k = 1, 4. \end{aligned} \quad (22)$$

Next, each stage burn time is required to be more than 2 sec. This prevents a stage propellant mass flow rate from becoming infinite. While an impulse can be an analytical optimum, it cannot be physically realized. Hence, additional inequality constraints are

$$C_{11+k} = t_{B,k}/2 - 1 \geq 0, \quad k = 1, 4. \quad (23)$$

The angle of attack is limited to 10 deg. Since angle of attack is negative, the constraint is expressed as

$$C_{15+i} = 1 + \alpha_i/10 \geq 0, \quad i = 1, 7 \quad (24)$$

with α_i in deg. This constraint is imposed at each node.

Because of structural considerations, it is necessary to limit the dynamic pressure $q = \rho V^2/2$ and the maneuver load $q\alpha$ where α is in deg. Path inequality constraints are converted to point constraints by computing the area of the constraint violation. For dynamic pressure, the constraint has the form

$$-\int_{t_0}^{t_f} [\max(q/q_{max} - 1, 0)]^2 dt \geq 0 \quad (25)$$

where the integrand is squared to make it look like a standard penalty function. A more logical form of the constraint is to require the constraint violation to be zero, that is, to require the integral to be zero. Unfortunately, that form does not always work in the optimization algorithm because both the constraint and its derivatives are zero along a path with $q \leq q_{max}$ everywhere. The form (25) requires a negative number to be nonnegative. Hence, the optimization algorithm makes it as close to zero as the accuracy of the computation allows. Finally, the integral constraint (25) is converted into a differential constraint, a boundary condition, and an algebraic constraint. If the variable P is defined as

$$\begin{aligned} \dot{P} &= -100 [\max(q/q_{max} - 1, 0)]^2, \\ P_0 &= 0, \end{aligned} \quad (26)$$

the constraint becomes

$$C_{23} = P_f \geq 0. \quad (27)$$

The factor 100 is a scale factor to make the constraint more visible to the optimization algorithm.

The maneuver load constraint is imposed in a similar manner. If the variable Q is defined as

$$\begin{aligned} \dot{Q} &= -100 [\max(q\alpha/[q\alpha]_{max} - 1, 0)]^2, \\ Q_0 &= 0, \end{aligned} \quad (28)$$

the constraint becomes

$$C_{24} = Q_f \geq 0. \quad (29)$$

5. Optimization Problem

The parameters used to minimize the launch mass are the stage payload mass ratios, the stage burn times, and the angle of attack nodes. These parameters are elements of the parameter vector

$$X = [\sigma_1 \dots \sigma_4 \ t_{B,1} \dots t_{B,4} \ \alpha_1 \dots \alpha_7]^T \quad (30)$$

which contains a total of 15 parameters.

If values are guessed for the 15 unknown parameters, the differential equations (1), (26), and (28) can be integrated from the initial conditions (18), (26), and (28) to the final time (12). This integration leads to values for the final states which make it possible to compute values for the performance index (20) and the constraint functions (21) through (24), (27), and (29). Hence, the optimization problem or the nonlinear programming problem is to find the elements of the parameter vector X which minimize the performance index

$$J = F(X) \quad (31)$$

subject to the constraints

$$\begin{aligned} C_i(X) &= 0, \quad i = 1, 3 \\ C_i(X) &\geq 0, \quad i = 4, 24. \end{aligned} \quad (32)$$

The nonlinear programming code used in this study is based on recursive quadratic programming and is discussed in Ref. 3. The partial derivatives needed by the algorithm are computed by central differences.

6. Numerical Results

Converged results for the problem defined in Sections 2 through 4 are presented in Fig. 1. The dynamic pressure and maneuver load limits imposed, that is, $q_{max} = 60,000$ psf and $(q\alpha)_{max} = 200,000$ psf deg, represent potential maximum values. For this missile, the payload mass ratios, the burn times, and the angle of attack nodes are given by

$$\begin{aligned} \sigma_k &= .295, .260, .243, 1.00 \\ t_{B,k} &= 2.00, 5.04, 16.1, 51.8 \text{ sec} \\ \alpha_i &= -5.25, -3.00, -1.42, -3.71, \\ &\quad -4.49, -8.71, -10.0 \text{ deg}. \end{aligned} \quad (33)$$

The minimum launch mass is 1,880 kg so that the vehicle weighs 4,115 lb at sea level. Note that the optimal missile has only three stages and that the actual coast time of the payload is 57 sec. Also, the angle of attack reaches the α_{max} limit at the end of the third stage, indicating that atmospheric effects are becoming small. Finally, the missile reaches the q limit at the end of the first stage but does not violate the $q\alpha$ limit because of the small angle of attack.

Fig. 2 shows the results of a trade study for different values of the standoff range, the interceptor flight time, and the q and $q\alpha$ limits. The lower q limits represent those required for a cylindrical missile such as a sounding rocket and show that such a missile would have an excessive launch mass. The higher q limits are possible for conical missiles. These limits have been chosen to bracket the design space. The results show that the launch mass is highly dependent on the standoff range and the flight time, in other words, the average interceptor velocity. For a flight time of 80 sec, the launch mass for the higher q limits varies between 650 kg for 350 km and 3,100 kg for 500 km.

A more detailed design has been carried out for a three-stage missile with a 70 lb payload and a 35,000 psf dynamic pressure limit. For this missile, the sizing code gives

$$\begin{aligned}\sigma_k &= .320, .276, .272 \\ t_{B,k} &= 6.0, 5.2, 12.8 \text{ sec} \\ \alpha_i &= -2.60, -3.12, -3.60, -4.37, \\ &\quad -5.25, -8.08, -10.0 \text{ deg}\end{aligned}\quad (34)$$

and a launch weight of 4171 lb.

First, the missile trajectory is used to determine the amount of heat protection needed to keep the internal temperature at a satisfactory level. Then, the mass fraction of each stage is obtained by starting with a motor mass fraction of 0.9 and adding the masses of the heat shield and the stage adapter to the structural mass. The resulting stage mass fractions and stage den-

sities are given by

$$\begin{aligned}\epsilon_k &= .886, .827, .745 \\ d_k &= 56.4, 60.6, 75.4 \text{ lb/ft}^3\end{aligned}\quad (35)$$

Finally, the payload heat shield mass and structural mass are subtracted from the payload mass so that the weight available for the kinetic kill vehicle is 33 lb. A layout of the interceptor is shown in Fig. 3.

Second, the performance of the resulting missile is evaluated. Aerodynamic tables are generated, and atmospheric effects on engine performance are determined for engines with vacuum $I_{sp}=290$ sec. Then, the performance is obtained by computing the maximum final velocity trajectory for a given flight time to a given intercept altitude. The prescribed final time plus 20 sec determines the position of the target at intercept and, hence, the final altitude of the interceptor. Trajectories have been computed for 60 through 90 sec and have been placed on a trajectory plot of the target, a single stage ballistic missile with a range of 500 km. As shown in Fig. 4, this missile can intercept the target within the atmosphere (60 km) from a standoff range of 400 km. Note, however, that the interceptor actually flies around 350 km since the target is moving toward it.

Trajectories of the ground launch design of Fig. 3 have been computed for the air launch initial conditions $h_0 = 35,000$ ft, $V_0 = 800$ ft/sec, and $\gamma_0 = 0.0$ deg. In general, the standoff range is increased approximately 100 km. Some of this range would be lost by having to make at least the first stage cylindrical for packaging the missile inside an aircraft. A sizing study has also been conducted for an air-launched interceptor. Even though the air launched missile must withstand the same q limits as the ground launched missile, the launch mass is reduced to approximately 1000 kg.

7. Discussion and Conclusions

A computer code has been developed to determine the size of a ground-launched, multistage

missile which can intercept a theater ballistic missile before it leaves the atmosphere. Typical final conditions for the inteceptor are 450 km range, 60 km altitude, and 80 sec flight time. Given the payload mass (35 kg), which includes a kinetic kill vehicle, and achievable values for the stage mass fractions (0.85), the stage specific impulses (290 sec), and the vehicle density (60 lb/ft³), the launch mass is minimized with respect to the stage payload mass ratios, the stage burn times, and the missile angle of attack history subject to limits on the angle of attack (10 deg), the dynamic pressure (60,000 psf), and the maneuver load (200,000 psf deg). For a conical body, the minimum launch mass is approximately 1900 kg. The missile has three stages, and the payload coasts for 57 sec. A trade study has been performed by varying the flight time, the range, and the dynamic pressure limits.

With the results of a sizing study for a 70 lb payload and $q_{max} = 35,000$ psf, a more detailed design has been carried out to determine heat shield mass, tabular aerodynamics, and altitude dependent thrust. The resulting missile has approximately 100 km less range than the sizing program predicted primarily because of the additional mass required for heat protection. On the other hand, launching the same missile from an aircraft increases its range by approximately 100 km.

Sizing the interceptor for air launch with the same final conditions as the ground-launched missile reduces its launch mass to approximately 1000 kg.

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STAGE	MP (KG)	MP (KG)	MS (KG)	MO (KG)	S (M**2)	R (M)	BETA (KG/S)	THR (N)
0	0.188E+04							
1	0.554E+03	0.113E+04	0.199E+03	0.188E+04	0.875E+00	0.528E+00	0.563E+03	0.160E+07
2	0.144E+03	0.349E+03	0.616E+02	0.554E+03	0.388E+00	0.351E+00	0.693E+02	0.197E+06
3	0.350E+02	0.924E+02	0.163E+02	0.144E+03	0.158E+00	0.224E+00	0.573E+01	0.163E+05
4	0.350E+02	0.000E+00	0.000E+00	0.350E+02	0.615E-01	0.140E+00	0.000E+00	0.000E+00

T (S)	X (KM)	H (KM)	V (KM/S)	G (DEG)	M (KG)	Q	QA	ALPHA	CD	CL
STAGE 1										
0.000	0.000	0.000	0.0305	85.0000	1879.7	12.	62.	-5.25	0.184	-0.168
0.500	0.037	0.120	0.4830	70.4966	1598.0	2935.	12110.	-4.13	0.238	-0.128
1.000	0.174	0.464	1.0111	66.7280	1316.4	12268.	36782.	-3.00	0.139	-0.098
1.501	0.451	1.063	1.6533	63.9300	1034.8	30202.	66700.	-2.21	0.079	-0.074
2.001	0.923	1.971	2.4810	61.3251	753.2	60003.	85123.	-1.42	0.058	-0.048
STAGE 2										
2.001	0.923	1.971	2.4810	61.3251	554.4	60003.	85123.	-1.42	0.058	-0.048
3.261	2.618	4.831	2.8132	57.0794	467.1	52013.	133468.	-2.57	0.055	-0.087
4.521	4.848	7.912	3.2464	50.9028	379.8	45300.	168217.	-3.71	0.055	-0.127
5.781	7.853	11.158	3.8122	43.5335	292.6	39932.	163731.	-4.10	0.057	-0.140
7.041	11.916	14.496	4.6015	35.1778	205.3	36721.	164772.	-4.49	0.058	-0.153
STAGE 3										
7.041	11.916	14.496	4.6015	35.1778	143.7	36721.	164772.	-4.49	0.058	-0.153
11.072	28.270	23.550	4.7600	23.7878	120.6	11280.	74404.	-6.60	0.082	-0.223
15.103	46.900	30.306	5.1476	16.3522	97.5	5198.	45246.	-8.71	0.108	-0.292
19.134	68.061	35.298	5.7347	10.3187	74.4	3242.	30317.	-9.35	0.121	-0.312
23.164	92.439	38.439	6.5924	4.2511	51.3	2778.	27783.	-10.00	0.131	-0.332
STAGE 4										
23.164	92.439	38.439	6.5924	4.2511	35.0	2778.	0.	0.00	0.078	0.000
36.123	175.863	44.426	6.4205	3.8968	35.0	1154.	0.	0.00	0.086	0.000
49.082	257.671	49.763	6.3343	3.5082	35.0	538.	0.	0.00	0.093	0.000
62.041	338.619	54.477	6.2862	3.1016	35.0	277.	0.	0.00	0.100	0.000
75.000	419.058	58.580	6.2566	2.6848	35.0	156.	0.	0.00	0.105	0.000
PAYLOAD										
75.000	419.058	58.580	6.2566	2.6848	35.0	156.	0.	0.00	0.105	0.000
76.250	426.798	58.943	6.2544	2.6442	35.0	148.	0.	0.00	0.106	0.000
77.500	434.535	59.301	6.2522	2.6036	35.0	141.	0.	0.00	0.106	0.000
78.750	442.269	59.653	6.2502	2.5629	35.0	134.	0.	0.00	0.106	0.000
80.000	450.000	60.000	6.2482	2.5221	35.0	128.	0.	0.00	0.107	0.000

Figure 1. Results of Sizing Computation

Design Parameters:

3-Stage Vehicle

Intercept Alt = 60 km

Payload Mass = 35 kg

Mass Fraction = 0.85

Cone Angle = 4.5 deg

Propellant Isp = 290

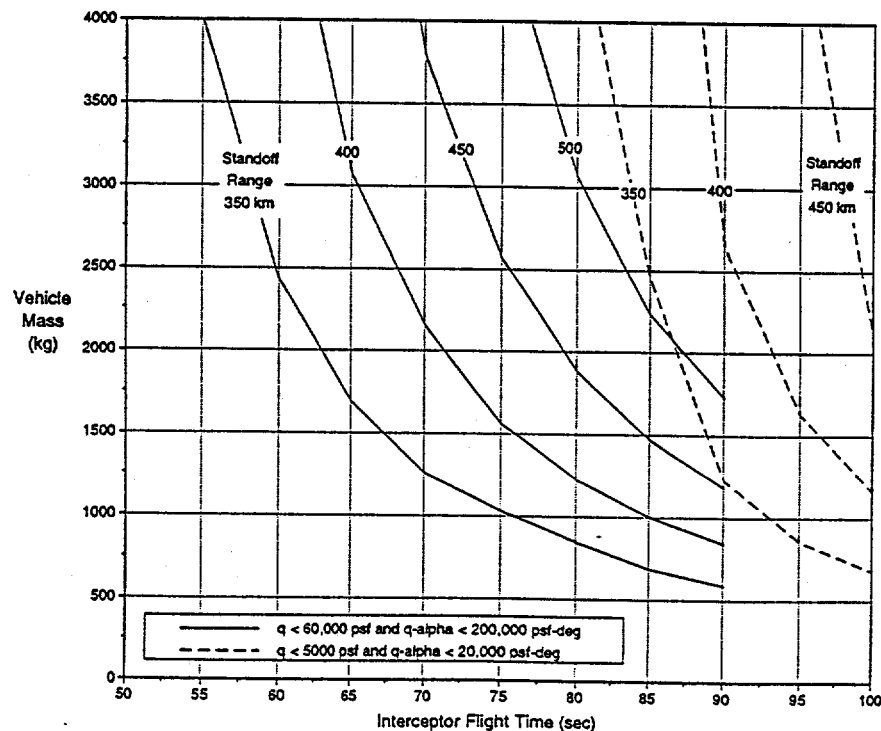
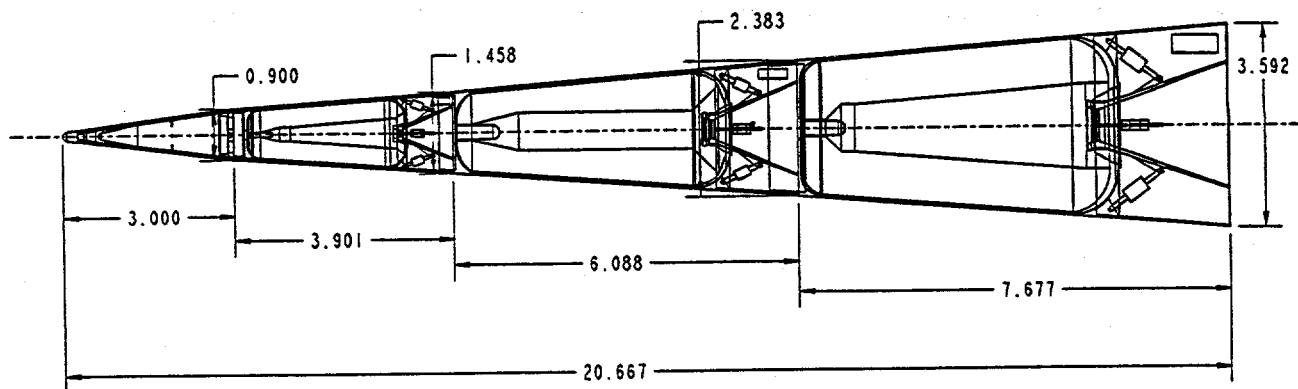


Figure 2. Sizing Results for a Ground-Launched Interceptor Missile



Weights

1st Stage	2983 lb
2nd Stage	1015
3rd Stage	310
Payload	70
Total:	4378 lb

All dimensions are in ft

Figure 3. Layout of Ascent-Phase Interceptor Concept

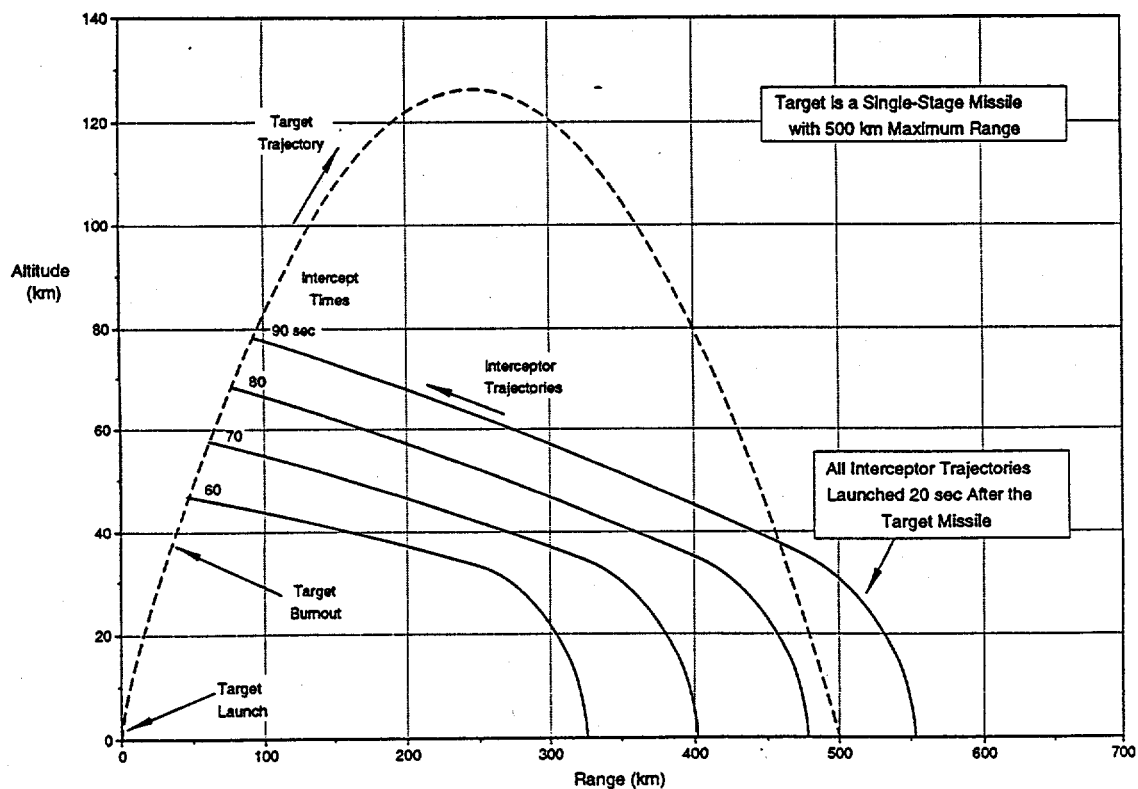


Figure 4. Intercept Trajectories for Ascent-Phase Interceptor Concept