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An Experimental Algorithm for Detecting Damage Applied to the I-40 Bridge Over the Rio Grande

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ABSTRACT

An algorithm originally used to locate errors in finite element models is applied to a full scale bridge damage detection experiment. The method requires experimental frequency response function data measured at discrete locations along the major bridge load paths. In the bridge damage application the algorithm is most effective when applied to static flexibility shapes estimated with a truncated set of six mode shapes rather than individual mode shapes. The algorithm compares "before damage" and "after damage" data to locate physical areas where significant stiffness changes have occurred. A damage indicator shows whether damage is detectable. Damage is correctly located in the two most significant damage cases using the driving point static flexibility estimates. Limitations of the technique are addressed. The damage detection experiment was performed on a three span steel girder bridge that was 425 feet long. This bridge was part of Interstate 40 across the Rio Grande. The New Mexico State University Department of Civil Engineering organized the experiment. The frequency response functions were collected by Los Alamos National Laboratories personnel. The bridge excitation was provided by Sandia National Laboratories.

NOMENCLATURE

FRF	Frequency response function
x	Displacement scalar
f	Force scalar
SR	STRECH ratio
M	Moment
z	Coordinate in direction of beam axis
E	Young's modulus
I	Area moment of inertia of a beam
θ	Rotation displacement
l	Beam span length between to sensors
Ψ_i^r	Mode shape at point i for r th mode
m_r	Modal mass of r th mode

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ω_r	Modal frequency
ω	Frequency
ζ_r	Critical damping ratio
DI	Damage indicator

INTRODUCTION

At the end of the summer in 1993, New Mexico State University directed a series of experiments on a full scale bridge designed to provide a data base for bridge health monitoring algorithms. Sandia National Laboratories participated with Los Alamos National Laboratories in the acquisition of dynamic measurements on the bridge. Sandia furnished and operated a shaker to provide both sinusoidal and random force inputs to the bridge while Los Alamos acquired the dynamic measurements. The modal test was originally designed for use in updating a finite element model of the bridge. However, subsequent to the testing, Sandia obtained the frequency response functions (FRFs) from Los Alamos to attempt to apply some damage detection algorithms to the data. These algorithms were based on a system identification algorithm originally applied in comparing modal test data to a finite element model to physically locate differences between the experimentally derived and analytically derived modal models[1]. This work was performed using funding from a laboratory directed research and development project in health monitoring at Sandia National Laboratories.

Many techniques using modal quantities have been used to attempt to locate damage, assuming that it is basically manifested as a local change in stiffness from the original structure. Frequency comparisons, global mode shape comparisons, and damping comparisons have often been disappointing in determining and locating damage[5]. It is this author's contention that global shape comparisons or even point to point comparisons are not the correct quantities to evaluate. If there is a change in stiffness, then there should be a change in displacement difference across that stiffness due to some forcing function. Damage detection techniques that assume a change in stiffness should consider displacement gradient type quantities. This approach is applied in this work.

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DESCRIPTION OF EXPERIMENT

Two papers in this conference [2],[3] describe the experiments in detail. A description for the purposes of this paper will now be given. Figure 1 shows a schematic view of the three span bridge that was tested. It was about 425 feet long and was one of three bridges that carried east-bound traffic across the Rio Grande in Albuquerque, New Mexico. The bridge was replaced by a new bridge immediately after the testing, which provided the opportunity to induce significant damage as well as test without traffic on the bridge. Two main steel plate girders (running the entire length) support the bridge, one on either side. This bridge is a fracture critical bridge, meaning that if one of the main plate girders was to fail, there is no redundant support to prevent catastrophic failure. Twenty-six vertical accelerometers were mounted near the neutral axis of the plate girders, 13 along each girder. They were evenly distributed along the length of each girder. Damage was induced with a cutting torch just west of center on the north plate girder. There was a series of five tests performed. The first test was performed on the as-used condition. The other tests were performed after each of four progressively severe vertical cuts were induced in the plate girder. The I shaped cross section of the girder is shown in Figure 2. The first cut was in the web centered about the neutral axis, and was two feet long. The second cut extended down to, but not into, the bottom flange. The third cut was halfway through the bottom flange. The final cut severed the bottom flange. Modal tests were performed at each stage using a random force input from the Sandia shaker mounted on the south side of the bridge in the center of the east span as shown in Figure 1. Los Alamos State University directed the dynamic testing and performed all the static testing as well (not discussed in this paper).

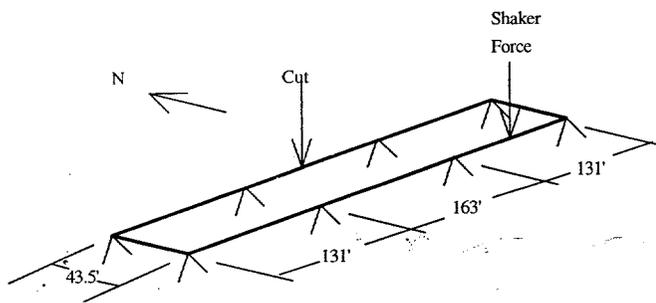


Figure 1 - Schematic of Three Span Bridge

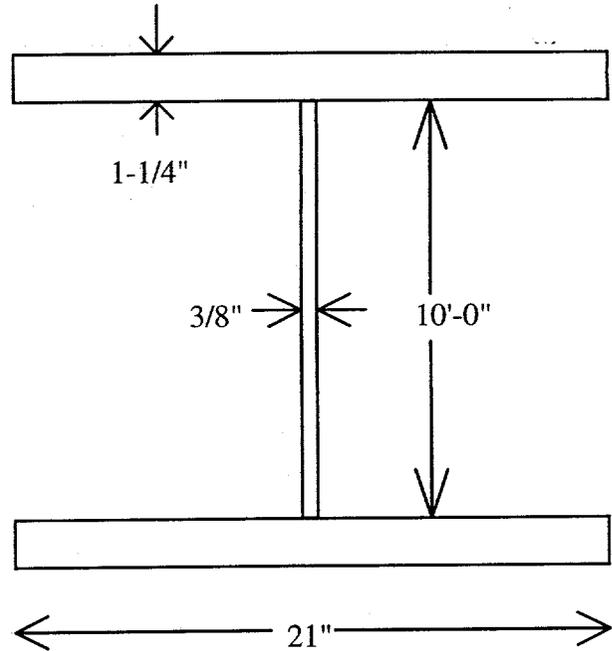


Figure 2 - Cross Section of the Steel Plate Girder
(Not to Scale)

STRECH CONCEPT

As stated in the introduction, an algorithm for error localization in a finite element model was published in an earlier IMAC[1]. The algorithm has been named with an acronym, Structural Translation and Rotation Error Checking or STRECH. STRECH is basically a static concept that has been applied successfully to locate soft or stiff areas of a finite element model by comparing the lowest cantilevered mode shapes from a modal test with the finite element model. A description of the algorithm will be given here utilizing static displacements from a two degree of freedom system as shown in Figure 3. The top figure would represent displacements in a "healthy" structure. The bottom figure would represent the displacements after spring 23 was damaged, that is, reduced in stiffness. For the purpose of this example, assume there is no damage to spring 12.

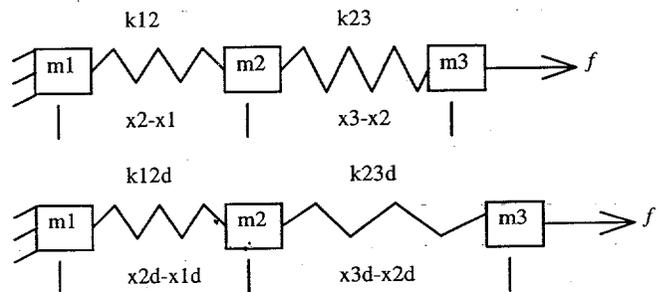


Figure 3 - Demonstration of the STRECH Concept

The simple static force displacement relations from the undamaged case are

$$f = k_{12} \cdot x_{12} = k_{23} \cdot x_{23} \quad (1)$$

where x_{12} is displacement x_2-x_1 and f is the applied force. For the damaged case (superscript d)

$$f = k_{12}^d \cdot x_{12}^d = k_{23}^d \cdot x_{23}^d \quad (2)$$

By equating the right hand sides of (1) and (2)

$$k_{23} \cdot x_{23} = k_{23}^d \cdot x_{23}^d \quad (3)$$

which can be rearranged as

$$\frac{x_{23}^d}{x_{23}} = \frac{k_{23}}{k_{23}^d} > 1 \quad (4)$$

Similarly, a relationship for spring 12 can be written

$$\frac{x_{12}^d}{x_{12}} = \frac{k_{12}}{k_{12}^d} = 1 \quad (5)$$

Theoretically, it would be easy to tell if there were damage to the springs and the extent of damage by applying a known force to both systems and measuring the displacements. In this case equation 4 would show that spring 23 had been damaged. This is the basic concept behind STRECH. The displacements can obviously be rotations and the forces in each element can be moments (which is how the relations will be used for the applications in this paper to the I-40 bridge). The displacement quotients given in equations (4) and (5) are known as the STRECH ratios. In general, additional degrees of freedom, constraints and load paths (i.e. parallel springs) may be included in real physical systems so that extent of damage to an individual spring may not be calculated, but the general trend of being able to detect damage and locate relative soft or stiff areas across the structure has been viable.

Although this concept is a static one, success has been realized by applying this to the first cantilevered mode shape when the mode shape looks a great deal like the static displacement shape. This has been utilized on a cantilevered robot arm, a cantilevered missile payload and a cantilevered third stage of a missile with payload. In each case significant stiffness differences between a finite element model and a modal test mode shape were identified, enabling the analyst to identify critical parameters to update in the finite element model.

NORMALIZATION AND DENOMINATOR FILTER

The realities of acquiring and fitting experimental data from a structure can cause some problems in the interpretation of the results of the STRECH ratios. One problem can occur if experimental data is accidentally taken with an incorrect global scale factor applied. To eliminate some of the confusion that might be caused by such a problem, a normalization has been applied. The STRECH ratio between two sensors are calculated

$$SR_{ij} = \frac{x_{ij}^d}{x_{ij}} \cdot \frac{\sum_{kl} x_{kl}}{\sum_{kl} x_{kl}^d} \quad (6)$$

The superscript d indicates data from the potentially damaged state. Data with no superscript is the baseline data which is considered undamaged. The summations are for all displacement differences defined along the load paths by the engineer. This basically defines the displacement difference x_{ij} as a fraction of the sum of all displacement differences measured for the structure's specific state. Although the average SR is not always exactly equal to one, it is generally very near one. This makes the interpretation of the data much easier, as a value much greater than one will indicate an area of the structure that has been significantly reduced in stiffness (i.e. damaged). The highest SR should correspond to the part of the structure most likely to be damaged. In practice, x is usually a displacement difference between two points on the structure, each of which has three coordinates. The algorithm calculates the square root of the sum of the squares of the three coordinate displacement differences, so that all x quantities shown in equation 6 are positive values. In this application, only vertical accelerations were measured, so the accelerations in the other two coordinate directions were considered zero.

From equation 6 it can be seen that if x_{ij} is very small, the SR can become very uncertain. Since all experimental data has noise associated with it, and data fitting algorithms are not perfect either, a false SR that is very large (because of a small denominator corrupted significantly by noise) may be calculated. A small value of x_{ij} in the denominator means that the structure is not being exercised between points i and j in the baseline structure. If this is the case, the true response should be insensitive to damage between those two points. Therefore, the engineer establishes a minimum denominator value for x_{ij} below which the SR is not calculated at all. In the algorithm, the minimum denominator value is set as a percentage of the largest displacement difference for the baseline structure.

APPLICATION TO THE I-40 BRIDGE

In this paper, the application is health monitoring with experimental data only. Processed experimental data for the I-40 bridge in its as used condition was the baseline data information (undamaged). Processed experimental data from four different

damage cases were the comparison data which were examined for evidence of softening between the sensor locations:

USE OF ROTATIONS

The SRs were calculated based on differences in rotation. The field measurements were accelerations in the vertical direction. Estimates of the rotations were obtained from displacement shape data by passing a parabola through three adjacent displacements on one of the plate girders. The slope of the parabola at the middle point was utilized as the estimate for the rotation of that point. The use of the rotation is justified based on force displacement relations of a beam.

$$M = EI \frac{\partial \theta}{\partial z} \quad (7)$$

where θ is the rotation of the plate girder in the plane of the web and z is in the direction of the neutral axis of the plate girder. M is moment, E is the modulus of elasticity, and I is the area moment of inertia. The partial can be approximated as a finite difference so that equation 7 now takes a form similar to equation 1.

$$\dot{M} = \frac{EI}{l} \cdot \theta_{ij} \quad (8)$$

θ_{ij} is $\theta_j - \theta_i$ and l is the distance between two sensors. Two load paths were chosen, one from one end to the other of each plate girder. SRs were calculated between each pair of adjacent accelerometer locations.

STRECH RATIOS USING MODE SHAPES

Initially, SRs were calculated comparing rotation differences for the first mode shape of the damaged and undamaged data. The modal frequency and damping were extracted with the Polyreference technique while real mode shapes were extracted using a technique devised by the author[4]. Six modes were extracted. The SR calculations were marginally successful when applied to the first mode for the third and fourth (most severe) cuts. Calculations applied to higher modes failed miserably. The comparisons for the third and fourth cuts had the worst indications of damage in members adjacent to the four inner pylons, with secondary indications in the damaged area. If the minimum denominator value was raised enough (20 percent or more of the maximum rotation difference in the undamaged bridge), the damaged member showed worst damage because all elements adjacent to pylons were excluded from calculations.

STRECH RATIOS USING STATIC FLEXIBILITY

Since the SR calculations were not extremely successful in detecting the location of damage with the first mode shape, another approach was utilized. Because the STRECH ratio is a static concept, a static deflection should work better for comparisons than a dynamic mode shape. An estimate of the static flexibility (the static deflection shape due to a unit load) can

be obtained from the modal parameters by use of the following well known formula for the frequency response function based on real modes.

$$\frac{x(\omega)}{f(\omega)} = \sum_{r=1}^{\infty} \frac{\Psi_i^r \Psi_k^r}{m_r (\omega_r^2 - \omega^2 + 2j\zeta_r \omega \omega_r)} \quad (9)$$

where $x(\omega)$ is displacement as a function of frequency, $f(\omega)$ is an applied point force as a function of frequency, Ψ_i^r is the mode shape at the response point for the r th mode, Ψ_k^r is the mode shape at the driving point for the r th mode, m_r is the modal mass, ζ_r is the damping ratio, ω is the frequency in radians/second, ω_r is the r th natural frequency and the summation is for all modes. An estimate of the static flexibility is achieved by evaluating equation 9 at zero frequency. In this case a truncation was made using only 6 modes.

$$\frac{x(0)}{f(0)} = \sum_{r=1}^6 \frac{\Psi_i^r \Psi_k^r}{m_r \omega_r^2} \quad (10)$$

Theoretically any driving point can be chosen, but the actual driving point appeared most accurate in this work. Figure 4 shows the estimate of the static flexibility shape for the undamaged bridge. The maximum displacement is at the point where the shaker was located. Recall that the damage was induced on the opposite side of the bridge from the shaker in the middle span. Although, this is far from the optimum location for the applied static force in terms of exercising the damaged portion of the bridge, the results were encouraging as compared to the calculations performed with individual mode shapes.

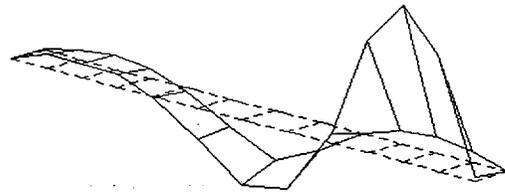


Figure 4 - Static Flexibility Shape of Undamaged Bridge

TRUNCATED STATIC FLEXIBILITY AS A DAMAGE INDICATOR

Figures 5 through 8 give the reader an intuitive feel for the value of the truncated static flexibility as an indicator of damage. The figures show an elevation view of the static flexibility shape of each of the main plate girders. The dashed lines are the undamaged plot. The solid lines are the damaged plot. The damaged girder is offset slightly above the other girder to separate the two. It is easier to separate the two by looking at the left side. The places where it appears there is very little deflection are where the girder ties into the pinned joints at the pylons. These

greatly exaggerated plots show the estimated static deflection as calculated from the modal parameters using equation 10. Notice how the damaged static flexibility shape progressively deviates from the undamaged (dashed) plot. For the most severe damage shown in Figure 8, the differences become very localized, but very pronounced in the center span on the damaged side. The very localized area moment of inertia was reduced by about 1 percent in cut 1, 13 percent in cut 2, 45 percent in cut 3 and 93 percent in cut 4. Remember that the effect is smeared over a significant distance as well. After these figures were obtained, the author attended the '94 IMAC where Aktan and others[5] presented convincing results that identified the static flexibility as a viable indicator of damage. They used 18 modes to increase the accuracy of the static flexibility estimate. The figures indicate that a less accurate static flexibility calculated with only 6 modes provides useful information for this case. This seems plausible, since the damage was introduced in a place that is exercised strongly by four of the first six modes.

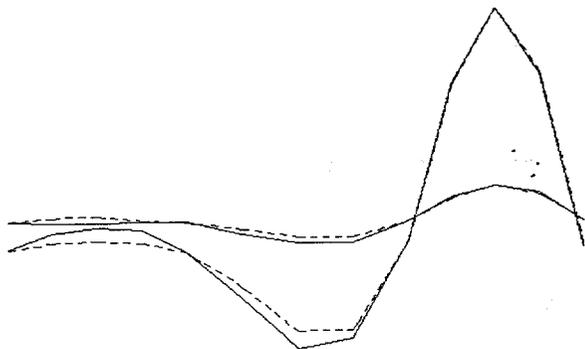


Figure 5 - Static Flexibility Comparisons for Both Main Plate Girders after Cut 1 (Dashed is undamaged - Solid is Damaged)

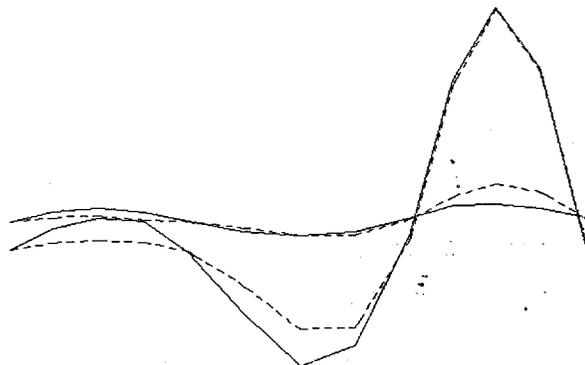


Figure 6 - Static Flexibility Comparisons for Both Main Plate Girders after Cut 2 (Dashed is undamaged - Solid is Damaged)

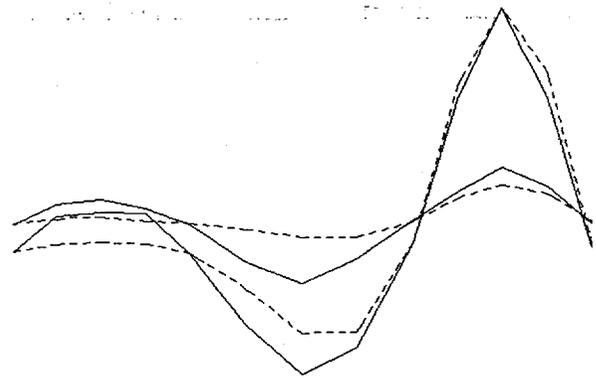


Figure 7 - Static Flexibility Comparisons for Both Main Plate Girders after Cut 3 (Dashed is undamaged - Solid is Damaged)

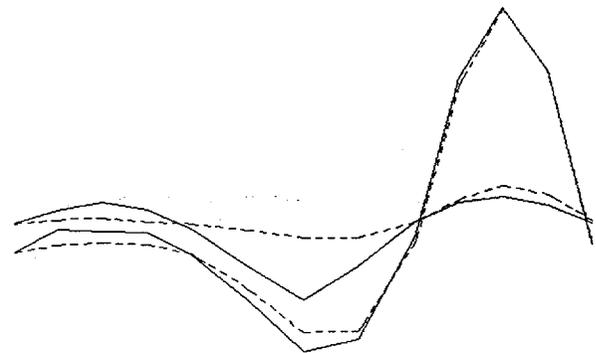


Figure 8 - Static Flexibility Comparisons for Both Main Plate Girders after Cut 4 (Dashed is undamaged - Solid is Damaged)

DAMAGE INDICATOR

Although the previous figures give some intuition into the progression of damage, a close examination would reveal at least the possibility that there is some noise or bias in the shapes. A quantity is needed that can be calculated to indicate the onset of recognizable damage. A threshold value for that quantity needs to be established which is high enough to discount the effects of noise, but low enough to sense significant damage. A quantity is proposed here using terms within the SR calculation as given below:

$$\text{Damage Indicator (DI)} = \frac{\sum_{ij} |x_{ij} - x_{ij}^d|}{\sum_{ij} x_{ij}} \quad (11)$$

where the terminology is the same as in equation 6. The damage indicator was calculated for each damage case using rotation differences. In addition, the modal parameters were extracted two more times on the undamaged bridge by two other common methods. Static flexibilities for the undamaged bridge were computed, and the damage indicator was also calculated for these two cases in which there was no damage to get a feel for the

effects of variation in modal extraction techniques on the damage estimates. The first extraction of undamaged modal parameters was used as the baseline. These results are printed in Table 1. The first two rows are the damage indicators for the undamaged bridge where the same data was used, but different modal extraction techniques were utilized to form the static flexibility. Then the damage indicators are calculated for each cut. Although this is not a statistically conclusive study, it appears that the damage indicator begins to rise significantly enough at cut 2 to indicate the presence of damage.

Table 1 - Damage Indicators

Case	Damage Indicator
Undamaged - Extraction Method 2	9%
Undamaged - Extraction Method 3	8%
Cut 1	14 %
Cut 2	28%
Cut 3	40 %
Cut 4	33%

DAMAGE LOCATION USING STRECH RATIOS ON STATIC FLEXIBILITY

The SR calculations were much more successful when applied to the static flexibility calculations, even though the damaged part of the structure was not exercised well. Using a minimum denominator value of only one percent (of the maximum rotation difference in the undamaged case) to filter the most noisy calculations, the location of damage was correctly identified for the two worst damage cases, cuts 3 and 4. For cut 1 the damaged location was the second choice of the algorithm. For cut 2 the damaged location was the fourth choice. Why does the calculation appear more successful for cut 1, where the damage was so minimal, than for cut 2? The answer may be in the fidelity of the data. Results from Los Alamos' report [6] show that the input force level was much higher for cuts 1 and 3 than for cuts 2 and 4. This would provide a better signal to noise ratio in the FRFs which could lead to a more accurate static flexibility shape for cut 1 than for cut 2. Even though the signal to noise ratio might not have been as good for cut 4, the damage was so significant that the noise did not matter so much. Note that the SR increases with increasing level of damage in the actual damaged element (number 107-108). Table 2 lists the results.

Table 2 - Predicted Damage Locations for Static Flexibility

Case/Element No.	STRECH Ratio	Comment
Cut 4/ Element 107-108	13.2	Correct 1st choice
Cut 3/ Element 107-108	10.5	Correct 1st choice
Cut 2/ Element 4-5	7.07	Wrong 1st choice
Cut 2/ Element 10-11	2.95	Wrong 2nd choice
Cut 2/ Element 12-13	2.89	Wrong 3rd choice
Cut 2/ Element 107-108	2.81	Correct 4th choice
Cut 1/ Element 4-5	4.18	Wrong 1st choice
Cut 1/ Element 107-108	2.53	Correct 2nd choice

*Note: Element 4-5 was adjacent to a pylon in the same span as the shaker. Elements 10-11 and 12-13 were on the opposite end of the bridge from the shaker where static responses were low. Elements 1-2 through 12-13 were on the south side (shaker side) of the bridge moving from east to west. Elements 101-102 through 112-113 were on the damaged north side of the bridge moving from east to west.

OTHER RESULTS

Although the results shown above are encouraging, in a practical sense, a minimum denominator value higher than 1 percent would probably be desirable for this set of data to reduce the potential of contamination of the static flexibility calculations from measurement and data analysis uncertainties. With the experience gained from past work with the STRECH algorithm, the minimum denominator value should probably be on the order of 5 to 10 percent. Using a more conservative level of 10 percent and applying it to this data, the damaged element is eliminated from the STRECH ratio calculations because the baseline rotation differences for the damaged portion of the bridge fall below this criterion. On the shaker side of the bridge, only measurements in the shaker span and the middle span had rotation differences large enough to qualify for calculation. On the damaged side of the bridge, only elements in the shaker span qualified for calculation. All others fell below the 10 percent minimum denominator requirement. However, in every damage case, for this minimum denominator value, the damaged element selected was element 7-8 which is directly across the bridge from the damaged element.

LESSONS LEARNED, PROPOSED IMPROVEMENTS AND ISSUES FOR FURTHER STUDY

In this experience, the STRECH algorithm performed much better on static flexibility data than on individual mode shapes. There are two possible causes for this. The most probable is that this bridge has eight constraint locations, whereas all structures to which this algorithm has been applied heretofore have had only a single constraint location (cantilevered). Although the static approach of the STRECH algorithm is certainly justified in its application to static flexibility shapes, it may not be applicable to individual mode shapes for structures as constrained as bridges. It is known that the STRECH algorithm is not applicable for high order mode shapes for any structure. A second possible cause might be that the rotation estimates are not accurate enough near the constrained points. However, the application of STRECH to

static flexibility shapes did not seem to suffer from this problem. A better algorithm for estimating the rotations might exist, or more measurements could be made. In addition to increasing the accuracy of the rotations, additional measurements also increase the sensitivity of these algorithms, since the effect of damage would not be smeared across such a long length of undamaged structure. The drawbacks to more sensors is increased test cost and increased possibility of faulty instrumentation.

Static flexibilities are more sensitive to damage in highly exercised parts of the structure. A future damage detection test series should have multiple excitation locations to exercise all parts of the bridge more fully. If only one location is possible, it should be in a place where as much of the structure is well exercised as possible. For this case, a location in the center span would have provided a better exercising of all parts of the two main girders. The shaker location was chosen to excite the first six modes well for finite element model reconciliation, not for damage detection. There is some technical advantage to placing the exciter away from the center of a span as well. If sensitivity to damage near the pylons is of interest, these areas are exercised only in higher modes of the structure (and some of these modes would need to be included in the static flexibility calculation). An exciter location away from the center of the span might be required to excite some of these higher modes better.

Noise on the measurements and uncertainty in the modal extraction process affect the calculations. Getting as much input force as possible for these large structures would be advantageous. If significant energy can be input at low frequencies, a fitting process might be developed to estimate the low frequency displacement/force FRF asymptote to achieve an extremely accurate static flexibility. This might remove the uncertainty of the modal extraction process as well as the errors in static flexibility due to modal truncation. The advantage to using accelerometers as sensors is that they can be placed directly on the bridge. They do not need a quiescent reference mounting location apart from the bridge as displacement or velocity devices require. The disadvantage is the long cabling required to bring the signals to the data acquisition system.

The setting of the minimum denominator for SR calculations is important for filtering out false indications of damage location. If this setting is too low there will be false indications due to noise. If the setting is too high, many possible locations for damage are eliminated from consideration. This value is probably dependent on data quality, modal extraction quality and relative displacement levels in the static flexibility shape. Engineering judgment is still required. A reasonable value for this test setup is around 5 to 10 percent of the largest rotation difference in the author's opinion.

The damage indicator provides some indication of the onset of damage. The big question is what is the threshold. Performing several different modal extractions on the undamaged data may be a reasonable way of establishing some threshold. A statistical analysis using the ordinary coherence function for the data carried through the extraction process would be more quantitative. The value of the damage indicator is possibly dependent on the

number and spread of sensors as well. The damage indicator will not be sensitive to damage at a particular location if the static flexibility is not sensitive to that damage.

CONCLUSIONS

This work adds strong supporting evidence to other referenced work that the static flexibility can be sensitive to damage. In addition, it provides some indication that a truncated set of modes in the static flexibility calculation may be acceptable for indicating damage. The value of a displacement gradient type quantity for use in assessing the onset of damage and the damage location has been strengthened. Algorithms for damage indication and damage location have been demonstrated using experimental data from a full scale bridge damage test series. Lessons have been learned to aid in the planning of future bridge damage detection testing.

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