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Wentzel-Bardeen singularity in coupled Luttinger liquids: transport properties

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The recent progress on 1 D interacting electrons systems and their applications to study the transport properties of quasi one dimensional wires is reviewed. We focus on strongly correlated electrons coupled to low energy acoustic phonons in one dimension. The exponents of various response functions are calculated, and their striking sensitivity to the Wentzel-Bardeen singularity is discussed. For the Hubbard model coupled to phonons the equivalent of a phase diagram is established. By increasing the filling factor towards half filling the WB singularity is approached. This in turn suppresses antiferromagnetic fluctuations and drives the system towards the superconducting regime, via a new intermediate (metallic) phase. The implications of this phenomenon on the transport properties of an ideal wire as well as the properties of a wire with weak or strong scattering are analyzed in a perturbative renormalization group calculation. This allows to recover the three regimes predicted from the divergence criteria of the response functions.

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I. INTRODUCTION

The developments in the field of Mesoscopic physics have generated a lot of excitement for more than a decade. One of the reasons for this is connected to the recent advances in fabrication techniques, which have allowed: a) the discovery of new phenomenon associated with quantum interference effects in small structures b) the verification of a plethora of theoretical predictions on the same systems. This relative balance between theory and experiment is very desirable in any field of physics. Apart from a few exceptions [1-3], it seems fair to say that until recently, the theoretical effort in Mesoscopic physics has dealt mostly with non interacting electrons: phenomenon associated with sample to sample averaging, interference effects in small disordered wires and rings, transport properties from a scattering matrix approach, chaotic phenomena in Mesoscopic conductors, to cite a few. The Coulomb blockade certainly deals with electron interactions, however, for large quantum dots, it can be argued in terms of classical capacitance effects [4], as opposed to a first principles treatment of interactions between electrons. There are several "kinds" of quasi one dimensional (1 D) systems: a) systems obtained by lateral confinement of a 2 D electron gas into

a narrow channel, b) micrometer metallic wires with a small aspect ratio, c) systems which have an intrinsic 1 D character like edge states in the integer (IQHE) and fractional (FQHE) quantum Hall effect, and of course, d) molecular chains and bundles of all sorts which have been studied for a long time. In the latter, rigorous methods from the 1970's [5,6] have long been applied to predict "phase" transitions from charge/spin density waves to singlet and triplet superconducting ground states as various coupling parameters are varied. By "phase", it is implied which type of fluctuations dominate, as we know that long range order is strictly not possible in 1 D. For "artificially tailored" systems in categories a) and b), disorder remains a technological challenge to this day. On the contrary, edge states in the quantum Hall regimes (c) have the advantage that the effect of impurities can be minimized because of the absence of backscattering.

Here we study a 1 D interacting electron system coupled to a low energy bosonic field. This system exhibits the interplay between the Coulomb repulsion and the attractive, retarded interaction mediated by the phonons. By varying the electron density alone, it is possible to reach a strong electron-phonon regime with dramatic consequences on the thermodynamic properties. Phase transitions in this system will be considered, and some emphasis will be put on the transport properties of ideal and disordered wires in the presence of coupling to a bosonic field. In practice, this boson field will be chosen to represent the phonons of the underlying lattice, but this is by no means a serious constraint. The bosonic field can just as well represent the charge field of another 1 D electron branch.

In 1D, the Fermi liquid ground state becomes unstable, as the interactions drive the systems into a new phase called the Luttinger liquid phase [7-9]. The jump of the momentum distribution function at the Fermi surface which is characteristic of Fermi liquids vanishes, leaving only a branch cut singularity at this point. The collective excitations of the electron near the Fermi level can adequately be described by a set of harmonic oscillators. At low temperatures, the single electron spectrum can be approximated by a linear spectrum, and considerable simplifications follow. The Fermion field operators are decomposed into right and left moving components, which in turn can be expressed as exponentials of bosonic fields chosen to reproduce the anti-commutation relations. One is then left with scalar, bosonic fields, which

are particularly suited for a functional integral representation of the partition function. Moreover, a renormalization group analysis can be coupled to this approach in order to determine which (and when) non linear operators are relevant. In the end (for specific range of electron density, coupling parameters) one has to deal essentially with an action which is quadratic in the bosonic fields, with parameters which are renormalized by the (irrelevant) operators.

The transport properties for a wire with ideal transmission yield a conductance which is no longer quantized in units of e^2/h , the non interacting result. Rather, the Luttinger liquid model predicts [10,11] $G = 2e^2 K_\rho/h$ below the free electron value, as $K_\rho < 1$ for electrons with repulsive interactions. In the presence of a single barrier the conductance vanishes as $T^{2/K_\rho-2}$. This applies for temperatures above the cutoff $T_c \equiv \hbar v_F/Lk_B$, where L is the length of the 1 D region. Below this cutoff, effects associated with the massive leads connected to the wire provoke a saturation of the conductance to a finite value. Despite this dramatic vanishing of the temperature, perfect transmission in two-barrier resonant tunneling structures can be achieved for a wide range of electrons couplings by tuning the gate voltage on the central region, because of constructive interference effects associated with the two barriers [10,11]. For resonant tunnelling in the FQHE regime [12] the resonance line shape is *universal* and deviates strongly from the Fermi liquid behavior. For free electrons, the line shape is supposedly Lorentzian and does not depend on the temperature. On the contrary, for a Luttinger liquid the peaks in conductance are delta functions at $T = 0$ and widen at finite temperature like $T^{1-\nu}$, with $\nu = 1/m$ (m integer) the filling factor. This remarkable result was recently tested experimentally [13] in a FQHE experiment with lateral confinement. Non-equilibrium current or voltage noise across a weak point contact in the FQHE has been shown (theoretically) to provide a direct measurement of the fractional charge: for a constriction with good transmission properties, the noise originates from the infrequent tunneling of Laughlin quasiparticles [14]. The Kondo effect in 1 D interacting electron systems has been studied by several authors [15]. Persistent currents for interacting electrons in the ballistic regime have been predicted [16,17].

1 D electron systems coupled to small-momentum phonons have been considered on several occasions. 40 years ago, Wentzel [18] and Bardeen [19] used a similar model as a tentative candidate to explain the theory of conventional superconductivity. For a critical electron-phonon coupling constant, the system becomes *unstable*, and the specific heat diverges as one approaches this critical point [20]: the retarded interaction mediated by the phonons induces a collapse of the system. Below, we refer to this singular point as the Wentzel-Bardeen (WB) singularity. Nevertheless, this mechanism was not considered too seriously, perhaps because of the relatively large

values of the electron-phonon coupling constant needed to reach the singularity? Also, it is rather difficult in an experiment to tune this coupling constant to a desired value. An exactly solvable model similar to that of Ref. [20] was studied recently [21], with the conclusion that these small-momentum phonons can be ignored in typical metals since their effect is of the order $c^2/v_F^2 \ll 1$ (v_F is the Fermi velocity and c the sound velocity). Traditionally [22], thermodynamic quantities have been expanded in terms of this ratio. Consequently, the interest [23-25] shifted towards larger-momentum ($2k_F$) processes, where an electron is scattered by a phonon across the Fermi surface.

The point we wish to make is that when the electron system includes Umklapp and backscattering processes, the small-momentum phonons *do* play an important role nevertheless. It is no longer the ratio c/v_F which characterizes the corrections associated with the phonons. Rather, the velocity u_ρ associated with particle-hole excitations close to the Fermi level replaces the Fermi velocity v_F . The Fermi velocity still determines the energy scales of the electron Hamiltonian. It is only the fluctuations near the Fermi level which are described by u_ρ . For the Hubbard model in particular, u_ρ vanishes at half filling due to the presence of the insulating phase. Consequently, the coupling to small-momentum transfer modes (such as acoustic phonons) is a *non-perturbative* effect in the ratio c/u_ρ as one reaches the WB singularity. For the Hubbard model, the WB singularity is accessed for arbitrary electron-phonon coupling constant by merely increasing the filling factor towards half filling. Near this singularity, CDW and SDW fluctuations are suppressed totally, and the system is pushed into a metallic and finally superconducting phase by slightly increasing the filling factor. The presence of an intermediate phase between the SDW/CDW and superconducting regimes constitutes a novelty.

II. MODEL

Our starting point is the Hamiltonian $H = H_e + H_{ph} + H_{e-ph}$, where

$$H_{ph} = \frac{1}{2} \int dx [\zeta^{-1} \Pi_d^2 + \zeta c^2 (\partial_x d)^2] \quad (1)$$

describes the free phonons, with d the displacement field, Π_d its canonical conjugate and ζ the mass density. The electron-phonon coupling is [26]:

$$H_{e-ph} = g \sqrt{\frac{\pi}{2}} \sum_s \int dx \psi_s^\dagger \psi_s \partial_x d, \quad (2)$$

where g is the coupling constant, and $\psi_s = e^{ik_F x} \psi_{s+} + e^{-ik_F x} \psi_{s-}$ is the electron field operator. In terms of boson fields, $\psi_{s\pm}(x) \propto \exp[i\sqrt{\pi}(\pm\varphi_s(x) -$

$\int^z \Pi_s(x', \tau) dx')$] (here $x = (x, \tau)$) representing right and left moving electrons. The slow spatial variation of the electron density in Eq. (2) reduces to $\sum_s \psi_s^\dagger \psi_s = \sqrt{2/\pi} \partial_x \varphi_\rho$, with $\varphi_{\rho, \sigma} = (\varphi_\pm \pm \varphi_\mp)/\sqrt{2}$ the charge and spin fields (similarly for the canonical conjugate fields $\Pi_{\rho, \sigma}$). We neglect the fast oscillating terms in the density which can be compensated by the $2k_F$ phonons. Therefore, phonon mediated backscattering which transfers an electron from one side of the Fermi "surface" to the other is not included here, but (non retarded) backscattering from electron electron collisions is otherwise included. Later on in this paper, we will show that the present model can also be applied to electron wave guides, where different modes associated with lateral confinement interact with each other. The problems associated with the $2k_F$ phonons are not present there. Following Ref. [27], the electronic Hamiltonian is $H_{el} = H_\rho + H_\sigma$, with

$$H_\nu = \frac{1}{2} \int dx \left[u_\nu K_\nu \Pi_\nu^2 + \frac{u_\nu}{K_\nu} (\partial_x \varphi_\nu)^2 \right], \quad (3)$$

with $\nu = \rho, \sigma$. In principle, non-linear terms should be added to H_{el} . However, below half filling, these terms are irrelevant and can be accounted for by renormalizing the parameters K_ρ , K_σ , and the charge (spin) velocities u_ρ (u_σ). The phonons couple only to the charge degrees of freedom, and this property is preserved in the propagators calculated below. For free electrons, $u_\rho = u_\sigma = v_F$ and $K_\rho = K_\sigma = 1$.

III. CORRELATION FUNCTIONS

To calculate thermodynamic quantities, we use a functional integral representation of the partition function. As we will be calculating Green functions which involve fermion operators only, the phonon degrees of freedom can be integrated out right away. The resulting effective action for the charge degrees of freedom, reads in Fourier representation: $S_\rho = (2\beta L)^{-1} \sum_{\mathbf{k}} D_\rho(\mathbf{k}) |\varphi_\rho(\mathbf{k})|^2$ ($\mathbf{k} = (k, \omega)$), with the inverse propagator:

$$D_\rho(\mathbf{k}) = \frac{1}{K_\rho u_\rho} \left(\omega^2 + u_\rho^2 k^2 - \frac{b^2 k^4}{\omega^2 + c^2 k^2} \right), \quad (4)$$

where $b = g\sqrt{K_\rho u_\rho}/\zeta$. The first two terms of Eq. (4) represent the contribution of the free charge field, and the retarded, attractive coupling associated with the phonons appears in the third term. At the WB point the charge density propagator, $k^2/D_\rho(\mathbf{k})$, becomes proportional to ω^{-2} and signals an instability towards long wave-length density fluctuations.

A. Green function

We first consider the single-particle Green function:

$$G_s(x, \tau) = - \langle T_\tau \psi_s(x, \tau) \psi_s^\dagger(0, 0) \rangle, \quad (5)$$

where T_τ is the imaginary time ordering operator. Using the decomposition of the Fermi operators into right and left moving components, and after normal ordering, the calculation of the Green function is reduced to an evaluation of Gaussian integrals. Analytic expressions for the Greens function can be obtained for this particular model, and are found in Refs. [28,29]. The coupling between the charge degrees of freedom and the phonons induces an hybridization of the two excitations with characteristic velocities: $v_\pm^2 = [u_\rho^2 + c^2 \mp \sqrt{(u_\rho^2 - c^2)^2 + 4b^2}]/2$. At large distances, the Green function decays as a power law,

$$G_s(x, 0) \propto |x|^{-1-\delta}, \quad (6)$$

with $\delta = K_\sigma/4 + 1/(4K_\sigma) - 1 + B/(4K_\rho) + AK_\rho/4$. The electron-phonon parameters are defined by $A = u_\rho(1 + c^2/v_+v_-)/(v_+ + v_-) \geq 1$, and $B = u_\rho(1 + v_+v_-/u_\rho^2)/(v_+ + v_-) \leq 1$. These parameters play a crucial role in the discussion of the ordering fluctuations below. In the limit $g \rightarrow 0$, $A = B = 1$. Let us now consider the case $g \neq 0$, where $A > 1$ and $B < 1$ [29]: As u_ρ/K_ρ approaches the critical value $u_\rho^*/K_\rho^* = g^2/(\zeta c^2)$ from above, v_+^2 tends to zero, and v_-^2 to $u_\rho^{*2} + c^2$. As a result, the exponent A diverges and B decreases to the finite value $B^* = u_\rho^*/\sqrt{u_\rho^{*2} + c^2} < 1$. For $u_\rho/K_\rho < u_\rho^*/K_\rho^*$ the velocity v_+ becomes complex and the model becomes unphysical. Thus we must require that $u_\rho/K_\rho \geq u_\rho^*/K_\rho^*$, or equivalently that $b/(cu_\rho) \leq 1$, the equality sign defines the WB singularity [30]. Because $\delta > 0$, if $g \neq 0$ [29], the Green function decays faster than $1/x$, the free electron result. From the large distance behavior, we find the momentum distribution function near k_F , $N(k) \simeq N(k_F) - \kappa \text{sgn}(k - k_F) |k - k_F|^\delta$, with κ some constant of order one, and $N(k_F) = \Gamma(1/2 + \delta/2)/[2\sqrt{\pi}\Gamma(1 + \delta/2)]$. The jump at the Fermi surface disappears. The presence of phonons induces this Luttinger liquid behavior even without electron-electron interaction ($K_\rho = K_\sigma = 1$).

We plot the quantity δ for the Hubbard model (see Sec. III C) coupled to phonons in Fig. 1a and 1b. We first plot δ for a quarter filled band as a function of the on site repulsion U in Fig. 1a, for several values of the phonon coupling parameter. For small phonon coupling, δ increases monotonically with increasing U . As the phonon coupling is further increased, δ acquires a minimum. This is an early signature of the competition between these two couplings, and will be more apparent in the discussion of the phase diagram.

Next, we choose $U = 2$ and vary the filling factor from zero to half filling in Fig. 1b. δ increases dramatically when these two extremes are reached, as the correlation effects between electrons dominate the physics in both cases. In particular, $\delta \rightarrow \infty$ at the WB singularity.

B. Ordering fluctuations

The definitions for the ordering correlation functions $N(\mathbf{x})$, $\chi(\mathbf{x})$, $\Delta_s(\mathbf{x})$ and $\Delta_t(\mathbf{x})$ which describe CDW, SDW, singlet (SS) and triplet (TS) superconducting fluctuations are given in Refs. [5,6,29]. At large times/distances, these response functions behave like power laws. The signature for a particular ordering fluctuation to be present is given by the divergence of the Fourier transform of the corresponding response function at low frequency and small momentum relative to $q = 2k_F$ ($q = 0$) for N and χ (Δ_s, t). We thus obtain the following criteria [28,29]:

$$K_\rho A + K_\sigma \leq 2 \quad (\text{CDW}) \quad (7a)$$

$$K_\rho A + 1/K_\sigma \leq 2 \quad (\text{SDW}) \quad (7b)$$

$$B/K_\rho + K_\sigma \leq 2 \quad (\text{SS}) \quad (7c)$$

$$B/K_\rho + 1/K_\sigma \leq 2 \quad (\text{TS}) \quad (7d)$$

Since $A > 1$ and $B < 1$ for $g \neq 0$, we see that the Cooper instability is always present for non-interacting electrons coupled to phonons.

C. Hubbard model

We specialize to a Hubbard model for the correlated electrons with on site repulsion U and hopping term t . In this case $K_\sigma = 1$ because of SU(2) symmetry, and as a result the CDW and the SDW (the SS and the TS) response functions have apparently the same exponents (except at half filling). However, logarithmic corrections originating from marginally irrelevant operators in the spin channel favor SDW (TS) over (CDW) SS fluctuations [31]. The remaining parameters u_ρ and K_ρ of the Luttinger liquid Hamiltonian are obtained for arbitrary on-site repulsion U and filling factor n by solving two integral equations [27,29] describing the ground state properties [32] and the spectrum of charge excitations [33].

Our results are plotted in Fig. 2 a) and b) where we have chosen $c = a_0 t / \hbar$ (a_0 is the lattice constant): for fixed U/t (Fig. 2 a), we determine which fluctuations dominate as a function of n and an effective electron-phonon parameter $b/(u_\rho c)$: for convenience we consider in these plots $n(U)$ and $b/(u_\rho c)$ as independent parameters. For small $b/(u_\rho c)$, SDW (i.e. antiferromagnetic) ordering fluctuations dominate for arbitrary filling factor. As this parameter is increased, we reach (away from half filling) a region for which no response function diverges: we refer to this "phase" as the metallic region. At low U and $b/(u_\rho c) \ll 1$ analytical results confirm the existence of this intermediate phase [29]. By further increasing $b/(u_\rho c)$, the region where superconducting fluctuations dominate is reached. On the other hand, near half filling, the correlation effects suppress the superconducting

phase. For larger values of U , the superconducting region shrinks towards the region where $b/(u_\rho c) = 1$, and the SDW region grows as expected [29].

Next, we choose quarter filling ($n = 1/2$), and plot in Fig. 2 b) the phase diagram as a function of U and the phonon coupling. At low U , $K_\rho \approx 1$, and the system is superconducting because of the Cooper instability. As U is further increased, one crosses the metallic and the SDW (CDW) phase, because the phonon-mediated attractive interaction is overcome by the instantaneous repulsion between electrons.

D. WB singularity

In both Figs. 2 a) and b), the upper line $b/(u_\rho c) = 1$ corresponds to the WB singularity. We now discuss the relevance of this singularity for the Hubbard model. Plotting u_ρ/K_ρ for several values of U as a function of the filling factor (Fig. 3), we notice that u_ρ/K_ρ vanishes as one approaches half filling. (for small U , the curve develops a peak located close to $n = 1$, and the fall to zero is all the more drastic). Thus, the WB singularity at u_ρ^*/K_ρ^* can be reached for arbitrary values of the coupling constant g ($\neq 0$). Near half filling, as one approaches u_ρ^*/K_ρ^* from lower filling factors, the divergence in A triggers a dramatic suppression of the SDW (CDW) fluctuations, and the system is driven into the metallic phase [34]. Moreover, since $B \rightarrow B^*$ as $u_\rho \rightarrow u_\rho^*$, and $B^* \propto g^2$, the SS/TS condition $B \leq K_\rho$ can be met near half filling for sufficiently small $g \neq 0$ (since $1/2 \leq K_\rho \leq 1$). Hence, the system can finally be driven into the superconducting phase by approaching half filling. In summary, the conjunction of the coupling to small momentum acoustic modes with the reduction of the charge velocity near half filling provides a very efficient mechanism to suppress SDW (CDW) order, and to drive the system into a metallic and finally superconducting phase. This mechanism need not be limited to the 1D Hubbard model, but could occur in other situations where strong correlations play an important role.

We note that this change of phases as a function of filling factor is reminiscent of the behavior of high T_c materials under doping. Since it is the low-momentum and not the $2k_F$ phonons which are important here and since strong correlations are also present in 2D, it might not be unreasonable to hypothesize that the mechanism discussed here is also realized in higher dimensions.

IV. APPLICATION TO ELECTRON WAVE GUIDES

We now illustrate our results with an application to Mesoscopic quasi-1D wires. It is well known that the transverse states of the electron wave function in a 2 D

electron gas with a point contact are at the origin of the conductance quantization in point contact experiments [35]. In a 1 D wire, one can also adjust how many of these transverse states are populated by tuning the voltage on an electrostatic gate.

A. Interaction between modes

Consider the Coulomb interaction between two different branches: if d is the distance from the 2D electron gas to the metallic gate, this interaction is screened for distances $x > d$. The interaction potential between mode “ i ” and mode “ j ” has the form:

$$V_{ij} = \sum_{s,s'} \int dx \int dx' \rho_{i,s}(x) V(x-x') \rho_{j,s'}(x'), \quad (8)$$

with $\rho_{i,s}$ the density operator for electrons of mode i and spin s , and $V(x-x')$ the screened Coulomb potential. As we are concerned with low temperature, long wave length properties, we can replace this interaction by a delta function potential $V_0 \delta(x-x')$ with [36].

$$V_0 \simeq 2 \frac{e^2}{\epsilon} \ln(k_F d), \quad (9)$$

where ϵ is the dielectric constant of the medium between the 2D gas and the gate. The global Hamiltonian describing the set of coupled Luttinger liquids is therefore:

$$H = \sum_i (H_{i\rho} + H_{i\sigma}) + g \sum_{ij} \int dx (\partial_x \varphi_{i\rho}) (\partial_x \varphi_{j\rho}). \quad (10)$$

The first term corresponds to the set of uncoupled charge and spin fields associated with each modes, and the coupling term has striking similarities with the electron-phonon case. The phonon field we had previously is simply replaced by the charge field associated with another mode, but the spin degrees of freedom are uncoupled (as long as there are no magnetic impurities). A similar model was used in Ref. [36] in the discussion of the transport properties of a multimode wire through a point contact, and is equivalent to the multicomponent Tomonaga Luttinger model [37]. Also, the present interaction does not allow a transfer from one mode to the other, but rather the direct interaction as well as the exchange interaction between these modes. We come back to this point below.

Note that the fast spatial variation ($2k_F$) of the electron density can be safely neglected here. There is no compensation between two rapid density oscillations in analogy with $2k_F$ phonons, because each mode has a fixed Fermi velocity specified by the lateral confinement and the electron density.

Next, we choose the simplest situation: an electron density such that only two modes, the transverse ground

state and the first excited state, are present. This system is now an exact analog of the electron-phonon situation, because the spin degrees of freedom of each modes are uncoupled. Consequently, by increasing the coupling between the two modes, we can expect to induce superconducting fluctuations in one or both branches. The coupling can be varied in several ways: by considering several samples with different separation between the 2D gas and the metallic gates, by increasing the electron density in the fundamental mode to the analog of half filling, or by lowering the density such that the Fermi velocity of the first excited state is adequately close to zero. The latter two methods exploit the WB singularity.

The Green function exponent associated with electrons from chain i is then given by $1 + \delta_i = K_{\sigma i}/4 + 1/4 K_{\sigma i} + B_i/4 K_{\rho i} + K_{\rho i} A_{ij}/4$. In analogy with the phonon case, we have defined $A_{ij} = u_{\rho i} \left(1 + u_{\rho j}^2 / (v_+ v_-) \right) / (v_+ + v_-)$,

$$B_i = u_{\rho i} \left(1 + v_+ v_- / u_{\rho i}^2 \right) / (v_+ + v_-), \text{ and } v_{\pm} = (u_{\rho 1}^2 + u_{\rho 2}^2) / 2 \pm \sqrt{(u_{\rho 1}^2 - u_{\rho 2}^2)^2 + 4b^2} / 2.$$

The criterion indicating strong superconducting fluctuations in a given chain i is then translated from Eqs. (7c) and (7d) using the definitions for A_{ij} and B_i . The strong superconducting fluctuations induced by the inter-chain coupling could be observed in wires where only two channels propagate, or alternatively in one-dimensional coupled electron-hole systems. There are two additional correlation functions describing superconducting fluctuations for this particular system. These occur when a singlet or triplet Cooper pair is formed with one electron from chain 1 and one electron from chain 2. They are ignored here because the criteria for Cooper pairs formation within one branch are in general satisfied for a lower threshold phonon coupling.

The possibility of superconducting transitions in two channel systems could in principle be probed e.g. by studying the persistent current of a two channel Mesoscopic ring. Rings with few channels built from GaAs/AlGaAs heterostructures have been recently fabricated to study the persistent current in the quasi ballistic regime [38]. In the normal metal regime, the Fourier transform of the power spectrum [39] yields in general two peaks: one associated with the flux quantum, signaling the twist in the boundary condition of the electron wave function due to the flux which threads the ring, and another, smaller peak at half the flux quantum associated with weak localization effects. The flux causes a twist in the boundary conditions, but the bulk properties (such as the correlation function exponents) are unaffected. It is plausible that by varying the electron density with an overall electrostatic gate, one could induce strong superconducting fluctuations using the WB mechanism. In this regime, the peak associated with the single flux quantum will decrease while the $\phi_0/2$ com-

ponent will be enhanced due to the presence of Cooper pairs. Whether or not a strictly two channel ring can be achieved is still a technological challenge. If disorder associated with impurities and irregularities in the electrostatic confinement can be minimized, a two channel ring is effectively achieved just by “pinching” the ring with a point contact positioned along the ring. Moreover, self inductance effects between the two modes can be safely neglected as these effects have been shown to be proportional to the fine structure constant multiplied by the ratio of the Fermi velocity divided by the velocity of light [17].

B. Relevance of hopping between modes

The relevance of hopping between two *chains* (as opposed to modes) has been discussed on several occasions [27,40] and there is some controversy regarding this. While Ref. [40] states that a critical coupling between chains must be overcome before the Luttinger liquid phase becomes unstable, Ref. [27] uses a renormalization group argument to show that transverse hopping is always relevant.

In the situation we are concerned with, however, the hopping between chains is *already* included: the ground state is a symmetric combination of the two chain states, and the first excited state is an antisymmetric combination of these states. As a result, transitions between modes will only occur as a result of local imperfections in the confinement potential, and scattering with impurities. These processes are thus described by the potential:

$$V_h = \sum_{i,j,s} \sum_k t_k (\psi_{is}^\dagger(x_k) \psi_{js}(x_k) + \text{H.c.}), \quad (11)$$

with x_k ($k = 1, 2, \dots$) indicating the position of the imperfection/ impurity along the wire. The relevance of hopping between these two modes does not seem to cause a problem. However, a rigorous treatment should be provided, and is presently under way.

V. TRANSPORT PROPERTIES

We discuss the transport properties of a 1 D interacting electron system coupled to phonons, first for an ideal wire, and then in the limit of a weak scatterer and a weak link. Our analysis follows the work of Ref. [10].

A. Conductivity of an ideal wire

The conductivity is given by the Kubo formula:

$$\sigma(q, \omega) = \frac{1}{i\omega} \left[i \int dx \int dt e^{i\omega t - iqx} \langle j_D(x, t) j_D(0, 0) \rangle - \frac{2}{\pi} u_\rho K_\rho \right], \quad (12)$$

With the second term representing the contribution of the diamagnetic part of the current operator, and $j_D \equiv \sqrt{2/\pi} u_\rho K_\rho \Pi_\rho$ the paramagnetic current. To calculate this quantity, it is useful to perform a Wick rotation to imaginary times. The calculation follows that of Ref. [41] for spinless Fermions. The diamagnetic part is cancelled by contributions coming from the second term in Eq. (12), and the only remaining contribution is proportional to $\langle \partial_\tau \varphi_\rho(x, \tau) \partial_\tau \varphi_\rho(0, 0) \rangle$. At zero temperature, the conductivity becomes:

$$\sigma(q, \omega) = \frac{2iu_\rho K_\rho}{\bar{\omega}} \left(\frac{\bar{\omega}^2(\bar{\omega}^2 + c^2 q^2)}{(\bar{\omega}^2 + v_+^2 q^2)(\bar{\omega}^2 + v_-^2 q^2)} \right)_{\bar{\omega}=i\omega} \quad (13)$$

which can be compared to the uncoupled case (no phonons):

$$\sigma_0(q, \omega) = \frac{2iu_\rho K_\rho}{\bar{\omega}} \left(\frac{\bar{\omega}^2}{\bar{\omega}^2 + u_\rho^2 q^2} \right)_{\bar{\omega}=i\omega} \quad (14)$$

Note that for large momentum, the conductivity does not depend on the phonon parameters:

$$\lim_{q \rightarrow 0} \sigma(q, \omega) = \frac{2u_\rho K_\rho}{\pi} \left[i\mathcal{P}(\omega) + \pi\delta(\omega) \right]. \quad (15)$$

In the DC case, a finite quantity can therefore be obtained by considering the quantity $\lim_{\omega \rightarrow 0} \omega \sigma(\omega)$.

The result for the zero temperature DC conductivity are rather dull: at finite temperatures, however, effects associated with the phonons should survive, even in the limit $q \rightarrow 0$. The finite temperature dependence can be obtained from conformal invariance arguments [42], but will not be considered here.

B. Conductance of an ideal wire

We now consider a quantity which can be obtained directly from an $I - V$ measurement on a small wire: the conductance. The Landauer formula [43] provides a simple way to relate the conductance of a Mesoscopic sample to the quantum mechanical transmission properties of this sample. While some generalizations of this formula to specific systems of interacting electrons have been proposed [44], its main application is for non interacting electrons. Fisher and Lee [45] have introduced a general expression relating the transmission properties of the sample to the Greens function: this approach assumes that a constant electric field is applied to a portion of wire with a finite length L , and calculates in the linear response regime the resulting current in the DC limit.

This approach has drawn some criticism [46] because it does not describe an open system (the leads to which the sample is connected are not taken into account). Nevertheless, the Fisher Lee formula will be used here, as it provides a systematic way to generalize single electron results to arbitrary many body systems. The end product can be compared to the free electron case at any stage of the calculation. The Fisher-Lee formula reads:

$$G = \lim_{\omega \rightarrow 0} \frac{1}{\hbar\omega L} \int d\tau \int_0^L dx \langle T_\tau \partial_\tau \varphi_\rho(x, \tau) \partial_\tau \varphi_\rho(0, 0) \rangle \times e^{i\omega\tau} . \quad (16)$$

The thermodynamic limit, if necessary, is taken only once the DC limit $\omega \rightarrow 0$ has been specified.

For a Luttinger liquid coupled to phonons, the result is:

$$G = \frac{2e^2}{h} K_\rho A . \quad (17)$$

Note that since $A > 1$, the conductance of this system is always enhanced by the phonon coupling. In particular, for free electrons with $K_\rho = 1$, the conductance associated with an electron channel is larger than that associated with the conductance quantum. Using once again the analogy with coupled electron systems of Sec. IV, we could envision an experiment performed in the integer quantum Hall regime, where the number of edge states propagating on the boundaries of the sample correspond to the number of filled bulk Landau levels. Edge states can be described by chiral Luttinger liquids [47], as for a given edge the direction of propagation is fixed by the $E \wedge B$ drift (E is the electric field associated with the confining potential). In the IQHE, each (isolated) edge state has a Fermi liquid behavior because the Luttinger liquid parameter K_ρ takes the marginal value $K_\rho^* = 1$. In a situation with only two Landau level filled, and assuming that the separation between edge states 1 and 2 is comparable to the magnetic length (so that the interaction between edge states is noticeable), Eq. (17) predicts a conductance which is *larger* than the free electron value!

C. Weak Barriers

The effects of impurity scattering for a weak barrier can be analyzed following the perturbative renormalization group treatment of Kane and Fisher [10]. Weak scattering is described by an additional term in the Hamiltonian:

$$\delta H = \sum_s \int dx v(x) \psi_s^\dagger(x) \psi_s(x) \quad (18)$$

For simplicity, we assume that $v(x)$ is short ranged and centered at $x = 0$, so that the fields away from the impurity can be integrated out. Moreover, it was argued

in Ref. [10] that the renormalization group will generate higher order terms in the perturbation series so that the most general effective action associated with the perturbation becomes:

$$\delta S = \int d\tau \sum_{n_\rho + n_\sigma \text{ even}} v(n_\rho, n_\sigma) \times \exp(\sqrt{\pi}(n_\rho \varphi_\rho(0) + n_\sigma \varphi_\sigma(0))) \quad (19)$$

where $v(n_\rho, n_\sigma)$ is the coupling strength associated with the transfer of n_ρ electron charges and n_σ electron spins across the weak barrier. A perturbative renormalization group analysis of the partition function then leads to the flow equations:

$$\frac{\partial v}{\partial l}(n_\rho, n_\sigma) = \left(1 - \frac{n_\rho^2}{2} AK_\rho - \frac{n_\sigma^2}{2}\right) v(n_\rho, n_\sigma) . \quad (20)$$

In particular, in the case where one electron is transferred, we see that v_{11} is relevant when $AK_\rho < 1$, and the system flows towards the strong coupling (large barrier) behavior. As was seen in Sec. III B, $AK_\rho < 1$ corresponds to the criterion for SDW (CDW). The temperature dependence of the conductivity can be obtained perturbatively:

$$G(T) = \frac{2e^2}{h} \left(AK_\rho - \sum_{n_\rho, n_\sigma} c(n_\rho, n_\sigma) T^{n_\rho^2 + n_\sigma^2 AK_\rho - 2} + \dots \right) \quad (21)$$

This expression seems to imply that the conductivity diverges at low temperature in the SDW (CDW) regime. However, this is the regime where the perturbation expansion breaks down, and we have to use a strong barrier theory to analyse this case.

D. Strong Barriers

The strong barrier situation can also be addressed because of the duality properties of the action. The action can be written in terms of the field φ_ν , or alternatively in terms of the fields $\theta_\nu(x, \tau) = \int^x dx' \Pi_\nu(x', \tau)$. A large barrier is described by the hopping term:

$$\delta h = \sum_s t(\psi_{+s}^\dagger(x=0) \psi_{-s} + \text{H.c.}) . \quad (22)$$

Including the multiple hopping processes, the additional term in the action becomes [10]:

$$\delta S = \sum_{n_\rho + n_\sigma \text{ even}} t(n_\rho, n_\sigma) \int d\tau \cos(n_\rho \sqrt{\pi} \Pi_\rho) \times \cos(n_\sigma \sqrt{\pi} \Pi_\sigma) , \quad (23)$$

with the fields evaluated at $x = 0$. The perturbative renormalization group analysis then generates the flow equations:

$$\frac{\partial t}{\partial l}(n_\rho, n_\sigma) = \left(1 - \frac{n_\rho^2 B}{2 K_\rho} - \frac{n_\sigma^2}{2}\right) t(n_\rho, n_\sigma) \quad (24)$$

Again for the case of single electron hopping, t_{11} becomes relevant when $B/K_\rho < 1$, which corresponds to the superconducting regime of Sec. III B. The temperature dependence of the conductance is given by:

$$G(T) = \frac{2e^2}{h} \sum_{n_\rho, n_\sigma} c'(n_\rho, n_\sigma) T^{(B/K_\rho)n_\rho^2 + n_\sigma^2 - 2} + \dots \quad (25)$$

E. Phase diagram

It is tempting to unify the weak and strong barrier regimes in the phase diagram of Fig. 4 a). In this figure, the dashed line at $K_\rho = 1$ represents the marginal line which separates the strong and weak relevant regime for electrons with no phonons in Ref. [10]. The dashed-dotted lines give the direction of the flow. The introduction of phonon coupling modifies the phase diagram significantly. First of all, the region where strong coupling is relevant shifts towards lower values of K_ρ , as the coupling to phonons compensates partly the repulsive interaction between electrons. Because $AB > 1$, the regions where v and t are irrelevant overlap, in sharp contrast with the case where no phonons are present. This is the manifestation of the intermediate (M) phase, where it was shown for the Hubbard model that no response function diverges. We therefore expect three different regimes: a) the SDW (CDW) regime, where weak scattering is relevant and strong barriers are irrelevant. The system is an insulator, as the conductance $G(T) \sim T^{B/K_\rho - 1}$ vanishes at low temperature. The I-V characteristic is then non linear, $I(V) \sim V^{B/K_\rho}$. b) the superconducting regime, where weak scattering is irrelevant, and strong barriers are relevant perturbations: at low temperatures, the conductance is essentially that of an ideal wire, and the I-V characteristic is linear. c) the intermediate (M) regime, where it is unfortunately not possible to get an analytic expression for the temperature dependence of the conductance. The conductance will behave either as in case a) or as in case b), depending on the value of K_ρ and the coupling constants v and t . For $1/A < K_\rho < B$, it is at present not possible to obtain the exact location of the marginal line, because no rigorous treatment seems possible in this region.

As one tunes the parameters of the system close to the WB singularity, some spectacular effects are expected. This case is illustrated in Fig. 4 b): at low v , the marginal line moves all the way to $K_\rho = 0$, while at low t it stops moving at $K_\rho = B^*$, with B^* defined in Sec. III A. Weak coupling becomes irrelevant regardless of the strength of the electron electron interaction, and the conductance diverges.

Note that by considering the transport properties of the coupled electron-phonon system, we have in fact recovered all the physics derived with the help of the correlation functions of Sec. III B. The transport properties thus provide us with a diagnostic tool to analyse the "phase" specified by the parameters of the model, as the perturbative renormalization group yields exact results in this particular case.

VI. CONCLUSION

We have analyzed the role of the WB singularity in a system of interacting electrons coupled to phonons in 1D. The Luttinger liquid description allowed us to get exact results for the correlation functions which in turn determine the dominant type of fluctuations. For the Hubbard model, the competition between the repulsive interaction and the attractive retarded interaction mediated by the phonons has been clearly displayed. An intermediate phase separates the SDW (CDW) from the superconducting regimes. By increasing the electron density towards half filling, the presence of the WB singularity suppresses SDW (CDW) order and pushes the system towards the superconducting phase. This could be checked in systems where two electron branches are coupled to each other, where complications associated with $2k_F$ retarded phonon processes are absent. The coupling to phonons affects the transport properties, enhancing the conductivity, and suggesting that it is possible to achieve in certain cases a conductance per channel larger than the conductance quantum. The analysis of the effect of disorder allows to recover the three regimes predicted by the correlation functions: an insulating regime and a superconducting regime (with finite conductance) separated by an intermediate phase. The insulating regime disappears totally at the WB point, and the conductance becomes infinite.

There are still many standing issues. First of all, is it conceivable that the WB singularity survives an additional $2k_F$ phonon interaction? For electron bandwidths which are sufficiently small that the retardation effects of $2k_F$ phonons can be neglected, the answer is yes, because then the on site repulsion parameter U is simply shifted towards lower values. Nevertheless, in the general case, no answer is available at the moment. Second, it seems quite plausible that the WB singularity could exist also in higher dimensional systems. However, in general 2 and 3 D systems do not possess the luxury of an exact solution, which renders the analysis more difficult. Finally, the analogy with electron wave guides suggests that it would also be interesting to study the WB singularity for a wire with an arbitrary number of coupled modes.

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FIG. 1. Green function exponent for the Hubbard model: a) at fixed filling factor, as a function of $u = U/t$ for $b/cu_{\rho} = 0$ (solid line), $b/cu_{\rho} = 0.2$ (dotted line), $b/cu_{\rho} = 0.4$ (dashed line) and $b/cu_{\rho} = 0.6$ (dashed-dotted line). b) for fixed u , as a function of filling factor and for the same values of b/cu_{ρ} as in a).

FIG. 2. a) Phase diagram of the Hubbard model coupled to phonons, as a function of filling factor and phonon coupling constant b/cu_{ρ} , for $U/t = 0.3$. b) Phase diagram of the Hubbard model coupled to phonons at quarter filling ($n = 1/2$), as a function of U/t and b/cu_{ρ} .

FIG. 3. Plot of u_{ρ}/K_{ρ} as a function of the filling factor n . On the left hand side, we have from top to bottom $U = 16, 8, 4, 2$. Note the abrupt change for small values of U as one approaches half filling.

FIG. 4. Flow diagram for the transport properties of a Luttinger liquid coupled to phonons: a) for an arbitrary electron-phonon coupling. $K (= K_{\rho})$ is the Luttinger liquid parameter, v is the weak barrier coupling, and t is the hopping strength. b) same as a), but for parameters corresponding to the WB singularity ($A \rightarrow \infty$ here).

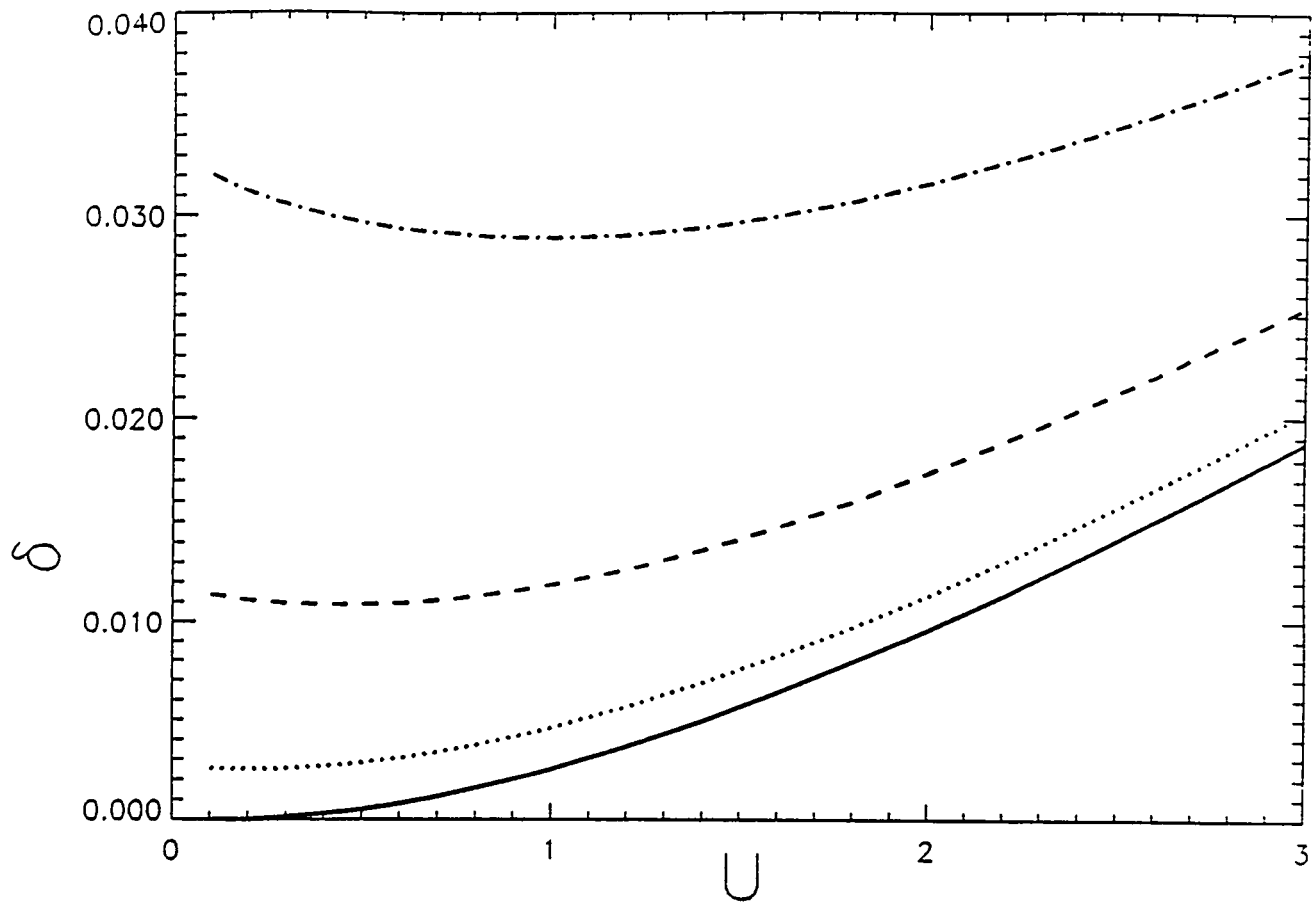


fig 1 a)

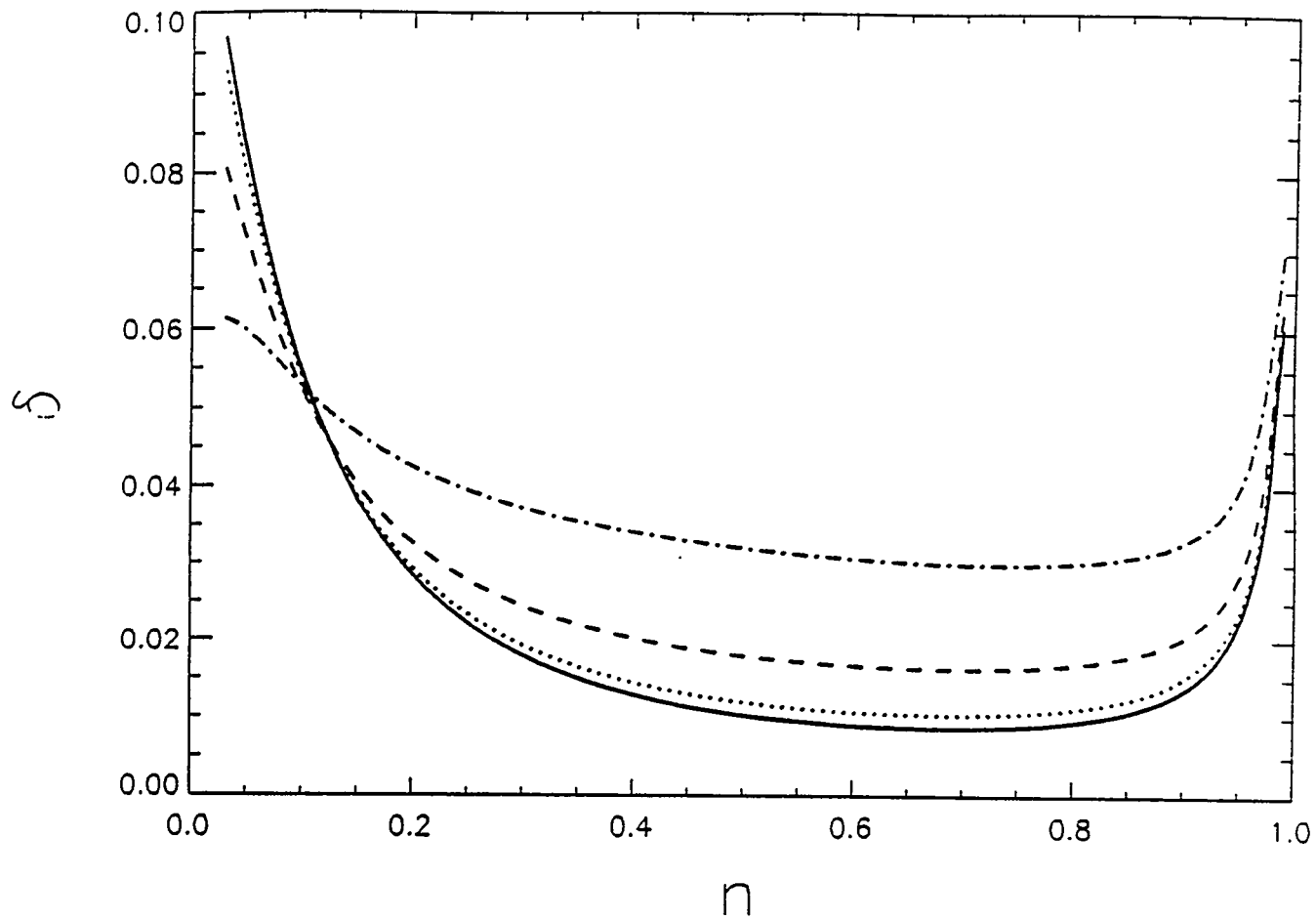
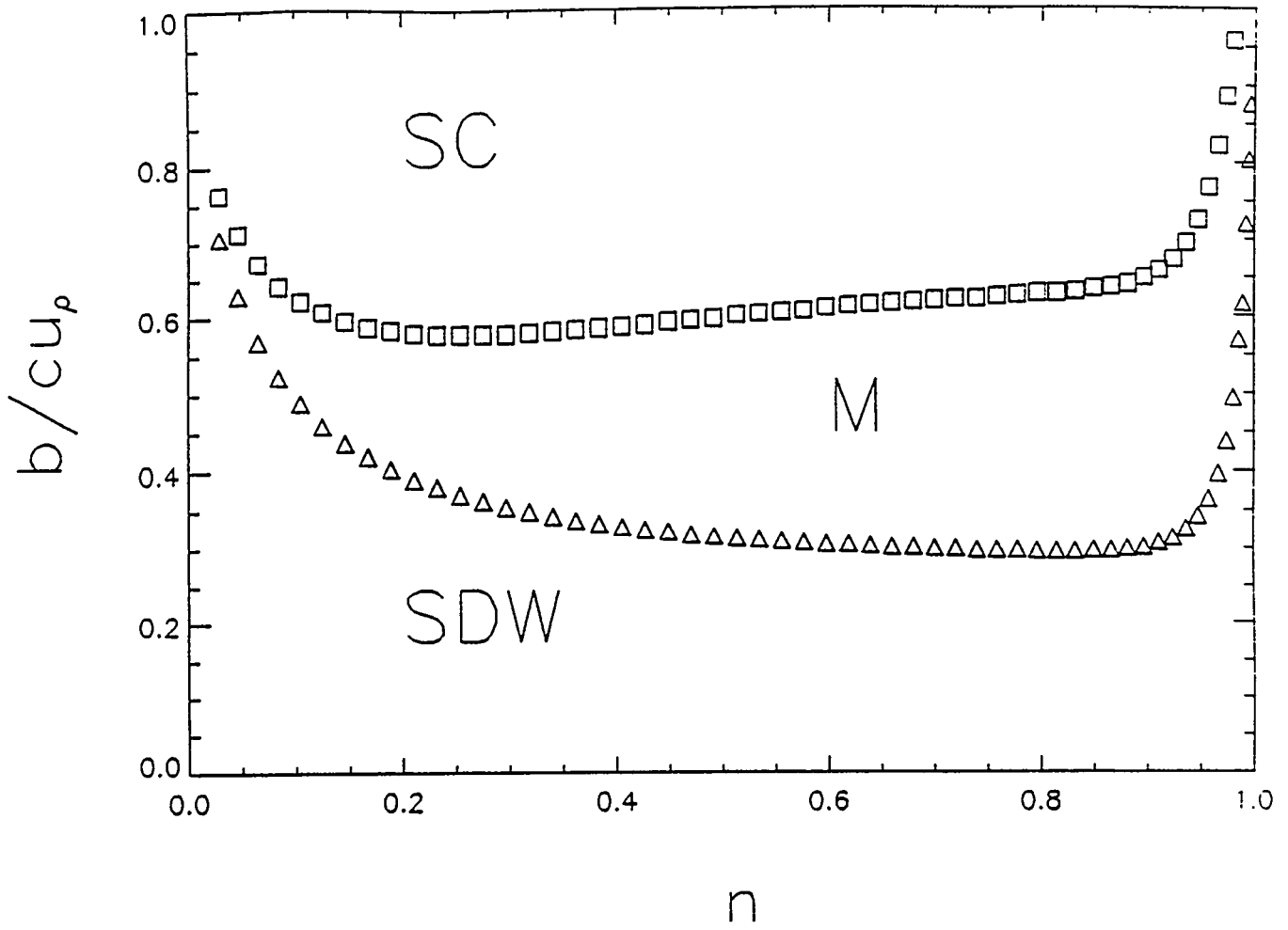
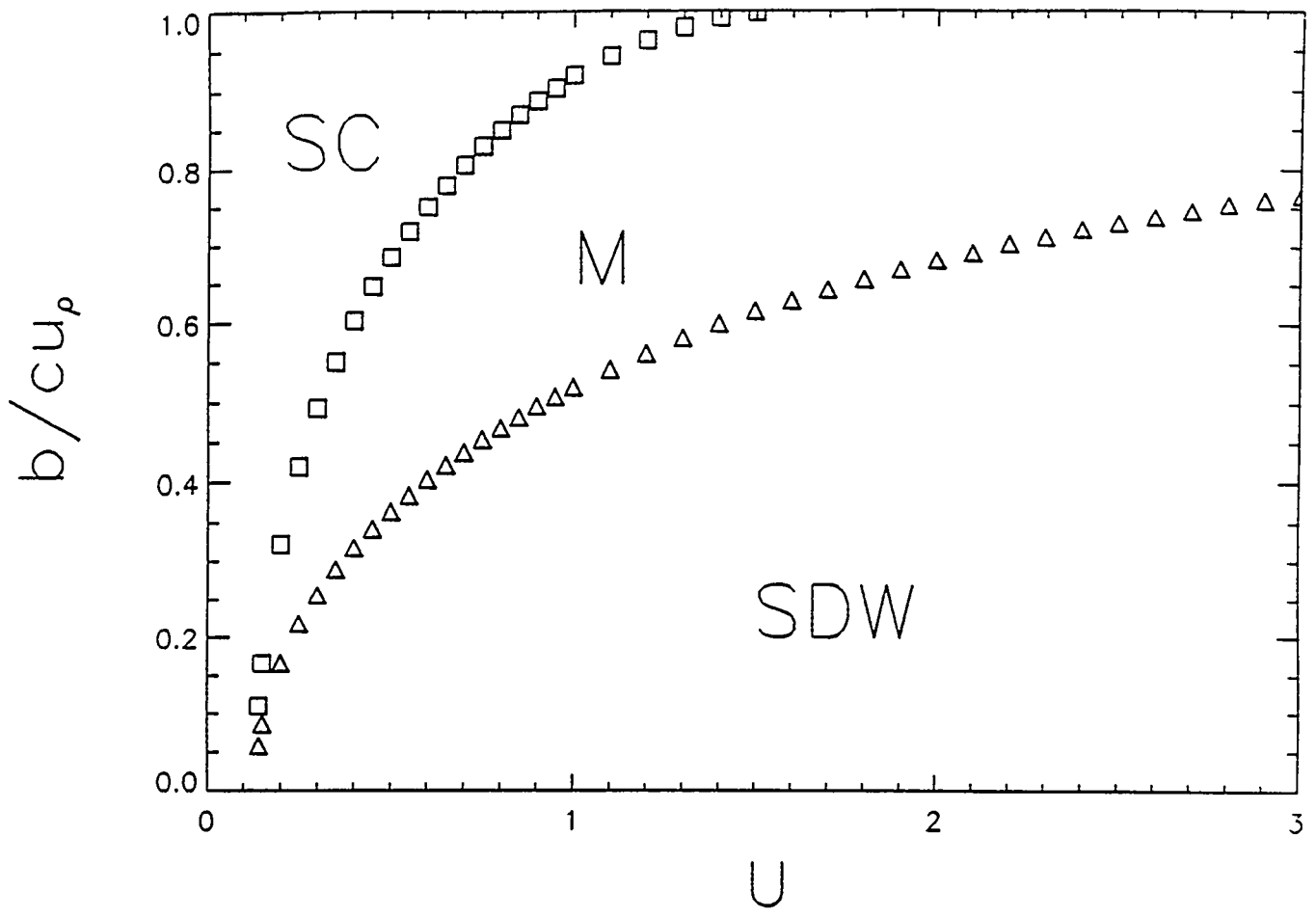


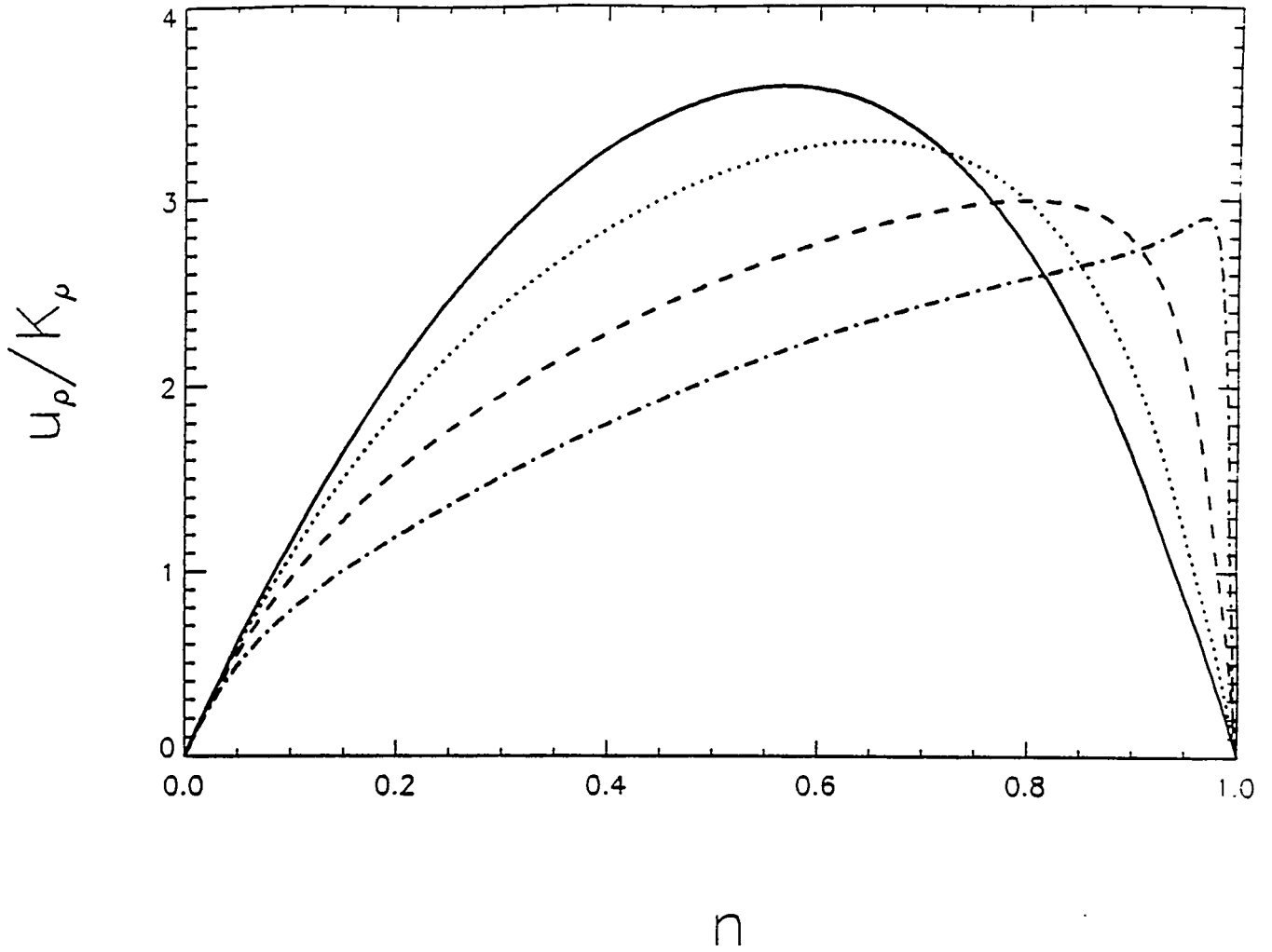
fig. 1 b)



2 a)



2 b)



3

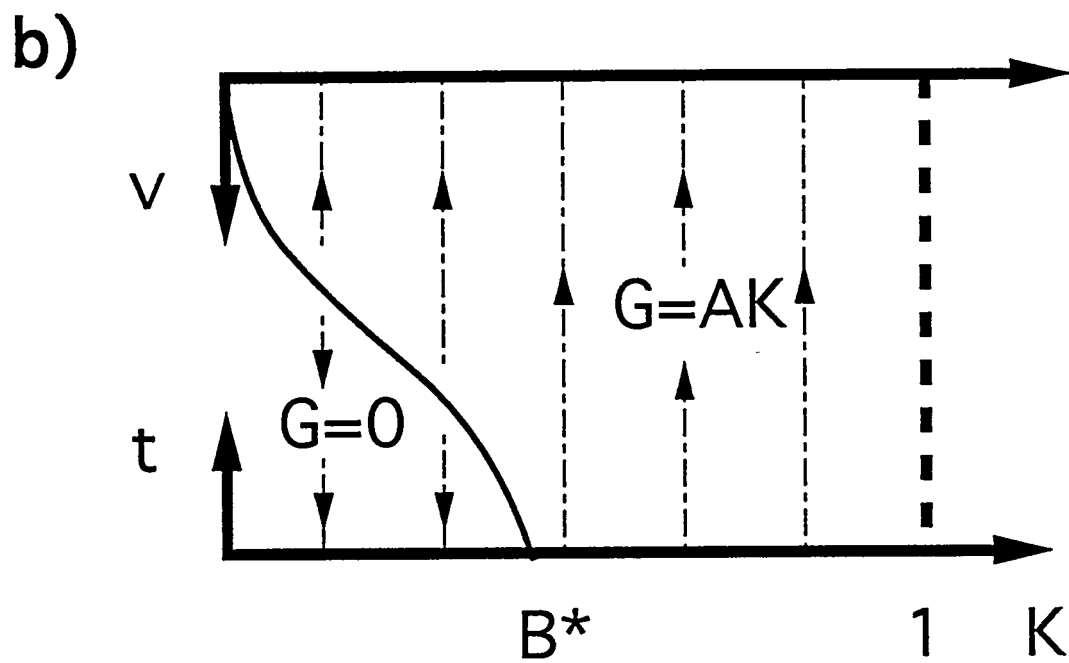
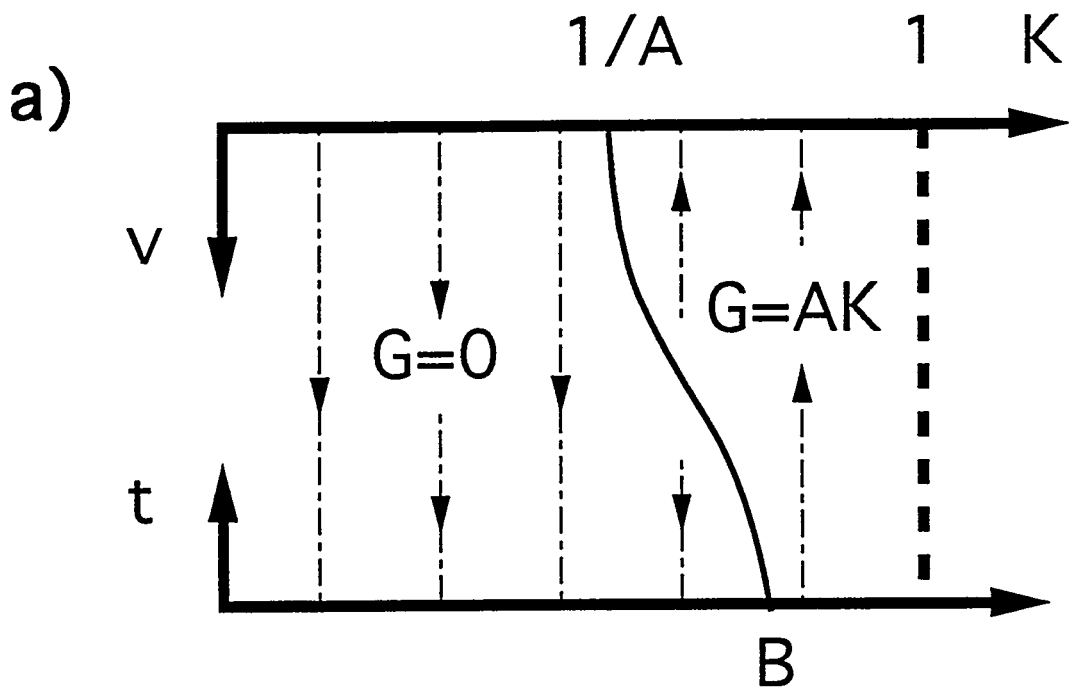


Fig 4.