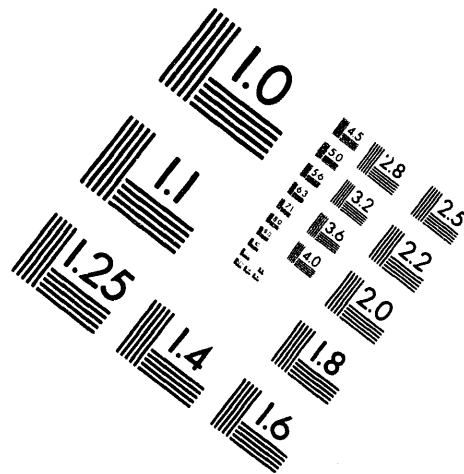
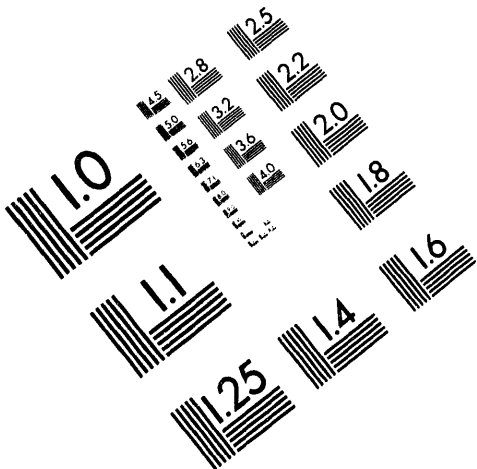




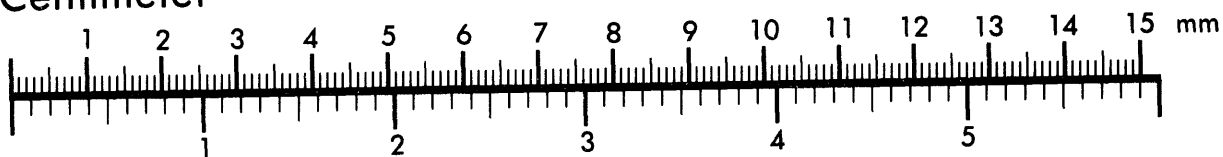
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Association for Information and Image Management

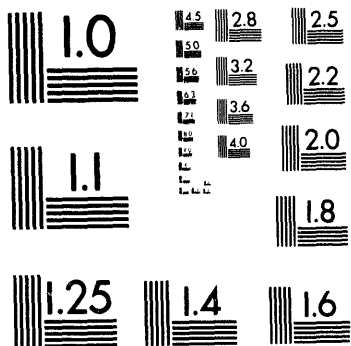
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Silver Spring, Maryland 20910
301/587-8202



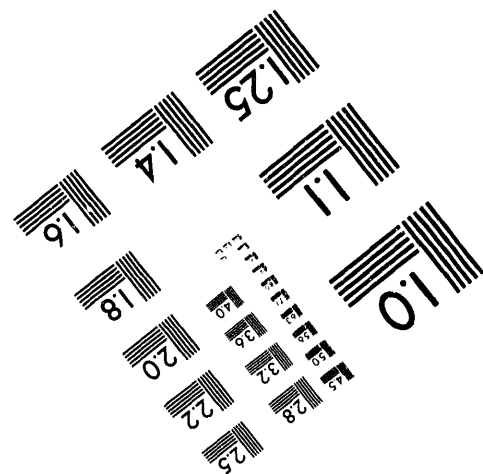
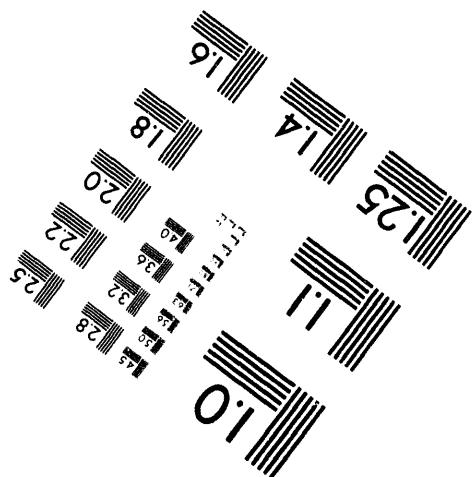
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1 of 1

NONLINEAR MAGNETOHYDRODYNAMICS

Progress Report

for Period July 1, 1993 - June 30, 1994

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July 30, 1994

Prepared for

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MASTER

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ABSTRACT

Work has continued to focus on resistive, viscous, magneto-hydrodynamic (MHD) steady states that model tokamak configurations. Recent emphasis has been on the subject of plasma rotation, and the stabilizing effects it has on the kind of MHD activity that results when current thresholds are exceeded in non-rotating configurations. We believe that relatively superficial consequences of the effects of rotation (e.g., the "velocity shear layer," which must result when any fluid of whatever nature is rotated in the presence of a material boundary) have been assigned causative effects that do not belong to them, in the presently-dominant perspective on the subject. Our three-dimensional spectral-method numerical code has shown how rotation may be made to suppress helical deformations of the current channel and paired helical vortices in a supercritical magnetofluid column. A velocity "shear layer" results if and when there is wall friction. The role of ion parallel viscosity (rather than shear viscosity) in determining stability boundaries in current-carrying magnetofluids is being investigated. A lattice-Boltzmann equation method of computing three-dimensional magnetohydrodynamic toroidal effects is under consideration.

ACTIVITY OVER THE PAST YEAR (1993-94)

Our effort continues to focus on resistive, viscous, MHD steady states which model tokamak configurations, and what happens to those steady states as currents are increased to values close to and above their stability thresholds. The new focus within the last year is on the combined effects of plasma rotation and wall friction. The rotation is seen as a consequence of slight charge non-neutrality, which may result from a variety of causes, and perhaps from several causes within a single device. The wall friction is an inevitable consequence of interaction of any rotating fluid, plasma or otherwise, with a material boundary. The resulting flow pattern has been well documented by our three-dimensional spectral-method computations in an upcoming paper in Journal of Plasma Physics [the abstracts of this and other papers, previously transmitted to DoE, are appended to this progress report; Abstract 10 in the list of "Refereed Publications" is the one now being alluded to].

This set of computations have been motivated by the observation that the gross dynamics of the "L to H transition" are

still manifestly not understood, despite the hundreds of publications on the subject that have appeared. The properties of the tokamak plasma in the "L mode" and the "H mode" are so drastically different, and one is so superior to the other from the point of view of confinement, that it has come to us to seem foolhardy to be building advanced, reactor-grade devices such as ITER without having understood the basic physics of the conditions which produce the "H mode." Our analytical theory and (fully 3D, spectral, MHD) numerical computation has been largely directed toward this end.

Our focus has been not on the microscopic kinetic theory origins of the departure from charge neutrality, but instead has been on the bulk MHD behavior it leads to: rotation and consequent stabilization. Rotation has long been known as a source of rigidity in Navier-Stokes fluids, and it should not have come as a surprise that something similar is true in MHD. The net effect of sufficient non-neutrality in the absence of wall friction we showed in a paper some time ago (1991) to be "spin up," or a rigid rotation of the plasma [see also Abstract 4, in the following]. However, that development was limited to the case of uniform resistivity and no wall friction, both necessary for realism. The work of the last year has removed both of those limitations. The papers we have generated in 1994 (in press at journals and given as papers at the Sherwood Theory Meeting in Dallas and the Tokamak Transport Task Force Workshop) have shown how to remove these limitations. We are now able to estimate the amount of rotation required to induce the MHD version of the "H mode" above threshold, and for the Hartmann numbers we are able to investigate, it turns out typically to require a very few per cent of the toroidal Alfvén speed. It seems imperative to us to begin the systematic experimental study of plasma rotation in confinement devices and the possibilities for systematically producing and manipulating it.

We have been strongly concerned with two other topics in global MHD: (1) the difficulties of finding fully toroidal (as opposed to cylindrical) resistive MHD equilibria in the absence of flow (velocity fields); and (2) possibilities for maximum-entropy current profile predictions of current-carrying MHD steady states. Accepted manuscripts on both [Abstracts 5 and 6 in the list of refereed publications] have already been transmitted to DoE. Our focus on these topics is continuing.

We have begun a series of numerical computations of the effects of "ion parallel" viscosity (in contrast to the much smaller "shear" viscosity) on the onset of MHD activity near critical current thresholds.

Xiaowen Shan is attempting to develop a toroidal lattice-Boltzmann numerical code for computing 3D MHD activity inside a toroidal boundary. He is currently spending three months at the Air Force Phillips Laboratory at Hanscom Field in Massachusetts, where he has access to the massively-parallel CAM-8.

David Montgomery, the Principal Investigator, intends to spend the month of September, 1994, working with Niek Lopes Cardozo at the Rijnhuizen Laboratory in the Netherlands, and October through December in Los Alamos, dividing his time there between the Center for Nonlinear Studies and the T-13 ("Complex Systems") group. One principal activity of this period will be to write an article on nonlinear MHD for Reviews of Modern Physics.

PLASMA PHYSICS ABSTRACTS, 1993-1994

In the nature of a progress report, the following is a collection of abstracts of publications supported in part by U.S. Dept. of Energy Grant DE-FG02-85ER53194 during 1993-1994. Copies of the listed papers and preprints have been transmitted in their entirety to D.O.E.

A. Refereed Publications

1. X. Shan and D. Montgomery
ON THE ROLE OF THE HARTMANN NUMBER IN MAGNETOHYDRODYNAMIC ACTIVITY
Plasma Phys. & Contr. Fusion 35, 619-631 (1993).

Magnetohydrodynamic activity near the threshold of instability is studied numerically for resistive equilibria in periodic cylinders. Attention focuses on the role of the viscosity (and its reflection in the Hartmann number) in determining the existence and nonlinear evolution of instabilities. Three identical axisymmetric resistive equilibria without flow are investigated, with three viscosities. The highest viscosity leads to stability of the axisymmetric state. The intermediate value leads to a helically deformed equilibrium with flow, with an $(m, n) = (2, 1)$ deformation. The lowest viscosity leads to a 'mixed' helically deformed, final, approximately steady state with flow, with $(m, n) = (2, 1)$ and $(3, 2)$ deformations and their harmonics, plus other, adjacent, modes near $m = 2n$. It is concluded that it is likely that the numerical value of the viscosity, and possibly the form of the viscous stress tensor, must never be ignored in discussions of near-threshold incompressible MHD activity in driven, dissipative MHD plasmas.

2. X. Shan and D. Montgomery
GLOBAL SEARCHES OF HARTMANN-NUMBER-DEPENDENT STABILITY BOUNDARIES
Plasma Phys. & Contr. Fusion 35, 1019-1032 (1993).

A numerical technique is developed for searching for the stability boundary for a resistive, straight-cylinder, magnetohydrodynamic equilibrium with spatially-dependent resistivity. For fixed aspect ratio, the boundary is a curve in the plane whose axes are Hartmann number and pinch ratio (or reciprocal of the safety factor at the wall). The technique is spectral and utilizes orthonormal eigenfunctions of the curl. Nonlinear behavior above the stability boundary is computed for a particular profile, using a nonlinear version of the code.

3. D. Montgomery
HARTMANN, LUNDQUIST AND REYNOLDS: THE ROLE OF DIMENSIONLESS NUMBERS IN NONLINEAR MAGNETOFLUID BEHAVIOR
Plasma Phys. & Contr. Fusion 35, B105-B113 (1993).
[Also presented as invited review lecture, European Physical Society Meeting, Plasma Physics Division, Lisbon, July 28, 1993].

Despite its limitations, magnetohydrodynamic theory remains the best possibility for a predictive framework for the large-scale dynamics of a magnetized plasma. The mathematical structure is very similar to that for Navier-Stokes fluids, and indeed contains fluid dynamics as a special case. Fluid dynamics would not have gotten very far without understanding the crucial role played by dimensionless numbers (such as the Reynolds number) in classifying its regimes of different kinds of behavior. The situation seems to be much the same in magnetohydrodynamics. In particular, the Hartmann number, familiar in the theory of MHD power generation, seems to be the crucial number describing the onset of MHD activity in voltage-driven dissipative equilibria that model such confinement devices as tokamaks. Stability thresholds are calculable and the supercritical behavior above those thresholds may be quantitatively compared with the numerical computations of Shan et al (1991a, 1993a,b).

4. H. Chen and D. Montgomery
EQUILIBRIUM PROPERTIES OF A ROTATING PLASMA: DIFFERENCES BETWEEN THE FLUID VELOCITY AND THE DRIFT VELOCITY
J. Plasma Phys. 49, 341-356 (1993).

An equilibrium state of a magnetized non-neutral plasma confined in a smooth cylindrically symmetric container is obtained. The particle number of each species is fixed, and total energy and total canonical angular momentum are conserved in the system. Moreover, the most probable state satisfies the stationary Vlasov equation exactly. The key finding from this result is that the $\mathbf{E} \times \mathbf{B}$ drift velocity can be significantly different from the true fluid velocity, which always corresponds here to a rigid rotation. It is suggested that this difference may also be experimentally important in situations that are not in thermal equilibrium.

5. D. Montgomery and X. Shan
TOROIDAL RESISTIVE MHD EQUILIBRIA (Los Alamos Report LA-UR-93-4446)
[accepted for publication by Comments on Plasma Physics & Controlled Fusion, 1994.]

The properties of ideal magnetohydrodynamic (MHD) equilibria without flow in periodic cylinders have recently

been shown to differ significantly from equilibria with small but finite resistivity and applied external voltages. It is shown here that these differences may be even more significant for magnetofluids in tori than they are for magnetofluids in periodic cylinders. Toroidal axisymmetric resistive equilibria without flow, under a standard set of assumptions and symmetries, are shown not to exist for the case of spatially-uniform resistivity.

Key Words: *resistive MHD equilibrium, toroidal magnetofluids.*

6. D. Montgomery and X. Shan
DETERMINATION OF CURRENT PROFILES IN CONFINED MAGNETOFLUIDS
[accepted for publication by Comments on Plasma Physics & Controlled Fusion, 1994.]

Steady-state solutions of the magnetohydrodynamic (MHD) equations are the most common mathematical models for the configurations of magnetically-confined plasmas. Considerable uncertainty remains, however, as to just what constitutes a physically acceptable "MHD equilibrium." There are at least three distinct candidates. The possibilities for ideal equilibria, with the electrical conductivity assumed infinite, are essentially unlimited: an infinity of possible (Grad-Shafranov) solutions exist corresponding to a given set of boundary conditions. Resistive equilibria, on the other hand, incorporating Ohm's law and Faraday's law from the beginning, appear to be unique for a given set of boundary conditions. The only non-uniqueness in the solutions seems to be associated with bifurcations at instability thresholds, when the static axisymmetric equilibrium branch may be replaced by preferred branches with finite velocity fields and different (helical) symmetries. Finally, there is the possibility that current profiles in MHD may select themselves as "most probable" states that maximize information-theoretic entropies, subject to constraints provided by exact or approximate conservation laws. Originally formulated as a way of discriminating among ideal equilibria, maximum-entropy methods have recently shown predictive power in the case of dissipative computations of turbulent relaxation to quasi-steady states in Navier-Stokes fluids and/or guiding-center plasmas. The intent of this note is to bring into focus the differences between, and the possible connections among, these three differing ways of selecting MHD equilibria.

Key words: *MHD equilibria, maximum-entropy methods, "most probable" states, profile consistency, resistive steady states.*

7. D. Montgomery, X. Shan, and W.H. Matthaeus
NAVIER-STOKES RELAXATION TO SINH-POISSON STATES AT FINITE
REYNOLDS NUMBERS
Phys. Fluids A₅, 2207-2216 (1993).

A mathematical framework is proposed in which it seems possible to justify the computationally-observed relaxation of a two-dimensional Navier-Stokes fluid to a "most probable," or maximum entropy, state. The relaxation occurs at large but finite Reynolds numbers, and involves substantial decay of higher-order ideal invariants such as enstrophy. A two-fluid formulation, involving interpenetrating positive and negative vorticity fluxes (continuous and square integrable) is developed, and is shown to be intimately related to the passive scalar decay problem. Increasing interpenetration of the two fluids corresponds to the decay of vorticity flux due to viscosity. It is demonstrated numerically that, in two dimensions, passive scalars decay rapidly, relative to mean-square vorticity (enstrophy). This observation provides a basis for assigning initial data to the two-fluid field variables.

8. D.O. Martinez, W.H. Matthaeus, S. Chen, and D. Montgomery
COMPARISON OF SPECTRAL METHOD AND LATTICE BOLTZMANN
SIMULATIONS OF TWO-DIMENSIONAL HYDRODYNAMICS
PHYS. FLUIDS 6, 1285-1298 (1994).

Numerical solutions of the two-dimensional Navier-Stokes equations are presented by two methods; spectral and novel lattice Boltzmann equation (LBE) scheme. Very good agreement is found for global quantities as well as energy spectra. The LBE scheme is, indeed, providing reasonably accurate solutions of the Navier-Stokes equations with an isothermal equation of state, in the nearly incompressible limit. Relaxation to a previously reported "sinh-Poisson" state is also observed for both runs.

9. W.B. Jones and D. Montgomery
FINITE AMPLITUDE STEADY STATES OF HIGH REYNOLDS NUMBER 2-D
CHANNEL FLOW
Physica D [~~in press, 1994~~]. 73, 227-243 (1994)

Two-dimensional steady states of plane Poiseuille flow are found by following the evolution of a two-dimensional Navier-Stokes fluid numerically for long times. Two runs are presented, one for Reynolds number $R = 4000$, the other for $R = 15000$. The Reynolds number is based on the pressure gradient and channel half-width. The final states are characterized by an array of alternating-sign vortices. The states are accurately time independent in a frame moving with the vortices. In this frame of reference the stream function

and the vorticity display an interesting correlation, showing that the flow is divided into three spatially distinct regions. Theoretical understanding of the state is sought in terms of the statistical mechanics of large numbers of discrete line vortices in the mean field limit, but without great success. Attention is devoted to characterizing the relaxed state.

10. X. Shan and D. Montgomery
ROTATING MAGNETOHYDRODYNAMICS
~~submitted to~~ J. Plasma Phys., 1994 (now in press)

It is shown that rotation of a periodic column of magnetohydrodynamic fluid can be stabilized against current-driven instabilities by rotation. The rotation is considered as resulting from a slight departure from overall charge neutrality, and is braked by wall friction (loss of angular momentum to the cylinder wall). Both effects are modelled phenomenologically in the equation of motion, and the emphasis is on their bulk dynamic effects rather than their microscopic kinetic theory origins. Much as rotation is known to stabilize a Navier-Stokes fluid against transverse displacements, it is shown that sufficient rotation will suppress the helical vortices and helical deformations of the current channel that are known to result when the axial current in the column exceeds its stability threshold.

11. X. Shan and H. Chen
LATTICE BOLTZMANN MODEL FOR SIMULATING FLOWS WITH MULTIPLE PHASES AND COMPONENTS
Physical Review E 47, 1815-1819 (1993).

A lattice Boltzmann model is developed which has the ability to simulate flows containing multiple phases and components. Each of the components can be immiscible with the others and can have different mass values. The equilibrium state of each component can be a nonideal gas equation of state at a prescribed temperature exhibiting thermodynamic phase transitions. The scheme incorporated in this model is the introduction of an interparticle potential. The dynamical rules in this model are local so it is highly efficient to compute on massively parallel computers. This model has many applications in large-scale numerical simulations of various types of fluid flows.

12. X. Shan and H. Chen
SIMULATION OF NON-IDEAL GASES AND LIQUID-GAS PHASE TRANSITIONS BY LATTICE BOLTZMANN EQUATION
[Physical Review E, ~~in press, 1994~~] 49, 2941-2948 (1994).

We describe in detail a recently proposed lattice-Boltzmann model for simulating flows with multiple phases and components. In particular, the focus is on the modeling of one-component fluid systems which obey non-ideal gas equations of state and can undergo a liquid-gas type phase transition. The model is shown to be momentum-conserving. From the microscopic mechanical stability condition, the densities in bulk liquid and gas phases are obtained as functions of a temperature-like parameter. Comparisons with the thermodynamic theory of phase transitions show that the LBE model can be made to correspond exactly to an isothermal process. The density profile in the liquid-gas interface is also obtained as function of the temperature-like parameter and is shown to be isotropic. The surface tension, which can be changed independently, is calculated. The analytical conclusions are verified numerically.

B. Conference and Workshop Presentations

1. D. Montgomery
TURBULENCE AND ENTROPY
Proceedings of the Second International Conference on Fluid Mechanics (Beijing, 1993), ed. by Qiu Shuqing; Peking Univ. Press, Beijing, 1993. (pp. 1113-1116) (invited review).

Computations of decaying, two-dimensional, Navier-Stokes turbulence indicate relaxation to a "most probable" or maximum-entropy state at high enough Reynolds numbers. A precise definition of entropy for the viscous continuum is required. Fundamental issues in statistical mechanics are involved. A two-fluid theory is developed.

2. D. Montgomery
THE ECLIPSE OF IDEAL MAGNETOHYDRODYNAMICS
Physics of Space Plasmas (1992), ed. by T. Chang and J.R. Jasperse; Scientific Publishers, Inc., Cambridge, MA, 1992 [did not appear until 1994], pp. 237-264 (invited tutorial).

Tractable mathematics for ionized media mostly do not fit the density and temperature regimes of space plasmas very well. Barring increasingly unlikely-looking breakthroughs, it seems probably that space data will forever bear only an occasional plausible relationship to convincing theory. A theorist's choice would be whether to: (1) continue decorating the data when possible, using the mathematics at hand; or (2) abstract a subset of well-proposed problems of intrinsic theoretical interest, for which observational data do not provide the eventual proof or disproof. Since it appears that no one has plans to do anything in space for which precise or convincing theory is essential, the suggestion here is that

the proper choice is (2). Two recent areas of activity and controversy are reviewed as possible examples: (i) the large differences between ideal and "slightly" non-ideal MHD steady states; and (ii) the applicability of statistical-mechanical methods to highly nonlinear regimes in "slightly dissipative," strongly-magnetized, guiding center plasmas. In both examples, the dominant processes are frozen out completely in the ideal limit, but are permitted by small but non-zero dissipative terms in the governing equations.

3. X. Shan and D. Montgomery
TRANSITIONS IN ROTATING MAGNETOFLUIDS
Paper 3C50, Proc. Int. Sherwood Fusion Theory Conference,
Austin, TX (March 1994).

This paper is based on two assumptions: (1) that the tokamak "L to H transition," as shown by the experiments of R.J. Taylor and others, has something to do with plasma rotation; and (2) it is worthwhile to model the bulk dynamics of the transition within a purely MHD framework, even though it seems certain that other non-MHD effects are prominent in the transition. We extend the previous spectral-method, periodic-cylinder, MHD computations of Shan, *et al*, to the rotating case. A toroidal vorticity source term is added to the equation of motion and models the effect of departures from charge neutrality inside the plasma, of whatever origin. A wall friction term, important only near the boundary $r = a$, models the mechanical drag that the plasma must experience from whatever interaction with divertor or limiter may be present. It is shown computationally that by increasing the poloidal rotation rate, we may suppress the supercritical states (involving paired vortices and helical deformations) above the stability threshold. A rotating axisymmetric state results, with poloidal flow which may or may not be sheared in the interior, but which does involve a boundary layer near $r = a$ that contains a large velocity shear. This boundary layer is viewed as a consequence, rather than as a cause, of the rotationally-induced transition. From this perspective, poloidal rotation is seen as highly desirable to confinement. Designing reactors without having fully understood the role of plasma rotation can be argued to be premature.

4. D. Montgomery and X. Shan
THREE KINDS OF MHD EQUILIBRIA
Paper 3C51, Proc. Int. Sherwood Fusion Theory Conf., Austin,
TX (March, 1994).

The most common mathematical models for the configurations of magnetically-confined plasmas are steady-state solution of the MHD equations. Some surprising uncertainty remains, however, as to just what constitutes a physically acceptable MHD equilibrium. There are at least three distinct

candidates. The possibilities for ideal equilibria, with electrical conductivity assumed infinite, are essentially unlimited: an infinity of possible (Grad-Shafranov) solutions exists corresponding to a single set of boundary conditions. Slightly resistive equilibria, however, which enforce Ohm's law and Faraday's law from the beginning, appear to be unique for a given set of boundary conditions. The only non-uniqueness in those solutions seems to be associated with bifurcations at instability thresholds; there, the static axisymmetric equilibrium branch may be replaced by preferred branches with finite velocity fields and different (helical) symmetries. (And resistive toroidal equilibria, unlike ideal ones, differ greatly from those in periodic straight cylinders). Finally, there is the possibility that current profiles may select themselves on statistical mechanical grounds as "most probable" states that maximize information-theoretic entropies, subject to constraints provided by exact or approximate conservation laws. First formulated as a way of discriminating among ideal equilibria, maximum-entropy methods have recently shown predictive power in dissipative computations of turbulent relaxation to quasi-steady states in Navier-Stokes fluids and/or guiding-center plasmas. The three methods are not the same and do lead to different answers. The intent of this paper is to bring into focus the differences between, and the possible connections among, these three differing ways of arriving at MHD equilibria.

5. X. Shan and D. Montgomery
ROTATING MAGNETOHYDRODYNAMIC COMPUTATIONS
Given at Tokamak Transport Task Force Workshop, Austin, TX
(March 1994).

The experiments of R.J. Taylor et al have seemed to reveal a connection between the L to H transition and poloidal rotation. A variety of approaches have been tried to make the relation more definite. It has seemed to us worthwhile to model the phenomenon as a bulk magnetohydrodynamic (MHD) one, even though it seems certain that additional non-MHD effects are prominent, such as density variations and radiation losses. We extend a previously-developed, spectral-method, MHD code which expands all vector fields in Chandrasekhar-Kendall orthonormal eigenfunctions of the curl. The immediately supercritical state of a cylindrical magnetofluid, as the axial (toroidal) current is raised across its stability threshold, is defined for this purpose as the "MHD L-mode." It is a resistive steady state containing paired helical vortices and helical distortion of the current channel which at still higher currents acquires a persistent quasi-periodic pulsation. The MHD equation of motion is now modified by adding two terms which model complex kinetic processes lying outside the MHD description: (1) a vorticity source term which leads to poloidal rotation with a (possibly) radially varying profile, and is thought of as arising from a small departure

from charge neutrality in the magnetofluid; and (2) a wall friction term which becomes effective only near the wall $r = a$, and is thought of as representing the mechanical drag that the plasma interaction with whatever wall or divertor may be there must lead to. It is shown computationally that the supercritical states, increasing the rotation rate will suppress the helical states and leave an axisymmetric rotating profile. The poloidal velocity may or may not be sheared in the interior, but a boundary layer containing considerable velocity shear emerges near the wall; this boundary layer is seen as a consequence, rather than a cause, of the rotationally-introduced "MHD H-mode."

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