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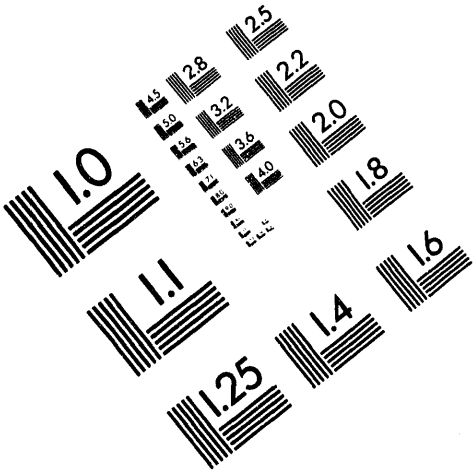
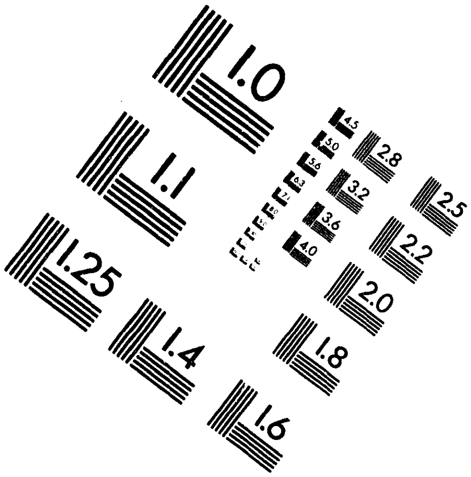

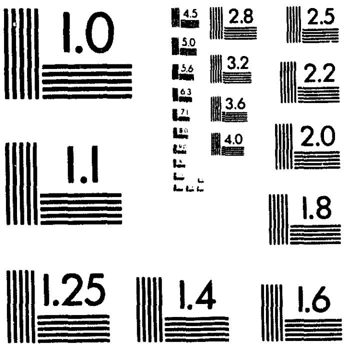

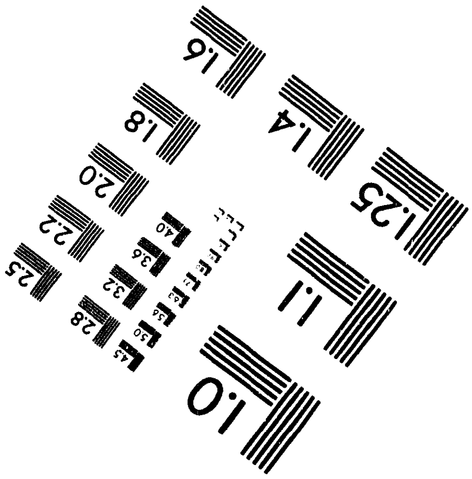
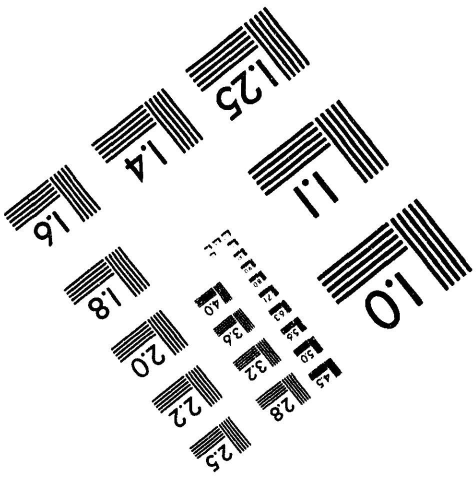

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TITLE: **THE NONLINEAR CHARACTERISTIC SCHEME IN X-Y GEOMETRIES**

AUTHOR(S): Wallace F. Walters
Todd A. Wareing

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THE NONLINEAR CHARACTERISTIC SCHEME IN X-Y GEOMETRIES

Wallace F. Walters and Todd A. Wareing

University of California
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

The Nonlinear Characteristic (NC) scheme for solving the discrete-ordinates form of the transport equation has recently been introduced and used to analyze one-dimensional slab transport problems.[1,2] The purpose of this paper is to determine the accuracy and positivity of the NC scheme as extended to solve two-dimensional X-Y problems. We compare the results obtained using the NC scheme to those obtained using the Bilinear Discontinuous[3] (BLD) scheme, the Bilinear Nodal[4] (BLN) scheme, Linear Characteristic[5] scheme, and the Diamond Difference with Fixup[6] (DD/F) scheme. As was found in one-dimensional applications, the NC scheme is strictly positive and as accurate or more accurate than the other schemes for all meshes examined. The accuracy of the NC scheme for coarse meshes is particularily outstanding compared to that of the other schemes.

The discrete ordinate equations,

$$\mu_m \frac{\partial \psi_m}{\partial x} + \eta_m \frac{\partial \psi_m}{\partial y} + \sigma_t \psi_m = S_m(x,y)\,, \tag{1}$$

are solved for each discrete ordinate *m* in each cell. From this point on we assume that $\mu_m, \eta_m > 0$. Now if the source $S_m(x,y) \geq 0$ for all x and y in the cell then the analytic solution of Eq.(1) obtained by the method of characteristics will be positive if the data on the inflow faces of the cell are greater than or equal to zero.

Using the methods of information theory[7,8], we can construct the least biased distribution $S_m(x,y) \geq 0$ such that the average value and two spatial moments of $S_m(x,y)$ are preserved. This distribution is given by:

$$S_m(x,y) = S_{A,m}\left[\frac{\lambda_x e^{\lambda_x P_1(x)}}{\sinh(\lambda_x)}\right] * \left[\frac{\lambda_y e^{\lambda_y P_1(x)}}{\sinh(\lambda_y)}\right]\,. \tag{2}$$

Here $S_{A,m}$ is the average source in the cell and $P_1(x) = \frac{2x}{\Delta x} - 1$ and $P_1(y) = \frac{2y}{\Delta y} - 1$ are the Legrendre polynomials on the cell. The following equation, which ensures that the first spatial moments of the source are preserved, is satisfied by λ_x and λ_y:

$$\frac{S_m^J}{S_{A,m}} = 3 * \left[\coth(\lambda_J) - \frac{1}{\lambda_J} \right] \quad J = x, y. \tag{3}$$

Here S_m^J is the Jth coordinate moment of the source.

In Fig.1 we show the singular characteristic associated with $\mu_m, \eta_m > 0$. In order to ensure positive inflow to the adjacent cells, we use information theory to construct edge distributions that preserve the zeroth and first spatial moments on each outflow edge of the cell in Fig.1. The equations for $\psi_{T,m}(x)$ and $\psi_{R,m}(y)$ are similar and so only the equations relating to $\psi_m^T(x)$ are included. These are:

$$\psi_{T,m}(x) = \psi_{T,m} \left[\frac{\lambda_T e^{\lambda_T P_1(x)}}{\sinh(\lambda_T)} \right] \text{ and ,} \tag{4}$$

$$\frac{\psi_{T,m}^x}{\psi_{T,m}} = 3 * \left[\coth(\lambda_T) - \frac{1}{\lambda_T} \right] . \tag{5}$$

Here $\psi_{T,m}$ is the average angular flux in direction *m* on the top edge, and $\psi_{T,m}^x$ is the first Legendre moment with respect to x of the angular flux in direction *m* on the top edge. The transcendental equations (3) and (5) are well represented by a polynomial fit and an asympotic expansion so that the computation of the parameter λ is not time consuming.

To compare the NC scheme with the other difference schemes already mentioned, we examine a difficult three group shielding problem with P_0 scattering using an S_4 quadrature set . The problem layout is seen in Fig.2. Both the total neutron leakage and the neutron absorption rate in the upper right corner of the problem are computed and displayed for several schemes in Table 1.

After examining Table 1 we see that the NC scheme is as accurate as any scheme examined for all mesh refinements. The NC results are even accurate and well-behaved in the limit of extremely coarse mesh where other high order schemes (LC, BLN, and BLD) fail badly. The inner iteration process does not even converge for the second order DD/F scheme at the coarsest (5x5) mesh!

There are several rather obvious conclusions. The behavior of the NC scheme is as impressive in two-dimensional x-y geomerties as it is in one-dimensional slab geometries. The NC scheme is strictly positive and more accurate than other commonly used schemes for this and all other test problems examined. This new NC scheme does not require modification of the sweeping algorithm used in most discrete ordinate codes. Since this is true this scheme can be inserted in current production codes with minor modification.

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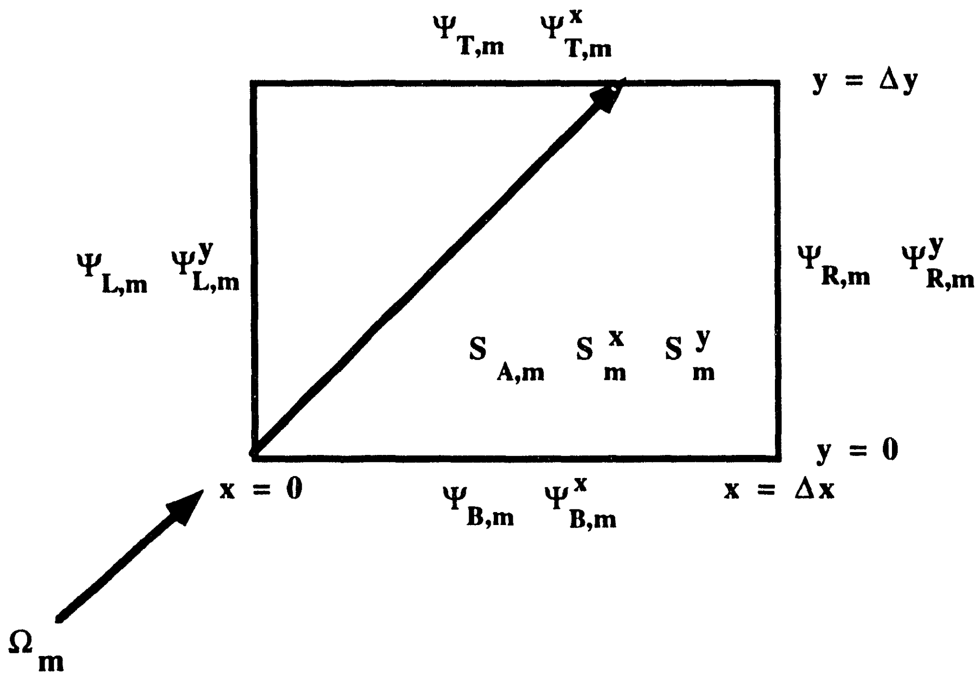

Figure 1: Cell Characteristic Ω_m - Inflow, Outflow and Source

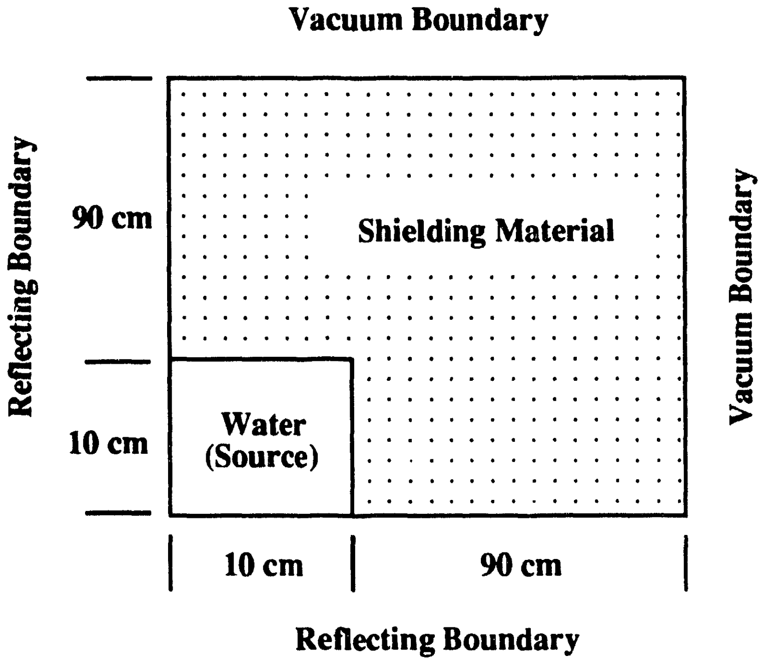

Figure 2 Shielding Test Problem

Table 1

	Total Leakage / Absorption Rate in Top, Right 22.5 cm x 22.5 cm				
Mesh	NC	LC	BLN	BLD	DD/F
5x5	9.10 / 0.85	60.2 / 20.9	359. / -5.98	2166 / -54.7	No Conv.
10x10	8.33 / 1.17	5.46 / 0.35	5.21 / 0.91	1.95 / 0.64	78.9 / 81.4
20x20	8.04 / 1.22	7.53 / 1.09	7.66 / 1.20	6.56 / 1.13	10.8 / 4.43
40x40	7.96 / 1.23	7.90 / 1.22	7.73 / 1.23	7.74 / 1.21	8.73 / 2.35