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Studies of Polarized Beam Acceleration and Siberian Snakes†

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ABSTRACT

We studied depolarization mechanisms of polarized proton acceleration in high energy accelerators with snakes and found that the perturbed spin tune due to the imperfection resonance plays an important role in beam depolarization at snake resonances. We also found that *even* order snake resonances exist in the overlapping intrinsic and imperfection resonances. Due to the perturbed spin tune of imperfection resonances, each snake resonance splits into two. Thus the available betatron tune space becomes smaller. Some constraints on polarized beam colliders were also examined.

1. Introduction

The spin equation of motion for a spin particle, governed by the magnetic interaction between the magnetic dipole moment of the particle and the static magnetic field in a synchrotron, is given by the Thomas-BMT equation [1],

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\vec{S}\times[(1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_\parallel + (G\gamma + \frac{\gamma}{\gamma+1})\frac{\vec{E}\times\vec{v}}{c^2}],$$

where $\vec{B}_\perp$ and $\vec{B}_\parallel$ are the transverse and longitudinal components of the magnetic fields with respect to the velocity vector, $\vec{\beta}$. In a planar synchrotron, vertical magnetic fields are needed to guide the orbiting particle around a closed path. Thus the spin vector is precessing with respect to the vertical axis at a frequency $G\gamma f_0$, where f_0 is the revolution frequency, $G = \frac{g}{2} - 1$ is the anomalous magnetic g-factor and γ is the relativistic Lorentz factor. The quantity, $G\gamma$, representing the number of spin precessions per revolution, is called the spin tune.

In synchrotrons, strong quadrupole fields are needed to focus the beam to a small size. Particles moving off-center vertically in quadrupoles will experience horizontal fields, which kick the spin vector away from the vertical axis. Since quadrupole magnets and particle closed orbits are periodic in a circular accelerator and the betatron and synchrotron motions are quasiperiodic [2], perturbing kicks to the spin vector can be decomposed into harmonics, K, given by $K = n + m\nu_z + \ell\nu_x + k\nu_{syn}$, where ν_z, ν_x and ν_{syn} are respectively the vertical betatron, the horizontal betatron and the synchrotron tunes, and k, ℓ, m, n are integers. The imperfection resonances, due to the vertical closed orbit errors, are located at integer harmonics, $K = n$. The intrinsic resonances, due to the vertical betatron motion, are located at $K = nP + \nu_z$, where P is the superperiodicity of the accelerator. Other depolarizing resonances arise from linear or nonlinear betatron coupling, vertical dispersion, synchro-beta coupling and random field errors.

When the spin tune equals to a harmonic of perturbing kicks, $G\gamma = K$, these spin perturbing kicks add up coherently. Beam depolarization may occur. To avoid a spin resonance condition, Derbenev and Kondratenko [3] proposed to use a local spin rotator, which rotates the spin vector 180° about an axis in

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the horizontal plane. These spin rotators are called snakes. Using snakes in an accelerator, the spin tune, ν_s, can become $\frac{1}{2}$ and independent of energy. The resonance condition can be avoided.

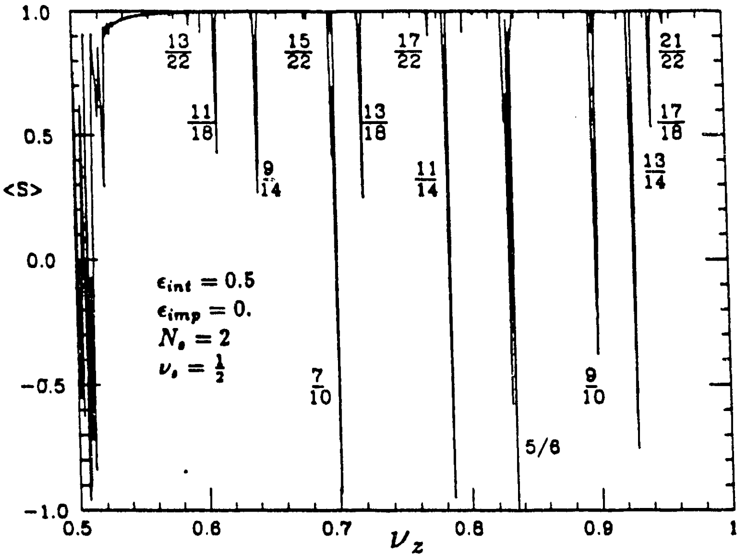

Fig.1. The final vertical spin vector after passing through an intrinsic depolarization resonance with strength $\epsilon_{int} = 0.5$ of an accelerator with two snakes is plotted as a function of the fractional part of the vertical betatron tune.

However, subsequent studies show that when the resonance strength is large, new spin depolarizing resonances occur at some fractional betatron tunes. These resonances, located at,

$$\nu_s + \ell K = \text{ integer,} \tag{1}$$

are called *snake resonances* [4], where ν_s is the spin tune, K is the spin depolarizing resonant harmonic and $\ell = 1, 3, 5, 7, \cdots,$. For $\nu_s = \frac{1}{2}$, we expect that snake resonances occur at fractional betatron tunes, $\nu_z = \frac{1}{2}, \frac{1}{6}, \frac{5}{6}, \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}, \cdots$, where the lowest order snake resonance has been observed [5]. Higher order snake resonances have been identified in numerical simulation shown in Fig. 1, where the final vertical spin vector, after passing through an isolated intrinsic resonance, is plotted as a function of the vertical betatron tune ν_z.

From Fig. 1, we note that numerical simulations show no apparent even order snake resonances at $\ell = 2, 4, 6, 8, \cdots$. Several reasons for the nonexistence of even order snake resonances were given in the past [4,6]. However, the situation has never been tested in the case of overlapping resonances. With overlapping resonances, cancellation of depolarization perturbation is not guaranteed and the coherent kicks due to the imperfection resonance may induce strong perturbation to the spin vector. This may lead to beam depolarization at even order snake resonances. Therefore careful studies are needed.

Overlapping resonances are important in high energy accelerators. Important intrinsic resonances are located at $K = nP \pm \nu_z \approx mPM \pm \nu_B$, where n and m are integers, P is the superperiodicity, M is the number of FODO cells per superperiod and ν_B is the total accumulated betatron tune of those FODO cells, which contain dipole magnets. The corresponding resonance strengths are given by

$$\hat{\epsilon}_{K,int} \approx \frac{G}{4\pi}\sqrt{\frac{\gamma\epsilon_N}{\pi}}PM\frac{\sqrt{\beta_z(D)}}{f}\left(1+\sqrt{\frac{\beta_z(F)}{\beta_z(D)}}\right),$$

where $\beta_z(F)$ and $\beta_z(D)$ are vertical betatron amplitude functions at the location of focusing and defocusing quadrupoles, f is the focal length of quadrupoles, ϵ_N is the normalized emittance, and γ is the Lorentz factor. The maximum resonance strength is about 0.45 at 250 GeV for RHIC, and is about 5 at 2 TeV at the SSC. These important intrinsic resonances are well separated.

On the other hand, important imperfection resonances will occur at integers near to an important intrinsic resonance. Thus overlapping intrinsic and imperfection resonances constitute the most important problem in the spin dynamics during polarized proton acceleration. The maximum imperfection resonance strengths are given by

$$\hat{\epsilon}_{K,imp} \approx \frac{G\gamma}{2\pi}PM\frac{\sigma_z}{f\sqrt{2\nu_z}}\left(1+\sqrt{\frac{\beta_z(F)}{\beta_z(D)}}\right),$$

where σ_z is the rms vertical closed orbit in the arc, ν_z is the vertical betatron tune. We expect that the imperfection resonance strength to be less than 0.05 for RHIC after a closed orbit correction with $\sigma_z \approx 0.2$ mm.

Previous studies [7] of overlapping resonances indicated that when the betatron tune is chosen *properly*, i.e. far away from low order snake resonances, the *tolerable* or *critical* intrinsic resonance strength is given by $\epsilon_{int,c} \le \frac{1}{5}N_s$, where N_s is number of snakes. However there are many open questions remaining, such as where is the proper tune? what is the depolarization mechanism for overlapping resonances? what are essential effects of imperfection resonances? etc. This paper is intended to investigate spin depolarization mechanisms of overlapping intrinsic and imperfection resonances. SEction 2 studies depolarization mechanisms. Section 3 reviews progresses of snake experiments. Section 4 gives the conclusion and requirements for a polarized collider.

2. Spin Depolarization Mechanisms in a Synchrotron

In a synchrotron, the Thomas-BMT equation can be cast into the equation for the two-component spinor [8], Ψ, as

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}\begin{pmatrix} G\gamma & -\xi \\ -\xi^* & -G\gamma \end{pmatrix}\Psi\ , \qquad (2)$$

where θ is the orbital bending angle, and ξ arises from nonvertical magnetic fields in a synchrotron and is the main source for beam depolarization. The spin vector is given by the ensemble average, $\vec{S} = \langle\Psi|\vec{\sigma}|\Psi\rangle$, where $\vec{\sigma}$ is the Pauli matrices. Because of periodic structure of a circular accelerator and the quasi-periodicity of betatron and synchrotron motions, one obtains $\xi = \sum_K \epsilon_K e^{-iK\theta}$, where ϵ_K is the resonance strength, and K is the resonance tune.

The spinor equation of motion can be solved analytically for a single resonance by transforming the reference frame to the resonance precessing frame with $\Psi_K(\theta) = e^{\frac{i}{2}K\theta\sigma_3}\Psi(\theta)$, i.e.

$$\Psi(\theta_f) = e^{-\frac{i}{2}K\theta_f\sigma_3}e^{\frac{i}{2}\lambda\hat{n}_{co}\cdot\vec{\sigma}(\theta_f-\theta_i)}e^{\frac{i}{2}K\theta_i\sigma_3}\Psi(\theta_i) = t(\theta_f,\theta_i)\Psi(\theta_i)$$

where $\hat{n}_{co} = \frac{1}{\lambda}[\delta\hat{e}_3 + \epsilon_R\hat{e}_1 - \epsilon_I\hat{e}_2]$ is the spin closed orbit in the resonance precessing frame, and $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$ are orthonormal bases corresponding to radially outward, longitudinal, and vertically upward axes, and $\lambda = (\delta^2 + |\epsilon|^2)^{1/2}$, $\delta = K - G\gamma$. The matrix elements of the spin transfer matrix are given by, $t_{11} = ae^{i[c-K(\theta_f-\theta_i)/2]}$, $t_{12} = ibe^{-i[d+K(\theta_f+\theta_i)/2]}$, $t_{21} = -t_{12}^*$, $t_{22} = t_{11}^*$, with parameters, $b = \frac{|\epsilon|}{\lambda}\sin[\lambda\frac{\theta_f-\theta_i}{2}] = (1-a^2)^{1/2}$, $c = \arctan[\frac{\delta}{\lambda}\tan(\lambda\frac{\theta_f-\theta_i}{2})]$, and $d = \arg(\epsilon^*)$. The parameter b is the effective resonance strength with a maximum amplitude $\frac{|\epsilon|}{\lambda}$.

2.1 Basic Requirements of Snake Configurations in Accelerators

Snakes are local spin rotators, which rotate particle spin by π radians about a horizontal axis locally without perturbing particle orbits outside a snake region. Thus a snake is characterized by the *spin rotation angle*, ϕ, and the *snake axis angle*, ϕ_s, with respect to $\hat{e}_1$. The spinor is transformed as, $\Psi(\theta^+) = e^{-i\frac{\phi}{2}\hat{n}_s\cdot\vec{\sigma}}\Psi(\theta^-) = T_s(\phi_s)\Psi(\theta^-)$, where $\theta^\pm$ depict azimuthal orbit rotation angles just before and after the snake. At $\phi = \pi$, or the 100% snake, we have $T_s(\phi_s) = -i\hat{n}_s\cdot\vec{\sigma}$.

Let us consider N_s snakes with snake axes, $(\phi_1, \phi_2, \cdots, \phi_{N_s})$, and let $\theta_{i,i+1}$ be the azimuthal orbit rotation angle between the i-th, and $(i+1)$-th snakes. The condition, $\sum_{k=odd}^{N_s}\theta_{k,k+1} = \sum_{k=even}^{N_s}\theta_{k,k+1} = \pi$, is needed to provide an energy independent spin tune [6,10]. If the odd (or even) orbital angle deviates from π, the spin tune is *shifted* away from $\frac{1}{2}$ by an amount, $\Delta\nu_s = G\gamma(1 - \frac{\theta_{odd}}{\pi})$. Similarly, the spin tune is obtained from the trace of the *one turn transfer matrix*, or the *one turn map* (OTM), i.e. $\nu_s = \frac{1}{\pi}\sum_{k=1}^{N_s}(-1)^k\phi_k$. The spin tune can be adjusted to the most favorable number in avoiding spin depolarizing resonances. For accelerators with two snakes, those two snakes should be located at π orbital angle apart and the snake axes of these two snakes should be orthogonal to each other to obtain a spin tune of $\frac{1}{2}$. For accelerators with a large number of snakes, proper snake superperiodicity and proper spin tune can be arranged.

2.2 Spin Tracking Hierarchy Equation

Let us consider an accelerator with N_s equally spaced snakes. The spin transfer matrix after passing through a pair of (ϕ_2, ϕ_1) snakes is given by

$$\tau(\theta_0 + \tfrac{4\pi}{N_s}, \theta_0) = T_s(\phi_2)t(\theta_0 + \tfrac{4\pi}{N_s}, \theta_0 + \tfrac{2\pi}{N_s})T_s(\phi_1)t(\theta_0 + \tfrac{2\pi}{N_s}, \theta_0).$$

The spin motion can be obtained iteratively by using the spin tracking equation through pairs of snakes, i.e. $T(\theta_{n+1}) = \tau(\theta_{n+1}, \theta_n)T(\theta_n)$, where $\theta_{n+1} = \theta_n + 4\pi/N_s$. The resonance strength parameter b becomes smaller due to a small orbital angle difference, $\theta_f - \theta_i$, between snakes. The spin tracking equation can be solved iteratively using a power series expansion in strength parameter b^2, i.e.

$$T_{11} = T_{11}^{(0)} + T_{11}^{(1)} + T_{11}^{(2)} + \cdots, \quad T_{12} = T_{12}^{(1)} + T_{12}^{(2)} + T_{12}^{(3)} + \cdots, \qquad (3)$$

where $T_{11}^{(i)} = O(b^{2i})$ and $T_{12}^{(i)} = O(ab^{2i-1})$. The final vertical spin vector is given by, $\langle S\rangle = |T_{11}|^2 - |T_{12}|^2 = 1 - 2|T_{12}|^2$.

2.3 The Perturbed Spin Tune and Snake Resonances

Without loss of generality, we discuss an accelerator with two snakes ϕ_1, ϕ_2, located at a π orbital angle from each other. The OTM is given by $\tau_{11} =$

$-e^{-i\pi\nu_s}(1-2b^2e^{i\Phi}\cos\Phi)$, $\tau_{12} = -2iabe^{-i(c-K\pi+\phi_2)}\cos\Phi$, where $\pi\nu_s = \phi_2-\phi_1$ and $\Phi = K\theta_0 + K\pi + d - \phi_1$ is the characteristic betatron phase of the orbital motion, $b = \frac{|\epsilon|}{\lambda}\sin\frac{\pi\lambda}{2}$, $\lambda = \sqrt{\delta^2+|\epsilon|^2}$, $\delta = K - G\gamma$ and $a = \sqrt{1-b^2}$.

The perturbed spin tune, Q_s, defined as the trace of OTM, is given by $\cos\pi Q_s = b^2\sin(2\Phi)$. Because of the betatron phase, the perturbed spin tune for an intrinsic resonance, Q_s, is oscillating around $\frac{1}{2}$ up to the maximum and the minimum given by $Q_{s,max/min} = \frac{1}{2} \pm \frac{1}{\pi}\arcsin[\sin^2\frac{\pi\epsilon}{N_s}]$

Thus if the resonance strength of a spin resonance is $|\epsilon| \approx mN_s/2$, $m = 1,3,\cdots$, the perturbed spin tune, Q_s, will cover a whole integer unit during the acceleration and cross the intrinsic resonance many times. The polarization may be lost. The final polarization is plotted as a function of the intrinsic resonance strength at $\nu_z = 0.81$ on the left side of Fig. 2, where the maximum and minimum perturbed spin tunes cover the entire tune space around $\epsilon = 1$ and 3 and where beam depolarization occurs. At the right side of Fig.2, the perturbed spin tune *shift* is shown as a function of $G\gamma$ for an imperfection resonance. The polarization after passing througth the resonance is independent of the vertical betatron tune.

Solving the spin tracking Eq. 3 to the first order in parameter b, the spin transfer matrix is given by,

$$\begin{aligned} T_{12}^{(1)}(\theta_{n+1}) =& iab(-1)^n e^{-i(c-K\pi+\phi_2)}\{e^{i(\Phi+nK\pi)}\zeta_{n+1}(K+\nu_s)+ \\ &+ e^{-i(\Phi+nK\pi)}\zeta_{n+1}(K-\nu_s)\} \end{aligned} \tag{4}$$

where the enhancement function, $\zeta_n(q)$, is given by, $\zeta_n(q) = \frac{\sin nq\pi}{\sin q\pi}$. At the first order snake resonance condition, $\nu_s \pm K =$ integer, the off-diagonal kicks add up coherently each turn through snake pairs. The beam can be depolarized easily as shown in Fig. 1 at $\nu_z = \frac{1}{2}$. Since betatron tunes of an accelerator are not half integers, the first order snake resonance condition can easily be avoided. A few useful observations is given below:

1. At an imperfection resonance, $K =$ integer, $T_{12}^{(1)}(\theta_{n=even}) = 0$. This means that imperfection kicks cancel each other every two revolutions. Thus snakes are effective in overcoming imperfection resonances.
2. When the betatron tune equals to a low order rational number, the linear terms in Eq. 4 cancel each other in the tracking equation. For $K = \frac{q}{p}$, we found that

$$T_{12}^{(1)}(\theta_m) = 0 \quad \begin{cases} m = p & \text{if } p \text{ is even,} \\ m = 2p & \text{if } p \text{ is odd.} \end{cases}$$

One might guess that the spin will be more stable against perturbation at a rational nmber betatron tune. At low order rational numbers, such as 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, etc., the spin vectors behave characteristically different from that shown on the left side of Fig. 2. When K is a low order rational number, the polarization is not much affected by the perturbed spin tune at $\epsilon = 1$ or 3 due to cancellation in perturbing kicks.
3. Avoiding snake resonances, the vertical spin vector across the resonance region will fall within the envelope of $\langle\langle S\rangle\rangle = 1 - 8a^2b^2$, $\quad b = \frac{|\epsilon|}{\lambda}\sin\frac{\pi\lambda}{2}$. The envelope function $\langle\langle S\rangle\rangle$ has many nodal points, where the depolarization driving term vanishes, i.e. $b = 0$ or 1. These nodal points corresponds to the

spin matching condition [10] where $\frac{\lambda}{N_s}$ = integer. Thus these nodal locations are separated approximately by N_s units of $G\gamma$. These nodal points plays an essential role in spin restoration during the passage through a depolarization resonance. Away from the central resonance location, the depolarization driving parameter b is usually small. Therefore after passing through the resonance, if the spin vector is not restored to the vertical position at the first nodal location, the spin is depolarized.

4. The width of envelope function is about $12|\epsilon|$ for 95% polarization. However, one can choose a nodal point to obtain 100% polarization.

5. Based on the linear response theory of Eq. 4, we expect that depolarization occurs when the betatron tune equals to a half integer shown clearly in Fig. 1. The snake resonance at $\nu_s = \frac{1}{2}$ had been observed [5].

From the above discussions, we might expect that the spin vector would be more stable when the betatron tune equals to a low order rational number. However, Figure 1 shows that there are many higher order depolarization resonances at a rational number betatron tune, e.g. 1/6, 5/6, 1/10, 3/10, etc. Solving the spin tracking equation beyond linear order in b gives rise to snake resonance conditions given by Eq. 1 [4].

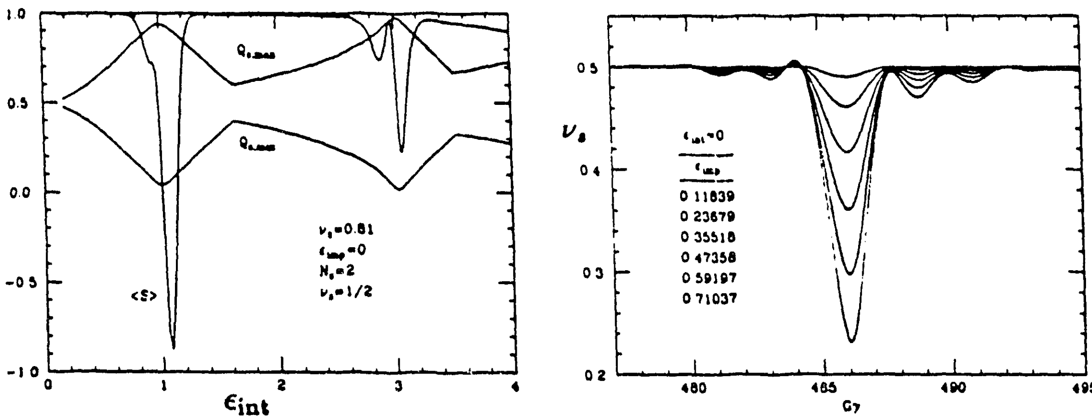

Fig.2 On the left side, the vertical spin vector and the perturbed spin tunes $Q_{s,max/min}$ obtained from a numerical tracking calculation at intrinsic resonance $\nu_z = 0.81$ are plotted as a function of the resonance strength. Note that the perturbed spin tune cover the entire tune space at $\epsilon = 1, 3$. On the right, the perturbed spin tune is plotted as a function of $G\gamma$

2.4 <u>Overlapping Resonances and Even Order Snake Resonances</u>

Basic accelerator theory [2] indicates that a closed orbit distortion has largest amplitude at a harmonic nearest to betatron tune. We thus expect a large imperfection resonance, located at an integer nearest to the important intrinsic resonance. The correlation remains important even after closed orbit corrections, which minimizes error harmonics nearest to betatron tunes.

At an even order snake resonance condition of Eq. 1, the spin vector is not much affected by the perturbative spin tune and is not depolarized at $\epsilon = 1, 3$ due to the cancellation of the linear spin kicks in Eq. 4. However, when the imperfection resonance is included, the vertical spin vector is perturbed strongly so that the spin vector can not retain its full polarization at the first nodal point. The memory on the vertical spin vector is lost. Including imperfection resonances in the spin tracking equation, the final vertical spin vector after passing through overlapping intrinsic and imperfection resonances, $\epsilon_{int} = 0.5, \epsilon_{imp} = 0.05$ is shown in Fig.3 as a function of ν_z, where *beam depolarization occurs at all even order snake resonances.*

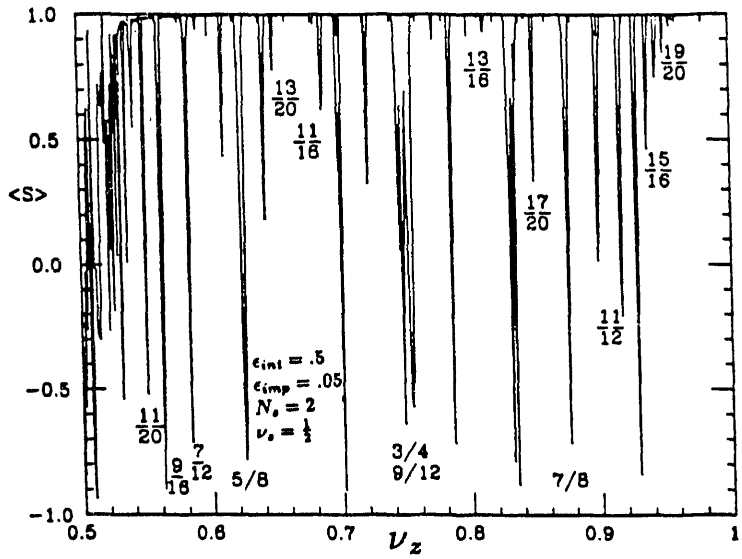

Fig.3 Beam polarization after passage through overlapping intrinsic and imperfection spin resonances is shown as a function of the fractional part of spin resonance tune. In comparison with that of Fig. 1, even order snake resonances appear while the odd order snake resonances are not much affected. At $\epsilon_{imp} = 0.05$ for two snakes, even order snake resonances are as important as odd order snake resonances.

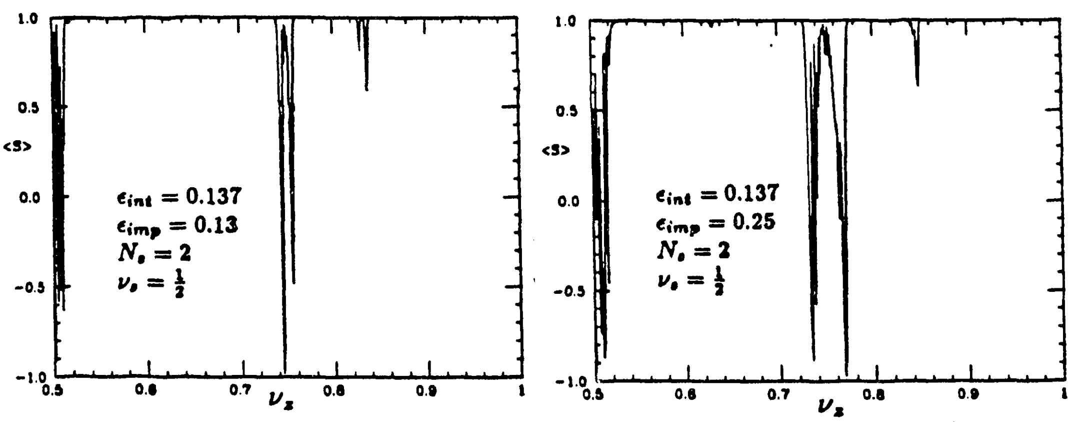

Fig.4 The effect of the imperfection resonance on the snake resonances is shown for $\epsilon_{imp} = 0.13$ (left) and $\epsilon_{imp} = 0.25$ (right). Note that even order snake resonances appear and each snake resonance splits into two resonance condition due to the unperturbed spin tune of the imperfection resonance.

To understand the effect of imperfection resonances on the spin motion, the intrinsic resonance strength is reduced to $\epsilon_{int} = 0.137$ so that only low order snake resonances at $\nu_z = 1/2,\ 1/6,\ 5/6$ are important. When an imperfection resonance at $\epsilon_{imp} = 0.13$ is included, even order snake resonances at $\nu_z = 3/4,\ 5/8,\ 7/8, \cdots$ appear shown in Fig.4. Furthermore, all snake resonances split into double dips. The distance between two dips increases with the strength of the imperfection resonance. The snake resonance condition becomes

$$\frac{1}{2} + \Delta Q_s \pm \ell\nu_z = \text{ integer}, \quad \ell = \text{ integer}, \tag{5}$$

where ΔQ_s is the perturbed spin tune shift from the imperfection resonance given by $|\Delta Q_s| \approx \frac{1}{\pi}\arcsin[\sin^2\frac{\pi\epsilon_{imp}}{N_s}]$. Because of spin tune shift, each snake resonance will split into two snake resonances separated by $\Delta\nu_z = \pm\frac{1}{\ell}\Delta Q_s$. The distance of splitting becomes smaller at higher order snake resonances.

Thus the spin depolarization mechanisms are due to the perturbed spin tune shift of the imperfection resonance and the snake resonance conditions of Eq. 1. Since betatron tunes of colliders, such as RHIC, SPS, Tevatron, and SSC, have to avoid similar low order betatron resonances for orbital stability, snake resonances do not impose further constraints to the operation of colliders. One can generalize the discussion to multi-snake accelerators, where the resonance condition of Eq.1 will be modified by snake superperiodicity, P_s. At higher snake superperiodicity, there are fewer snake resonances, yet resonance width is also increased. Basic physics remains unchanged [7] .

Let us consider a model of overlapping intrinsic and imperfection resonances with a small local spin precessing kick, χ, about the $\hat{e}_1$ axis. The OTM becomes, $\tilde{\tau} = e^{-i\frac{\chi}{2}\sigma_1}\tau(\theta_0 + 2\pi, \theta_0)$, The resonance strength of the imperfection resonance is given by $\epsilon_{imp} = \chi/2\pi$ at all integer harmonics. The off diagonal matrix elements of the OTM is given by,

$$\tilde{\tau}_{12} = -2iabe^{-i(c-K\pi+\phi_2)}\cos\Phi\cos\frac{\chi}{2} + ie^{i\pi\nu_s}(1 - 2b^2e^{-i\Phi}\cos\Phi)\sin\frac{\chi}{2},$$

Note here that the off-diagonal matrix elements now contain a term oscillating at two times the betatron frequency with an amplitude proportional to $b^2\sin\frac{\chi}{2}$. Thus the tolerable even order snake resonance strength will decrease inversely with respect to the imperfection resonance strength. Following the same procedure in deriving Eq. 4, one obtains a snake resonance condition, $\nu_s \pm 2K =$ integer. By performing similar higher order analysis, one can obtain all even order snake resonances. Since the first term of $\tilde{\tau}_{12}$ depends on the imperfection resonance in $\cos\frac{\chi}{2}$, the odd order snake resonance is not much affected by the overlapping imperfection resonance.

2.5 Critical Snake Resonance Strength

Let us define the critical resonance strength for the snake resonance of the order ℓ of Eq. 1 as the maximum resonance strength such that the final polarization is 98% or higher after passing through the resonance. The critical resonance strength depends on the order of snake resonance, ℓ, the acceleration rate, the imperfection resonance strength. We will assume that the imperfection resonance strength is small so that the spin tune is not substabtially shifted, e.g. $\epsilon_{imp} \leq 0.05$ for two snakes. Fig.5 shows the critical resonance strength tolerable as a function of the order of the snake resonance at an acceleration rate of $\Delta p = 5$ MeV/c per turn. Note here that the critical resonance strength increases with the order of the snake resonance. The critical snake resonance strengths for even order snake resonances follow a similar behavior. At a constant imperfection resonance strength, the dependence of the critical resonance strength on the accelration rate Δp in [MeV/c per turn] is shown in the middle of Fig.5. Finally, the dependence of the critical resonance strength for an even order snake resonance is shown on the right as a function of the imperfection resonance resonance strength at a fixed acceleration rate of $\Delta p = 0.3$ [MeV/c/turn]. Here we observe that the critical even order snake resonance strength depends inversely with respect to the imperfection resonance.

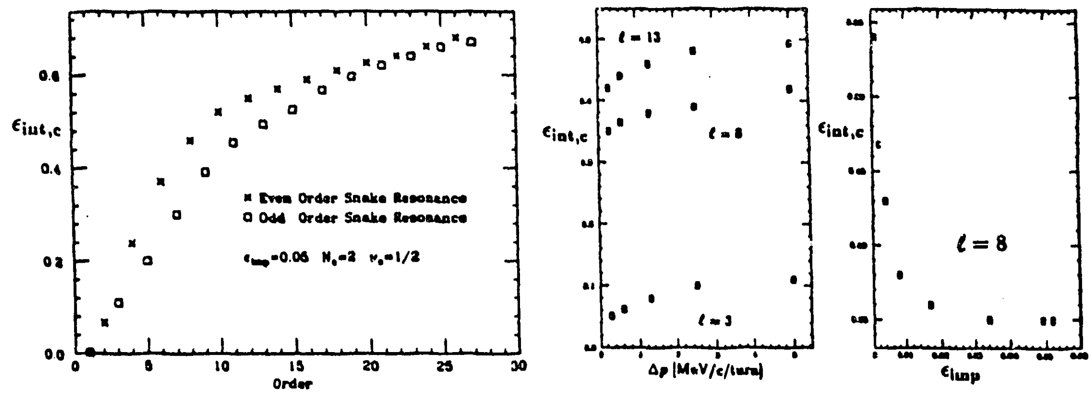

Fig.5 The critical snake resonance strength vs the order ℓ of the snake resonance at the acceleration rate of 5 MeV/turn is shown on the left. The dependence of critical resonance strength on the acceleration rate is shown in the middle for $\ell = 3, 8, 13$. The dependence of the critical resonance strength for an even order snake resonance ($\ell = 8$) on the imperfection resonance strength is shown on the right. Because betatron tunes of RHIC are chosen to lie between 0.8 and 0.8333, therefore possible snake resonances are the 8th order at $\nu_z = \frac{13}{16}$ and the 13th order at $\nu_z = \frac{21}{26}$

3. Snake Experiments at the IUCF Cooler Ring

Through a series of pioniering snake experiments at the IUCF Cooler Ring led by A.D. Krisch, much of spin physics experiments have been performed, where a superconducting solenoid snake and two pairs of skew quadrupoles and tuning quadrupoles, to achieve a local linear coupling corrections, are installed in the S section of the Cooler Ring. The experiments starts with polarized proton at 80% polarization kicked injected into the cooler. The polarized proton is cooled and then the polarization is measured using a skimmer target. The radial and vertical polarizations were measured in the polarimeter detector with up-down and left-right symmetry. One can summarize results of these experiments as follows:

1. The spin closed orbit vector in the present of snake and partial snake had been verified. The electron beam confinement elements introduced a spin tune shift, $\Delta\nu_3$, equal to the product of the spin precession angles due to the vertical orbit bump and the main solenid field divided by 2π [5,11]. This gives rise to an apparent $G\gamma = 2$ resonance energy shift of $\Delta E = mc^2\frac{\Delta\nu_3}{G}$. The interference between the longitudinal field error and the horizontal transverese field error of the vertical closed orbit was used to determine the strength and phase of the imperfection resonance strength.
2. The synchrotron sidebands around an imperfection resonance was observed. This indicate that the depolarization due to synchrotron motion is not negligible in storage rings. Fortunately, the synchrotron tune for proton storage

ring is of the order of 10^{-3}. Therefore, they plays little role in depolarization mechanism.
3. Using an rf solenoid, the spin tune of the orbiting proton was measured. The first order snake resonance had been confirmed. At the full snake strength, the synchrotron depolarization resonance did not exist. This confirms that the spin tune is energy independent with a full snake.
4. By ramping the rf solenoid through the spin tune, the Froissart-Stora formula was also confirmed. Operating the rf solenoid at the spin tune, the spin will precess about the spin closed orbit $\hat{n}_{co}$ at a rate determined by the strength of the solenoid. The measured spin tune of partial snake agrees very well with $\cos\pi\nu_s = \cos\pi(G\gamma - \Delta\nu_3)\cos\frac{\phi}{2}$, where $\Delta\nu_3$ is the spin tune shift due to magnetic fields in electron cooling region.

4. Conclusions and Requirements of a Polarized Collider

We find that even order snake resonances exist in the presence of overlapping intrinsic and imperfection resonances. and the depolarization mechanisms arise essentially from the perturbed spin tune of imperfection resonances and snake resonances. Because of the perturbed spin tune shift induced by the imperfection resonance, each snake resonance is split into two resonances. The available tune space becomes smaller. The depolarization mechanisms can be used to derive requirements for polarized colliders. Possible depolarization sources are (1) spin tune modulation so that the spin tune overlaps with snake resonances, (2) betatron tune modulation so that snake resonances overlap with the spin tune, (3) the effects of beam-beam interactions, higher order nonlinear resonances, and synchrotron depolarization resonances, (4) an uncompensated solenoid field at IP of experimental detectors and the effect of spin rotators for helicity experiments, (5) effects of linear coupling, and (6) effects of rf noises. Careful analysis of these effects on the spin in needed to obtain high energy spin collision.

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