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# Complete Mode-Set Stability Analysis of Magnetically Insulated Ion Diode Equilibria

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**Abstract**

*We present the first analysis of the stability of magnetically insulated ion diodes that is fully relativistic and includes electromagnetic perturbations both parallel and perpendicular to the applied magnetic field. Applying this formalism to a simple diode equilibrium model that neglects velocity shear and density gradients, we find a fast growing mode that has all of the important attributes of the low frequency mode observed in numerical simulations of magnetically insulated ion diodes, which may be a major cause of ion divergence. We identify this mode as a modified two-stream instability. Previous stability analyses indicate a variety of unstable modes, but none of these exhibit the same behavior as the low frequency mode observed in the simulations. In addition, we analyze a realistic diode equilibrium model that includes velocity shear and an electron density profile consistent with that observed in the numerical simulations. We find that the diocotron instability is reduced, but not fully quenched by the extension of the electron sheath to the anode. However, the inclusion of perturbations parallel to the applied magnetic field with a wavelength smaller than the diode height does eliminate growth of this instability. This may explain why the diocotron mode has been observed experimentally with proton sources, but not with LiF, since the turn-on of LiF is not uniform.*

## Background

High intensity light ion beams are being developed to drive inertial confinement fusion (ICF)[1]. Propagation and focussing requires these beams to have low ion divergence, ~ 6 mrad, as compared to ~20 mrad that has been experimentally achieved. Therefore understanding the mechanisms that generate ion divergence is critically important.

Three dimensional particle-in-cell simulations (3-D PIC) have been used to study the divergence generated in magnetically insulated diodes[3-4]. In these simulations, electrons stream into the diode along the magnetic field lines and form an electron sheath. Initially this sheath is quite thin with electrons flowing primarily in the **ExB** direction. There is considerable shear to this flow due to the large variation in the **E** field across the sheath and surface waves can flow at the edges of the sheath, see Fig. 1. The interaction of these surface waves leads to a fluid instability called the "diocotron", which has a very fast growth rate and saturates in a few nanoseconds.

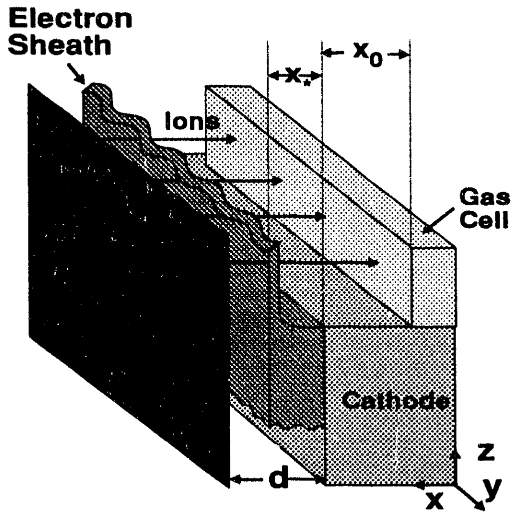


Fig. 1. A schematic of a magnetically insulated ion diode. The applied field is in the z direction.

The growth and saturation of the diocotron instability is fairly well understood. Linear stability theory gives a good estimate of the initial growth rates[4,5] and a simple trapping model[3] yields a saturated amplitude for the mode that is in good agreement with the simulations. The period of this instability is small compared to the ion crossing time and consequently it generates a relatively small ion divergence (<10 mrad). More importantly it allows electrons to cross the gap by breaking symmetry in the **ExB** direction. As the electron sheath broadens ($x_*$ goes to d), the ion current density increases and simulations indicate that a transition to a low frequency instability occurs. This instability generates considerable divergence (>30 mrad), because the period is roughly equal to the ion transit time.

The physics of the low frequency mode is not well understood. Linear analyses[6], using a Brillouin model of the electron sheath, exhibit unstable modes, but not with the correct wavelength and frequency when realistic diode parameters are used. However, these analyses did not include perturbations in the direction of the applied magnetic field. It is known[7] that the frequency of the two-stream instability in the presence of a magnetic field depends strongly on the direction of propagation with respect to the magnetic field. Recently, Sudan and Longcope[8] have presented a stability analysis including mode structure in the direction of the applied magnetic field. They found instability at low phase velocities, ($V_{ph}$~c/15) which is characteristic of the low frequency mode observed in the numerical simulations, but in the opposite direction. In the simulations the waves travelled parallel to the electron drift velocity. The authors suggest that this may be due to their approximate treatment. In particular, their analysis was electrostatic and not fully relativistic. In this paper we remove these approximations and find an unstable mode that is low frequency, low phase velocity, and propagates in the same direction as the low frequency mode observed in the simulations. Furthermore, the growth rate of the mode increases with ion current density consistent with the simulation result that the transition to the low frequency mode depends on the ion current enhancement, defined as the ratio of the ion current density over the Child-Langmuir ion current density.

## Stability formalism

The dynamics of a magnetically insulated ion diode can be described by Maxwell's equations and the equations governing the conservation of mass and momentum for the ions and the electrons. We treat both the ions and the electrons as laminar fluids. The ions propagating across the gap (-x direction) while the electrons drift parallel to ExB (y-direction), see Fig. 1. It is appropriate to treat the ions nonrelativistically since the ion velocity is about c/30. However, the electron velocities approach the speed of light so we have used the relativistic form of the momentum equation for the electrons. From these equations we have derived a system of linear equations that describe the stability of an arbitrary planar laminar fluid ion diode equilibrium[9]. The analysis is fully electromagnetic and accounts for wave propagation in a general direction relative to the applied magnetic field. This represents a generalization of previous work.

## The constant parameter model

The constant parameter model, CPM, is an idealization of an ion diode equilibrium in which all electron and ion parameters are constant. Sudan and Longcope[8] used this model in their electrostatic stability analysis. In our relativistic electromagnetic analysis of this model, we find an unstable mode which travels in the proper direction, see Fig. 2.

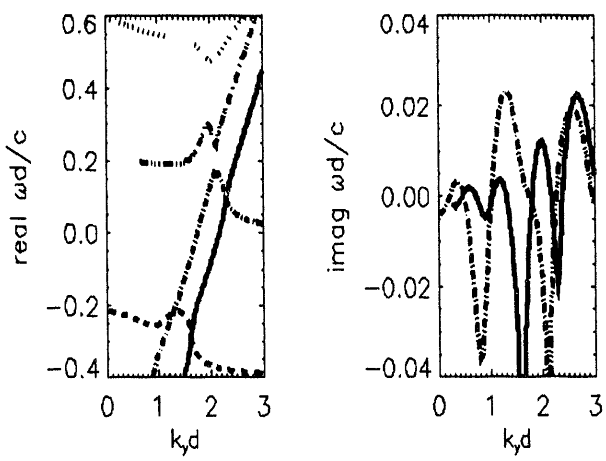


Fig. 2. The dispersion results for the CPM with $k_z d=1.9$, a $Li^+$ velocity of 0.05c, an electron drift velocity of $v_0=0.5c$, $B_{app}d=2.5$ T-cm, $\omega_p d/c=4.7$, and $\Omega_c=12.8$. Note the solid curve has maximum growth at a phase velocity of roughly c/9.

To understand this difference, we derived a relativistic electromagnetic dispersion relation for the electron fluid. In the limit of infinite applied magnetic field and phase velocities much less than the speed of light, the result is

$$\omega = v_0 k_y \pm \frac{(\omega_p/\gamma) k_z}{\sqrt{\frac{\omega_p^2}{c^2} + (m\pi/d)^2 + k_y^2 + k_z^2}}, \quad (1)$$

where m is a nonzero integer and $\omega_p$ is the plasma frequency. This equation shows that there are an infinite number of discrete electron space charge waves, all of which originate on the beam line in $k_y$ space for $k_z = 0$. This is due to the large applied magnetic field in combination with relativistic electron velocities, which hinder the electron response to transverse perturbations. However, electrons can easily respond to perturbations in the direction of the applied field and thus the degeneracy is removed for finite $k_z$. Equations similar to Eq. (1) were derived by Krall and Liewer[7], and Sudan and Longcope[8] using the electrostatic approximation. Their results did not have the plasma frequency term in the denominator which slows the rate that the y-phase velocity is reduced as $k_z$ is increased. Instability results when the real part of the electron and ion space-charge waves are approximately equal. Therefore, the relativistic terms have an important effect on the phase velocity of the unstable waves. Furthermore, it is clear from the form of Eq. (1) that the instability of Fig. 2 is fundamentally the same as the modified two-stream instability studied by Krall and Liewer[7]. This identification is consistent with the increase in the growth rate that we obtain as the ion and electron densities are increased.

## A realistic diode equilibrium

The 3-D numerical simulations show that electron density and velocity vary smoothly across the accelerating gap. These profiles are well approximated by letting the electron density be the simple function of the electric potential, $N(\phi) = n_0 + n_1\phi^\alpha + n_2\phi^\beta$. Values of $\alpha < 1$ and $\beta > 1$ insure that Poisson's equation and Amperes law can be satisfied along with the boundary conditions, E=0 at the virtual cathode and at the anode, $N(0)=N_i$, and $N(\phi_*)=0$. It is interesting to note that even when the electron distribution extends completely to the anode the diocotron instability still has a significant growth rate. However, the next figure shows that the growth rate of the diocotron goes to zero as the value of $k_z$ is increased.

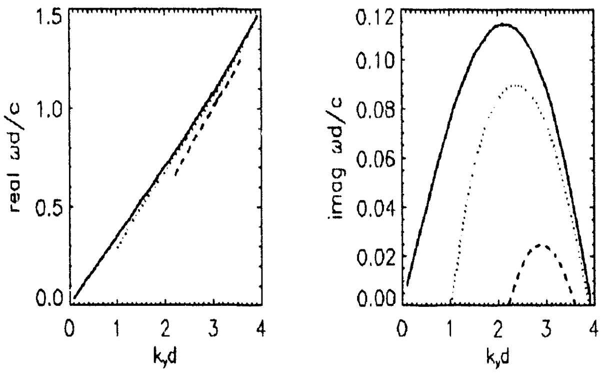


Fig. 3. The dispersion for $\phi_*=1$ and only electron perturbations, $k_z=0$ (solid), $k_z=0.3$ (dotted), $k_z=0.6$ (dashed).

This behavior may explain why the diocotron phase has not been observed when a LiF anode source is used, since it is known that the uniformity of ion current density from these sources is poor.

Figure 4 shows the behavior of the dispersion relation when ion perturbations are included.

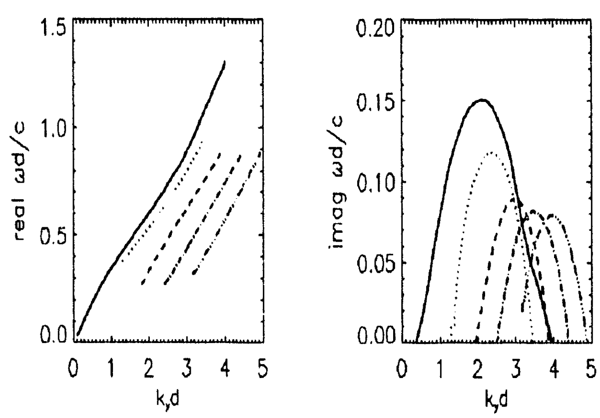


Fig. 4. The dispersion for $\phi_*=1$ with both electron and ion (proton) perturbations, $k_z=0$ (solid), $k_z=0.5$ (dotted), $k_z=1.0$ (dashed), $k_z=1.5$ (dash-dotted), and $k_z=2.0$ (dash-dot-dot).

Notice that the phase-velocity decreases as $k_z$ is increased in the same manner as observed from the constant parameter model. Further studies of this instability revealed the following behaviors that are consistent with the numerical simulations. The growth rate increases with the ion current density as determined by the equilibrium parameters $\alpha,\beta$, and $\phi_*$. The period of the instability is roughly equal to the ion transit time. Thus heavier ions result in a lower frequency. A space-charge limited ion source has the largest growth rate, i.e. a finite electric field at the anode lowers the growth rate. Finally, we found that injecting the ions with an initial velocity reduced the growth rate, which may explain the divergence reduction that has been observed both experimentally and numerically in two-stage diodes.


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