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Two and Three-Dimensional Magnetotelluric Inversion

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Summary

Our overall goal is to develop efficient techniques for high resolution imaging of the electrical structure of the Earth's subsurface. We have focussed on natural source techniques, such as magnetotellurics (MT). The main accomplishment under our past DOE funding has been to implement a new algorithm to invert MT data for multi-dimensional structure which is orders of magnitude faster and more memory efficient than competing algorithms. In our most recent work, we have substantially extended the capability of our two-dimensional code: completed basic implementation of a three-dimensional code and investigated holographic techniques able to rapidly extract images without solving for material properties. The principal new goal of our proposed research is to extend our methods to the controlled source electromagnetic (CSEM) techniques used in many industrial applications.

Introduction

The decade from about 1979-89 saw a dramatic increase in the quality and quantity of magnetotelluric (MT) data. This was the result of the micro-electronics and computer revolutions, innovative time series processing techniques and new field strategies. With these improvements has come new interest in using low-frequency electromagnetic (EM) methods in oil exploration in areas such as New Guinea that are are difficult for seismic work. Recent societal interest in environmental problems such as the invasion of aquifers by salt water and the cleanup of toxic waste sites has provided further impetus for improved EM remote sensing methods. However, the new data have made it obvious that Earth electrical structure is very often multi-dimensional and interpretation without taking this into account can easily lead to wrong or misleading results.

A modern MT profile can involve 10,000 data (fifty sites with four complex impedance estimates at twenty-five frequencies) and a bandwidth of five decades. Statistical errors are commonly as low as 1%. The "standard" algorithms for inverting such data for two-dimensional (2D) structure available prior to our work were based on calculating the derivatives of the data with respect to a large set of 2D model parameters. With perhaps 1000 cells being required to adequately parameterize the structure, the matrix of these derivatives becomes very large and the solution of the linear systems necessary to improve the model and then solve the forward problem for the fields are very time-consuming. Furthermore, since the MT inverse problem for conductivity is non-linear, large inverse problems have to be solved for model and data perturbations multiple times to reach the final model. Computer limitations meant that these algorithms could only be used by decimating the data and severely restricting the model space and they had little hope of being usefully extended to fully three-dimensional (3D) structure.

We have developed an algorithm to directly invert large multi-dimensional magnetotelluric data sets that is orders of magnitude faster than standard algorithms and is equally applicable to 2 or 3D (Smith and Booker, 1991). It is called the Rapid Relaxation Inverse (RRI). This algorithm is iterative. At each iteration, the effects of lateral structural variations are held fixed, while vertical variation of the structure is improved by doing a one-dimensional (1D) inversion at each site. Improved vertical structure is laterally interpolated to improve the the multi-dimensional structure and provide the necessary lateral gradient information necessary for the next iteration. Stabilizing convergence of this process in the presence of data noise proved to be the most difficult hurdle in developing a practical 2D algorithm. Sufficiently accurate solution of the forward problem turned out to be the biggest difficulty in 3D.

Overview of Accomplishments

A complete program package including the basic 2D RRI for continuous models with a flat air-Earth interface plus a series of utilities for color animation of model convergence was released in late 1992. It can be obtained by anonymous ftp or we will mail a diskette. Users with UNIX workstations can execute a script provided with the package that will completely install the software, run a test example and plot the output model on a laser printer. Thirty-one groups worldwide have registered that they have obtained the package. There are undoubtedly others that we do not know about. We have received many compliments about the ease of getting the package operational. A second release is planned in the near future which will include a variety of advances that have come to fruition during our present grant:

(1) The possibility of three types of model constraints: (a) inserting discontinuous boundaries (faults, known basin depth, etc.) within the model; (b) freezing the model value at any node (based, for instance, on well logs); and (c) allowing different measures of roughness in specified local areas. These allow one to incorporate virtually any kind of direct or indirect prior information about the conductivity structure. The ability to use *a priori* constraints is a major desire of potential industrial users and can dramatically increase resolution. (See Figure 1.)

(2) Vertical to horizontal magnetic field transfer functions. The RRI philosophy requires data sensitive to structure beneath each site. To convert vertical field data to this form, we construct spatial averages of the vertical field at adjacent sites. However, we also found it necessary to carry more terms in the perturbation expansion than required for MT data. A simple example is shown in Figure 2. We have not yet tested this enhancement on field data, but expect to shortly.

(3) Two-dimensional resolution windows. Although RRI does not use the full 2D derivatives of the data with respect to the model, we have code to calculate them and use them to construct Backus-Gilbert type resolving windows for any RRI model. This is not done at every iteration, because it involves a severe time penalty.

(4) Improved static shift calculation. The dominant 3D effect is often the electric charges that build up on small scale near-surface structure. Their effect is called static (or sometimes Galvanic) distortion. One of the most important recent advances in MT is impedance tensor decomposition. This allows the response of an underlying 2D structure to be recovered form a statically distorted measured tensor within two frequency-independent constants. The unknown constants multiply the data magnitudes and are often called static shifts, because they shift the level of the log apparent resistivities. These shifts are arbitrary in 1D, but not in 2D. Our original 2D RRI implementation finds the static shift that makes the misfits of the log data magnitude least at any iteration. However, particularly for the TM mode (the polarization with the electric field perpendicular to the strike),

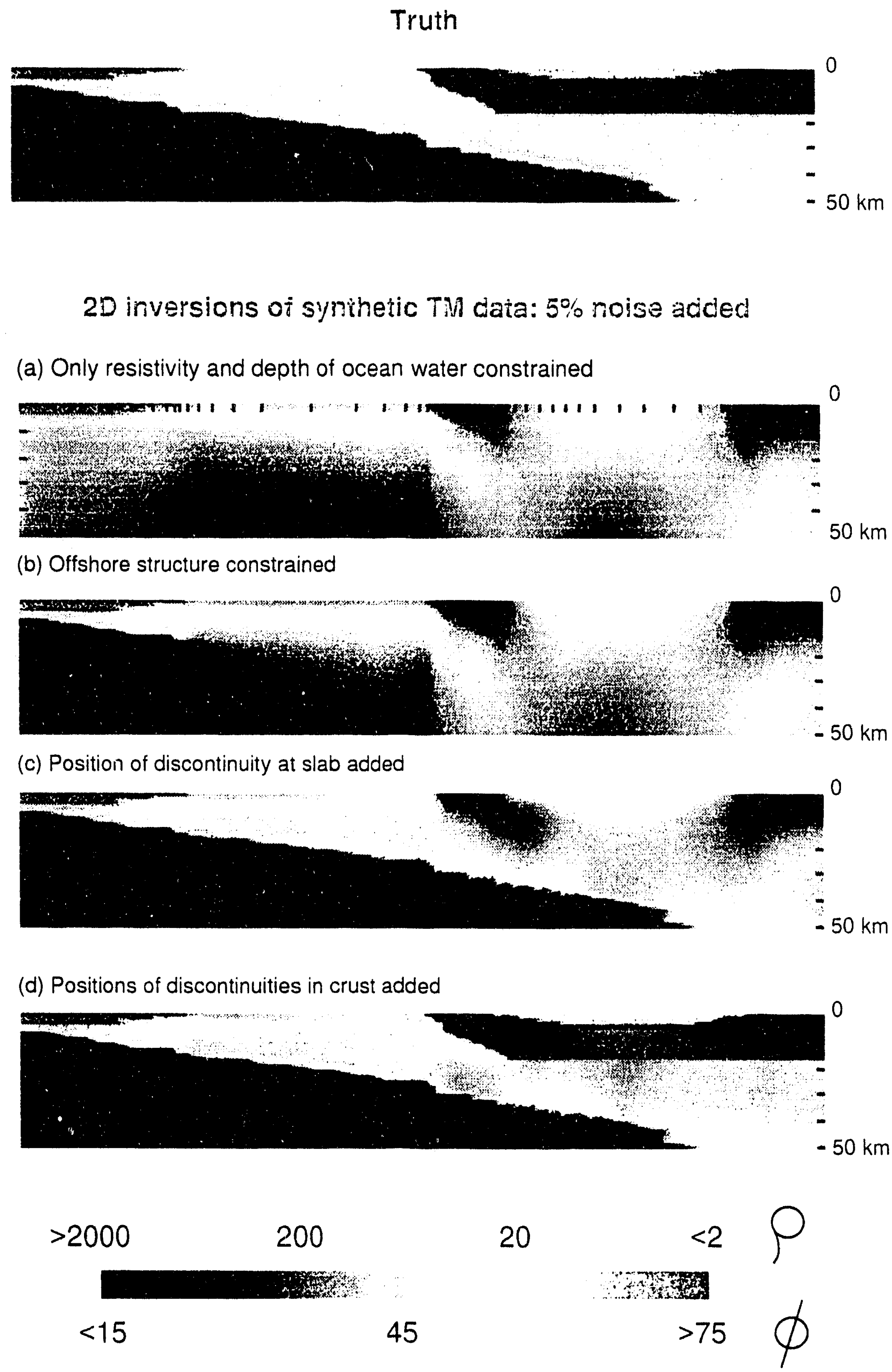
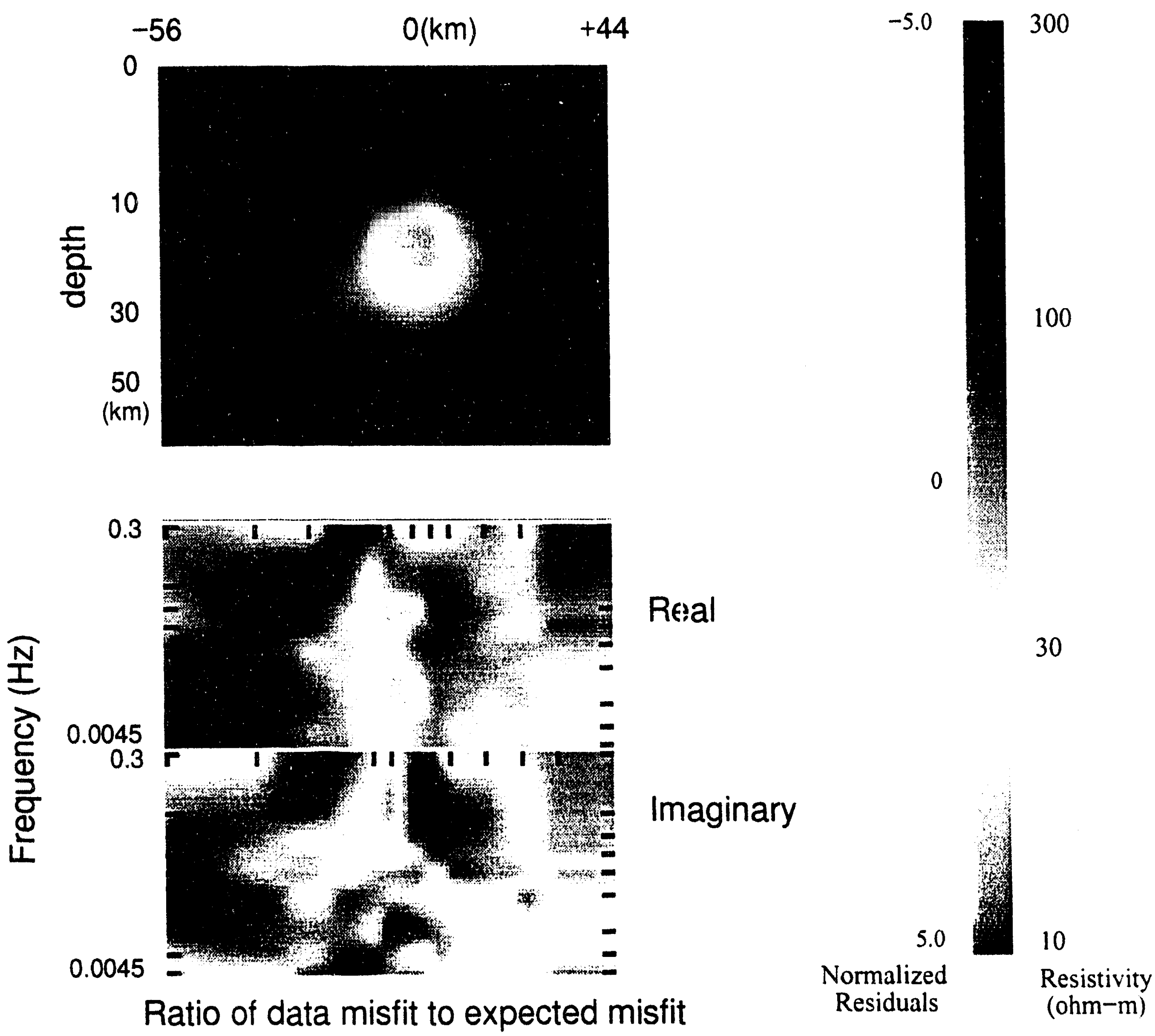


Figure 1. Inversion of data calculated for a hypothesized model of the Cascadia subduction zone through the Olympic Mts. and the Puget Lowland near Seattle. The site locations are shown by the small tick marks on the top of panel (a). Only the 35 sites on the land (to the left of the red, highly conducting ocean) were used in the inversion. The data are TM (electric field perpendicular to the coast) phase and apparent resistivity computed for the truth at 20 frequencies fro 20 to 13.600 seconds. The 5% added noise is Gaussian. Note the considerable increase in structural fidelity as different constraints are added to the inversion.

Figure 2. RRI inversion of vertical to horizontal magnetic field transfer function at the sites shown as tick marks along the top edge of the model. The data were generated from a 10 Ohm–m prism in a 100 Ohm–m halfspace. The dashed line shows the true location of the prism. The maximum data magnitude is 0.09 and the added noise has an expected value of 0.01.



there is almost a complete trade-off between internal model structure and static shifts. The result is that an inversion of TM data can be trapped in a local minimum of the object function (which is a weighted combination of the data misfit and the model roughness) in which magnitude data are very well fit, but phase data are almost ignored. This does not appear as important a a problem when the starting model already fits the phase data well and we have developed a protocol for determining static shifts that uses this fact. Multiple inversions are done. Phase is fit first and then the static shifts are determined. However, recently we appear to have almost entirely overcome the local minimum problem by damping the static shift estimation. Figure 3 compares static shift coefficients calculated automatically using RRI (Wu, et al., 1993), for sites along a profile in the Williston Basin, with those derived by Alan Jones of the Geological Survey of Canada (GSC) using well log data (Jones, 1988). The discrepancy near -70 km is a region where 2D structure probably violates one of Jones' assumptions.

(4) Improved 2D finite difference approximations (FDA). An important element in implementation of air-Earth topography, sea floor sites and the possibility of internal sloping interfaces was our extension of finite difference modelling to triangular meshes (Aprea, et al., 1994). This permits much of the flexibility traditionally ascribed to finite element modelling. This work also resulted in an FDA for the TM mode which considerably improved forward modelling accuracy for continuous models with large gradients and beneficially affected convergence of inversions including TM data. The capability of inverting sea floor sites has recently become of interest in the oil industry because there appears to be a realistic possibility of finding subsalt hydrocarbon traps using sea floor MT and CSEM (i.e. Hoversten and Unsworth, 1994). The topography feature has not yet been used on field data, because in the cases we have looked at, the effects of topography can be absorbed in static shift with considerable saving in computer resources. A related development is a 2D forward code that uses the pseudo-Tchebyshev spectral technique (see Wu, 1993) rather than finite difference approximation to achieve very high accuracy. It is not used by our inversions, because it is rather slow, but is useful for verifying the accuracy of other codes.

(5) Introduction of virtual sites. By its nature, RRI is capable of changing structure only directly beneath each site. Structure between sites is interpolated from improved vertical profiles under the sites. This is clearly a disadvantage if the structure has a major feature located between sites. A simple and effective way to overcome this problem, which does not materially increase computational complexity, is to introduce what we call virtual sites (Wu, et al.}, 1993; Wu and Booker, 1994) above the region where we want added model flexibility. These are sites at which the modified 1D inversions are performed in exactly the same way as real sites. The roughness of the model below each virtual site is included in the object function that measures the fitness of a model, but the data misfit at the virtual site does not (i.e. the data values at the virtual site are arbitrary). This allows structure to build up under the virtual sites if it can help minimize the object function, which does include the data misfit at real sites.

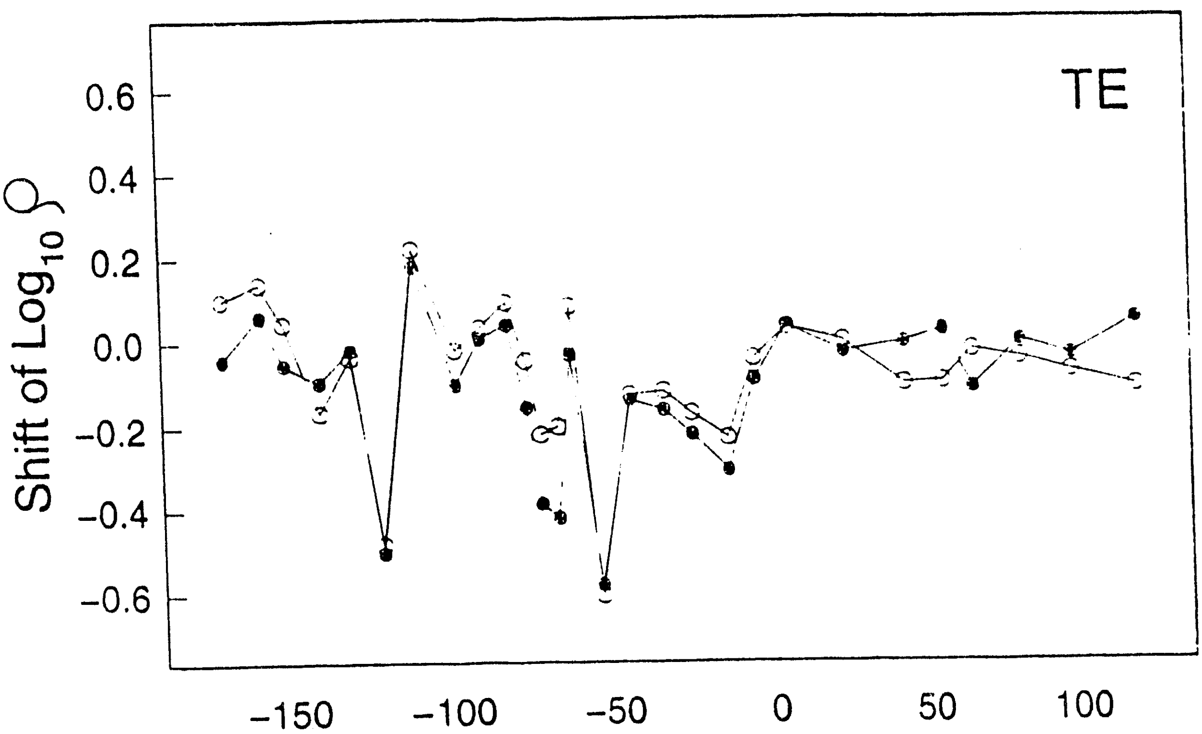


Figure 3. Static shifts: $\log_{10}(\rho_{measured}/\rho_{undistorted})$ calculated by Jones (1988) (closed circles) and by Wu, *et al.* (1993) using RRI (open circles). Jones chooses the static shift to make the second layer in a 1D inversion of high frequency data at each site be 3 Ohm-m. The choice of this layer resistivity is based on information from numerous well logs. RRI inverts for the static shifts as free parameters. The biggest disagreement between the two methods is at the sites near -75 km. It is almost certainly because local, shallow lateral variations in this area are not properly accounted for in Jones' use of local 1D inversions.

(6) Improved convergence control. To solve the initial instabilities associated with RRI, we introduced a variety of damping schemes (some of which can be turned on or off by a sophisticated user). We now know that some of these are too aggressive and can be relaxed. One of these called "null space error limiting" in the released version detects the rapid rise in model roughness that accompanies overfitting data and stalls the fitting process. In many cases, this works so well that one can simply declare the desired misfit to be very small and let the program estimate the actual level of misfit that can reasonably be achieved and hence estimate the actual errors in the data. This is very useful for field data that report errors that are too optismistic (i.e. too small).

(7) Input and output files that are a subset of those required for our 3D code. 3D input files require more information and this is simplified by a change in input philosophy that involves using "include" commands that allow actual input information to reside in many places. Ultimately, our goal is to have a single 3D code that can be used equally effectively for 2D work. At present, however, many of the advanced features of the 2D code, such as the ability to freeze nodes, are not implemented in the 3D code. Changing the input format at this time will simplify user transition to the ultimate code at a later time. The change in input philosophy is driven by the desire to make the codes interface in a simple way with a commercial MT interpretation package called Geotools-MT. (This is fundamentally a data base product for transparently exchanging MT data, manipulating MT data, preparing input files for interpretation software such as RRI and visualizing the results. It frees the user from onerous bookkeeping. Geotools-MT is becoming widely used in the exploration industry and NSF has recently acquired an unlimited cpu license for the U.S. academic research and education community.) The new input files are not backward compatible, but the conversion is straightforward.

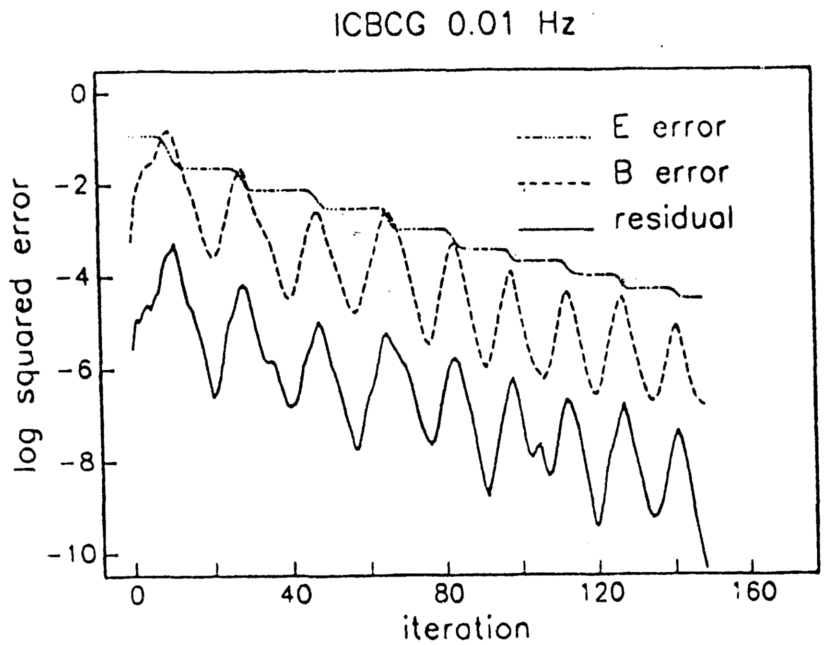
We have also developed a new impedance tensor decomposition (Smith, 1994; Chave and Smith, 1994) (closely related to that of Bahr (1988) in Germany) whose calculation is almost linear and whose physical interpretation is simpler than the more widely used decomposition of Groom and Bailey (1989; 1991) (G-B). The important parameters are the angles through which two perpendicular electric fields in the underlying 2D structure are rotated by the shallow 3D structure and the ratio of the amplification of these two fields. The G-B parameters are transcendental functions of our parameters, which explains their greater non-linearity and means that the G-B parameters can be trivially recovered. The near linearity of the Smith decomposition considerably simplifies estimation of parameter errors and stablizes the strike estimation. It also simplifies adding additional frequencies or sites to further stablize strike estimation. We have implemented our decomposition and validated it on synthetic examples discussed by G-B and others. Tests on field data are in progress.

An important achievement in connection with our code for calculating the fields in 3D models is an *ex post facto* estimate of the errors in the fields using a spatial filtering technique that is both easy and accurate (Smith, 1992). Our most important result, however, is a method of guaranteeing conservation of electric current that goes beyond the use of a staggered grid (Smith, 1992). We use an iterative scheme to solve the sparse linear system associated with the 3D forward problem. Every few iterations, we solve a 3D Poisson equation for the erroneous electric fields due to charges associated with the failure of current conservation. These are used to correct the overall electric field. Figure 4 demonstrates the startling improvement in the rate of convergence that can occur. Without the correction, the iterative process takes more than a hundred iterations to achieve a modest reduction in residual. With the correction, the residual decreases to machine precision in tens of iterations.

Having developed a fast and accurate 3D forward code, we have made rapid progress on 3D RRI. Figure 5 shows 3D inversions of synthetic data with different polarizations. Only three frequencies (.2 .02 .002 Hz) and a rather small mesh (31x: 27y: 24z) were used to permit rapid turn-around (about half an hour on an HP750 or Sparc 10) during this de-bugging phase. This limits the resolution, but the results are obviously very encouraging.

Finally, EM Migration is a holographic technique in which "wavefronts" are traced back to scatterers and foci. Such images can potentially be generated much faster than inverting the data for material properties. However, one cannot proceed as in seismic migration and simply reverse time flow, because the diffusion equation obeyed by electromagnetic fields is unstable in the presence of noise when time is reversed. Our collaborator Michael Zhdanov has shown that treating the received signal with time reversed as a source and diffusing this signal downwards, one should be able to recover the phase (but not the magnitude) of the backward propagated signal inside the earth in a stable way. Funded by a small subcontract from our grant, he has been examining the properties of EM migration of noisy data. Figure 6 is an example from Zhdanov, et al. (1994) that clearly demonstrates the stability of migration in the presence of noise. Research continues on the effects of noise due to uncertainty in the background structure in which the migrated fields are calculated.

Figure 4. Convergence history for three-dimensional (3D) electromagnetic forward computations. Both cases use a finite difference (FD) mesh in which the electric (**E**) and magnetic (**B**) fields are defined at nodal meshes that are staggered with respect to each other. This staggered grid is designed so that FD approximations of $\nabla \cdot$ and $\nabla \times$ obey $\nabla \cdot \nabla \times (\mathbf{E} \text{ or } \mathbf{B}) = 0$. These relations are a statement of the physical necessity for conservation of electric current. Failure of current conservation is one of the major sources of error in 3D FD calculations. The top figure relies only on the staggered grid to enforce current conservation. In the lower figure, the iterative method (Incomplete Cholesky preconditioned Bi-Conjugate Gradient) used to solve the sparse linear system is alternated with solution of a Poisson equation for the erroneous **E** of charge accumulations due to residual failure of current conservation. This so-called "static correction" becomes more important as frequency becomes lower and can easily result in an order of magnitude imprevement in the rate of convergence.



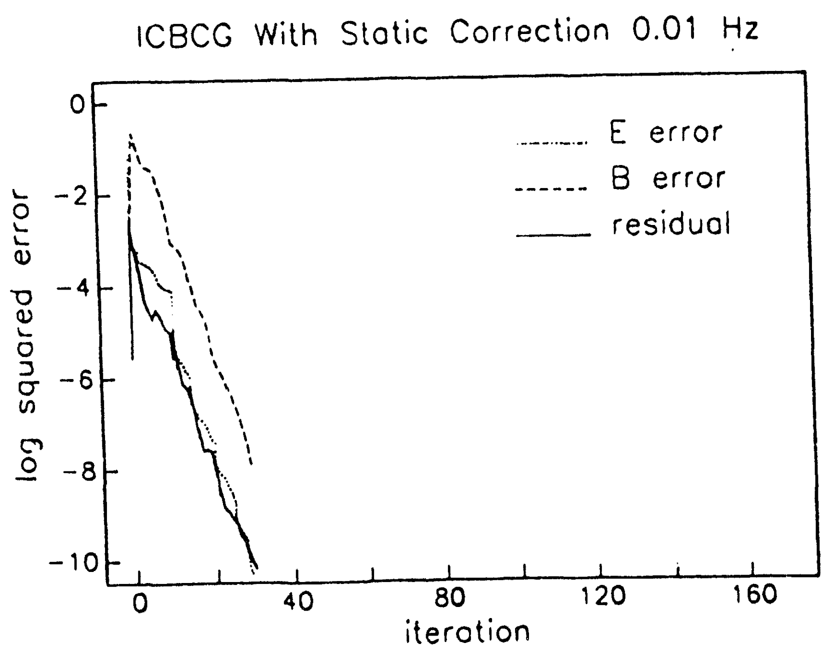


Figure 5

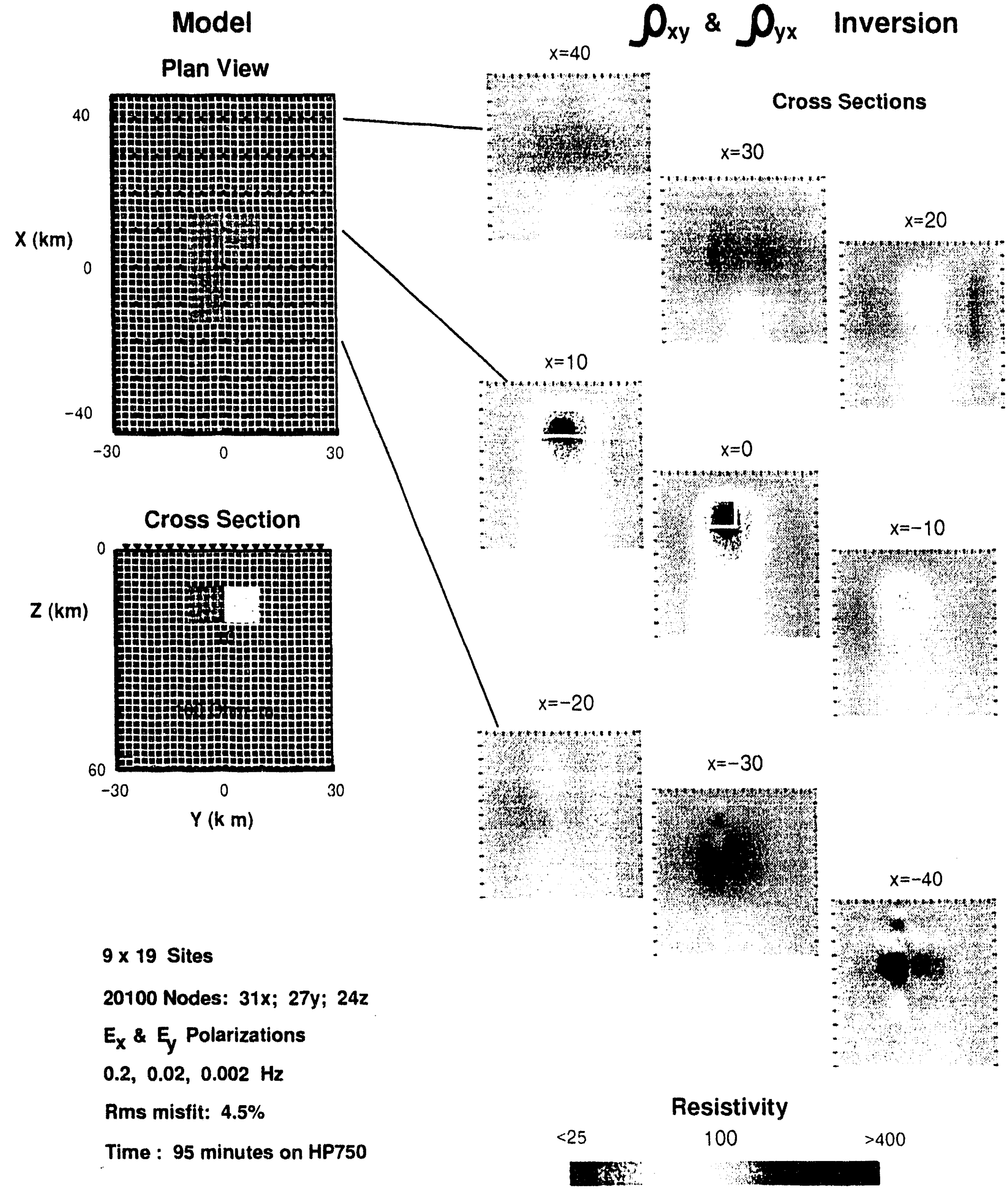
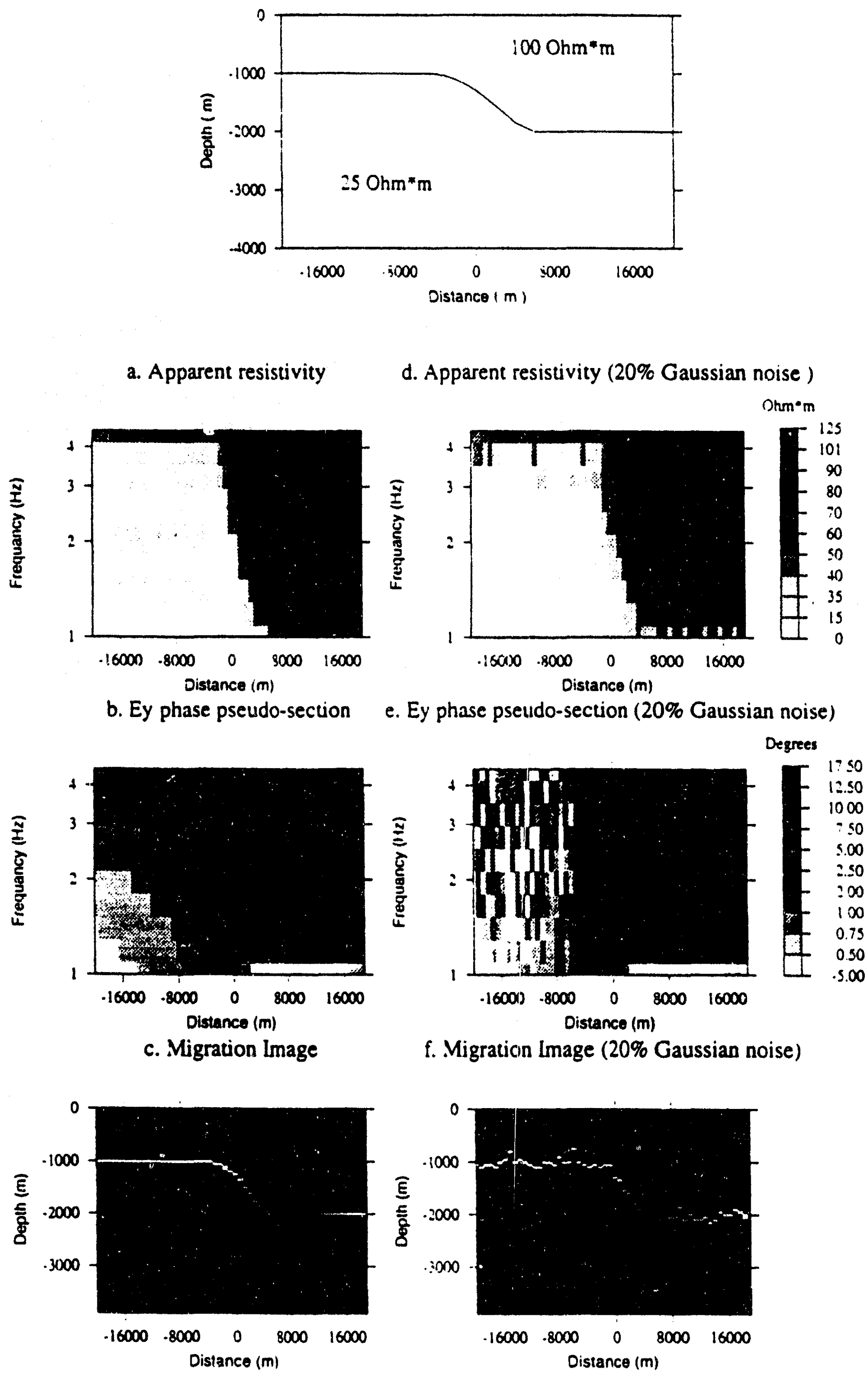

Model
Plan View
40
X (km)
0
−40
−30
0
30
Cross Section
0
Z (km)
60
−30
0
30
Y (k m)
9 x 19 Sites
20100 Nodes: 31x; 27y; 24z
Ex & Ey Polarizations
0.2, 0.02, 0.002 Hz
Rms misfit: 4.5%
Time : 95 minutes on HP750
ρxy & ρyx Inversion
Cross Sections
x=40
x=30
x=20
x=10
x=0
x=−10
x=−20
x=−30
x=−40
Resistivity
<25
100
>400

Figure 6

Resistivity model



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