

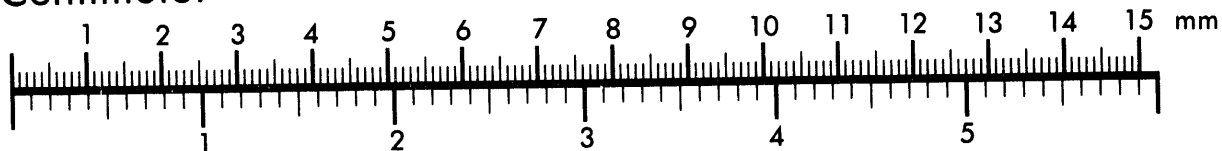


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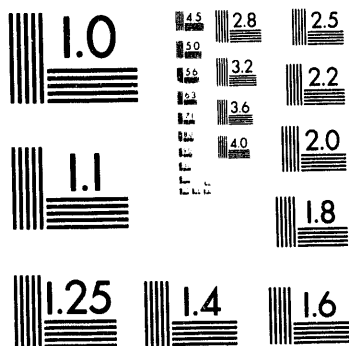
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**INTERPOLATION OF SCATTERED TEMPERATURE DATA  
MEASUREMENTS ONTO A WORLDWIDE REGULAR GRID  
USING RADIAL BASIS FUNCTIONS WITH  
APPLICATIONS TO GLOBAL WARMING**

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# Interpolation of Scattered Temperature Data Measurements Onto a Worldwide Regular Grid Using Radial Basis Functions With Applications to Global Warming

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## Abstract

Using temperature data from over 5,500 airports worldwide, we calculated estimates of temperature on a regularly spaced grid of terrestrial points using a method based on radial basis functions. We will discuss the methodology used to map the interpolated values and compare our results to the results using other methods.

## Introduction

Our current research into the response of natural ecosystems to a hypothesized climatic change (e.g. global warming) requires that we have estimates of various meteorological variables on a regularly spaced grid of points on the surface of the earth. Unfortunately, the bulk of the world's meteorological measurement stations is located at airports that tend to be concentrated on the coastlines of the world (see Fig. 1) or near populated areas. We can also see that the spatial density of the station locations is extremely non-uniform with the greatest density in the USA, followed by Western Europe. Furthermore, the density of airports is rather sparse in desert regions such as the Sahara, the Arabian, Gobi, and Australian deserts; likewise the density is quite sparse in cold regions such as Antarctica Northern Canada, and interior northern Russia. The Amazon Basin in Brazil has few airports. The frequency of airports is obviously related to the population centers and the degree of industrial development of the country.

We address the following problem here. Given values of meteorological variables, such as maximum monthly temperature, measured at the more than 5,500 airport stations depicted in Fig. 1, interpolate these values onto a regular grid of terrestrial points spaced by one degree in both latitude and longitude. This is known as the scattered data problem, and it has received considerable attention in the literature.

## Methodology

In Franke's [1] classical paper, he evaluated 29 different algorithms for scattered data interpolation using six known bivariate test-functions over randomly generated data sets containing 25, 33, and 100 scattered data locations over a unit square. Only the function values were given at the data locations. Using each of the methods to be tested, he interpolated the results onto a 33x33 uniformly spaced grid. Surprisingly, the standard methods of triangulation, finite elements, Shepard algorithm performed poorly as compared to radial basis function methods that include Hardy's [2,3] multiquadrics and reciprocal multiquadrics. From these data sets, he interpolated the solutions onto a 33x33 grid, and graded the performance of the various interpolants based on accuracy, visual appeal, ease of implementation, and effort expended.

Of the methods tested, he graded Hardy's [2,3] multiquadrics (MQ) the best. He gave much lower grades to more traditional methods such as triangulation, Shepard's method, etc.

Given a domain,  $\Omega$ ,  $\mathbb{R}^d \supset \Omega$ , containing  $N$  loci,  $\underline{x}_i$  and dependent values,  $f(\underline{x}_i)$ ,  $i=1,2,\dots,N$  any piecewise continuous function can be expanded in terms of  $N$  basis functions,

$$f(\underline{x}) = \sum_j \alpha_j g(\underline{x} - \underline{x}_j)$$

where  $g$  represents the  $C^\infty$  MQ basis function given by

$g(x - x_j) = [ (x - x_j)^2 + \Delta^2 ]^{1/2}$  where  $\Delta^2 > 0$  is an input parameter.

The set of expansion coefficients,  $\{\alpha_j\}$ , are the solution to the following set of linear equations,

$$f(x_i) = \sum_j \alpha_j g(x_i - x_j)$$

Micchelli [4] proved that the MQ interpolation is always solvable. Madych and Nelson[5] proved that as the density of the data centers increased, the rate of convergence increases exponentially. Buhmann[6] proved that as the dimension,  $d$ , of the space increases, the rate of convergence also increases.

The set of linear equations using the MQ basis functions is full; the number of operations required to find the expansion coefficients varies as  $O(N^3)$ . Likewise, the condition number of likewise increases as  $N$  increases. Foley[7] showed that domain decomposition and blending across subdomain cuts was preferable because the exceptional accuracy and convergence rates of MQ could be maintained without the need for dealing with large systems of linear equations. Note, a very large problem decomposed into many overlapping subdomains is readily solvable on parallel machines.

Other authors have proposed methods of extrapolating station data of environmental or climatic data to a gridded set. Leemans and Cramer [8] have developed a database of average monthly temperature, precipitation, and cloudiness. The weather records used by Leemans and Cramer were constrained to the years 1930 to 1960 and were gridded to the terrestrial portion of a global grid of  $0.5^\circ$  for the grid cell lengths. They used triangulation of all data points developed by Green and Sibson [9] and then a smooth surface fitting algorithm developed by Akima [10] to produce the final gridded set. They rejected spherical interpolation on the basis of great circle influences across the poles should be small compared to within-latitudinal influences. Legates and Willmott [11,12] gridded temperature and precipitation station data, respectively. They use a spherical interpolation method derived from Shepherd's [13] weighted interpolation method. The resulting data sets were for the  $0.5^\circ$  by  $0.5^\circ$  lattice. They use a spherical method because earlier work of Willmot et al., [14] suggested that significant errors can occur when Cartesian-based method are used for climate fields.

We followed the recommendations of Foley[7] in our problem of interpolating the scattered data airport temperature onto a grid of every degree of longitude and latitude. First, we note that the MQ scheme is valid only where the monthly averaged temperatures are continuous within landmass. We deliberately avoided extrapolating land based temperatures into water.

The procedure we used was to collect data locations into overlapping rectangular bins. If a bin within a land mass contained no locations, it was enlarged until it contained at least three points. For bins containing small islands with only one or two airport locations, a constant averaged temperature was assumed. Within those bins containing 5 or more airport locations, we used MQ to interpolate the temperatures onto a grid of every degree of latitude and longitude. In the overlap region between two or more bins, we used Foley's blending scheme. For the most part, the MQ solutions in the overlap regions amongst the different bins were nearly identical when the data density was high, and for undersampled regions such as in the polar regions or deserts, the mismatch was in the worst cases about 4-5 degrees F.

Because some of the coast lines are very irregularly shaped, especially the peninsulas it was not possible to always enclose a land mass within a rectangular bin without doing some extrapolation over a body of water. When this occurred, we nulled out those regions over which extrapolation occurred. We have presented our interpolation of land based airport monthly averaged temperatures for the month of January where it is winter in the northern hemisphere and summer in the southern hemisphere. Note, we did not interpolate the temperatures over Antarctica because the data was vastly undersampled over such a large land mass.

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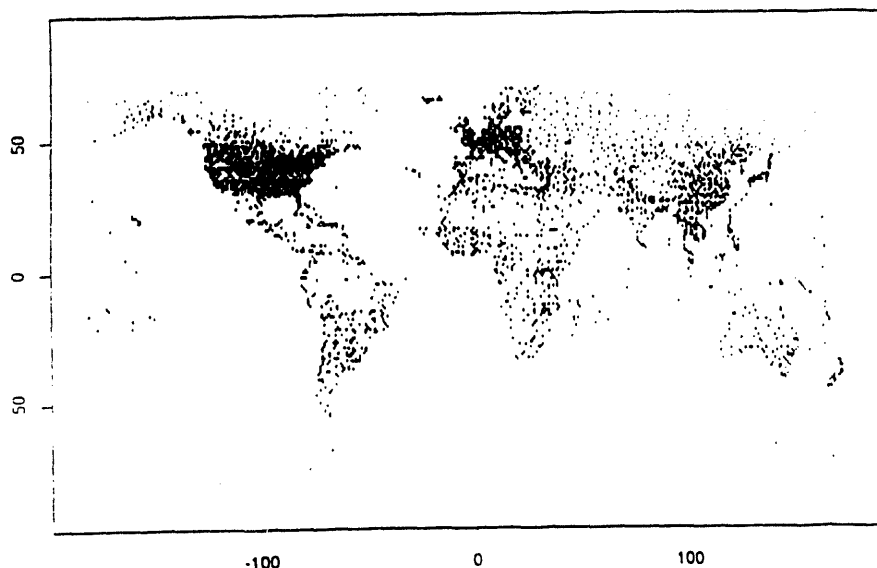


Fig.1. The scattered data set of 5500 airport temperatures.

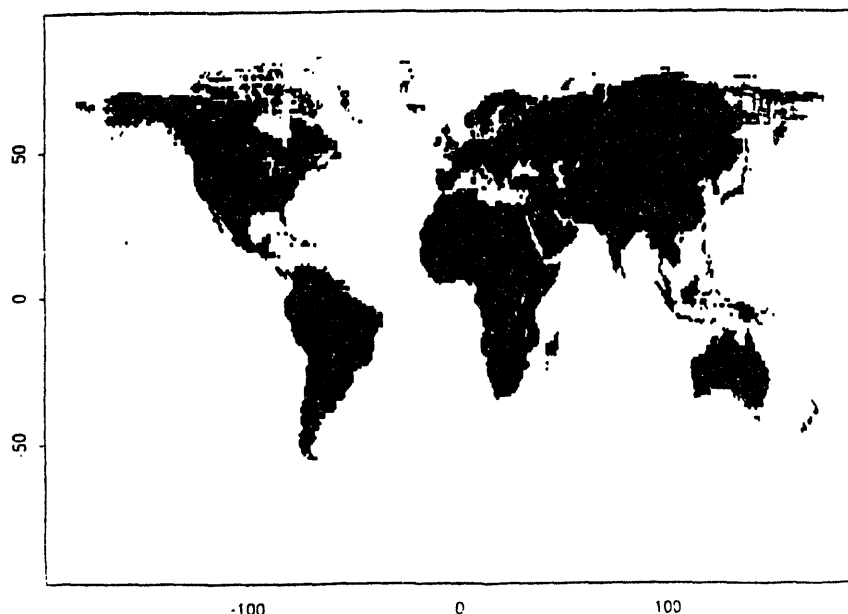


Fig.2. The global interpolated temperature map using MQ.

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