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Intermittency via Moments and Distributions in central O+Cu collisions at 14.6 A·GeV/c

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ABSTRACT

Fluctuations in pseudorapidity distributions of charged particles from central (ZCAL) collisions of ^{16}O+Cu at 14.6 A·GeV/c have been analyzed by Ju Kang using the method of scaled factorial moments as a function of the interval $\delta\eta$ An apparent power-law growth of moments with decreasing interval is observed down to $\delta\eta \sim 0.1$, and the measured slope parameters are found to obey two scaling rules. Previous experience with E_T distributions suggested that fluctuations of multiplicity and transverse energy can be well described by Gamma or Negative Binomial Distributions (NBD) and excellent fits to NBD were obtained in all $\delta\eta$ bins. The k parameter of the NBD fit was found to increase **linearly** with the $\delta\eta$ interval, which due to the well known property of the NBD under convolution, indicates that the multiplicity distributions in adjacent bins of pseudorapidity $\delta\eta \sim 0.1$ are largely **statistically independent**.

1. Normalized Factorial Moments Ju Kang has been making an intermittency analysis of TMA data in central ^{16}O+Al, Cu and Ag collisions. The centrality cut was made using ZCAL and requiring that the forward energy be less than one projectile nucleon (i.e. $T_{ZCAL} <$ 13.6 GeV). In order to study fluctuations as a function of size of rapidity interval, Bialas and Peschanski proposed to use Normalized Factorial Moments and further proposed that "spikes" in single-event rapidity distributions were evidence of **fractal** or **"intermittent"** behavior. Ju uses the 'classical' method of Normalized Factorial Moments in which the "Vertical" Factorial Moments are defined

$$< F_q >_V = \frac{1}{M} \sum_{m=1}^{M} \frac{< k_m(k_m-1)\ldots(k_m-q+1) >}{< k_m >^q} \quad , \tag{1}$$

where M is the number of bins of size $\delta\eta$ into which the distribution is divided, k_m is the multiplicity in the m'th bin on a given event and the $< \; >$ brackets indicate averaging over all events.

The scaled factorial moments $F_2, \ldots, F_6$ are shown in Fig. 1 in a log-log plot vs $\delta\eta$. All the moments show a clear power law behavior down to $\delta\eta \sim 0.1$. The observed slopes $\phi_q = -d(lnF_q)/d(ln\delta\eta)$ obey two scaling rules illustrated in Fig. 2:

$$\phi_q = \frac{q(q-1)}{2}\phi_2 \, , \quad \phi_2 \sim 1/[(\Delta\phi/2\pi)(dN/d\eta)] \quad . \tag{2}$$

The Factorial Moment with the clearest interpretation is

$$F_2 = \frac{< n(n-1) >}{< n >^2} = \frac{< n^2 > - < n >}{< n >^2} = \frac{\sigma^2 + < n >^2 - < n >}{< n >^2} = 1 + \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} \quad (3)$$

where $\mu \equiv < n >$. Note that the Normalized Factorial Moments are all equal to unity for a Poisson Distribution. The negative binomial distribution of an integer m is defined as

$$P(m) = \frac{(m+k-1)!}{m!(k-1)!} \frac{(\frac{\mu}{k})^m}{(1+\frac{\mu}{k})^{m+k}} \quad (4)$$

where $P(m)$ is normalized for $0 \leq m \leq \infty$, $\mu \equiv < m >$.

2. Negative Binomial Distribution Fits

I always wondered what "intermittent" behavior would look like in terms of the distributions and I asked Ju Kang to send me the multiplicity distributions as a function of $\delta\eta$ so that I could try to figure out what intermittency meant for the distributions, while he did the "classical" analysis. He kindly obliged by sending me the multiplicity distributions for central ^{16}O+Cu in bins of $\delta\eta$= 0.1,0.2, 0.3...,0.5,1.0, where the bin of 1.0 covers $1.2 \leq \eta \leq 2.2$ in the laboratory. These are shown in Fig. 3 together with the results of NBD fits. The fits are all excellent. Note that the exact details of the centrality cut are important for Fig. 3 and presumably also for the intermittency analysis by moments. If the usual 7% TMA cut, or even a PbGl cut had been made, the multiplicity distributon for $\delta\eta = 1.0$ would have been truncated by a sharp cut near the peak. This would have wildly distorted the moments. This is easy to understand by thinking of the moments of a Gaussian compared to the moments of the upper half of a Gaussian truncated at the mean value. Of course, the key to the intermittency analysis is how the distribution develops as $\delta\eta$ is reduced and it isn't clear how a sharp cut would have affected that analysis. The excellent fits of the NBD to all the distributions makes it credible that the ZCAL centrality cut, being an indirect cut on multiplicity, may in fact give a shape for the rising (lower) half of the distribution that has a real physical meaning.

The k parameter for the NBD fits is shown in Fig. 4 and indicates a totally unexpected and striking **linear** dependence on $\delta\eta$. The linear evolution of the NBD fit paramater k with $\delta\eta$ is an indication that the multiplicity in each rapidity bin is largely statistically independent of that in the next bin for central ^{16}O+Cu collisions. We learn this because the linear evolution of the NBD parameter k means that the multiplicity distributions convolute as the bin size is extended. The fact that our measured multiplicity distributions are excellently fit by distributions (NBD) with well known properties under convolution enable us to make this observation by inspection. This explains and demystifies "intermittency". **Intermittency is nothing more than the indication that the multiplicity in each pseudorapidity bin of size $\delta\eta \sim 0.1$ is largely staticically independent of that in the next bin for central O+A collisions.**

The analysis is only briefly noted in this report. Extensive further analyses and conclusions exist and should be published shortly.

3. Acknowledgements

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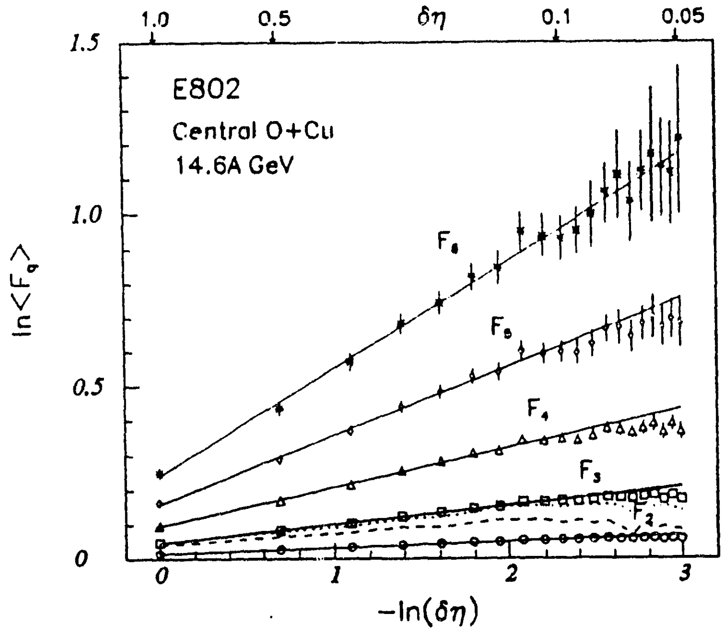

Fig. 1: Scaled factorial moments for central collisions of ^{16}O+Cu. Solid lines represent linear fits to the moments up to $\delta\eta = 0.1$, which are then extrapolated to $\delta\eta = 0.05$.

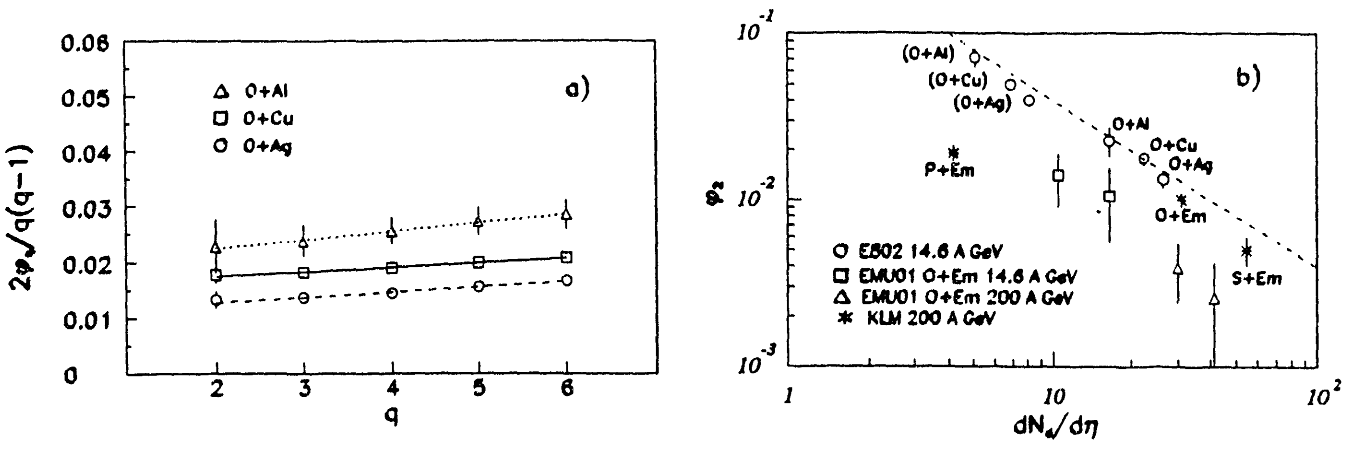

Fig. 2: Tests of scaling rules: a) $2\phi_2/q(q-1)$ as a function of q. Lines are drawn to guide the eye. b) Slopes of F_2 as a function of the average detected multiplicity density $(\Delta\phi/2\pi)(dN/d\eta)$ within $\Delta\phi = 200^o$ (and $\Delta\phi = 60^o$ in parentheses). Previous data are shown as indicated. The dashed line is a power law with exponent -1.

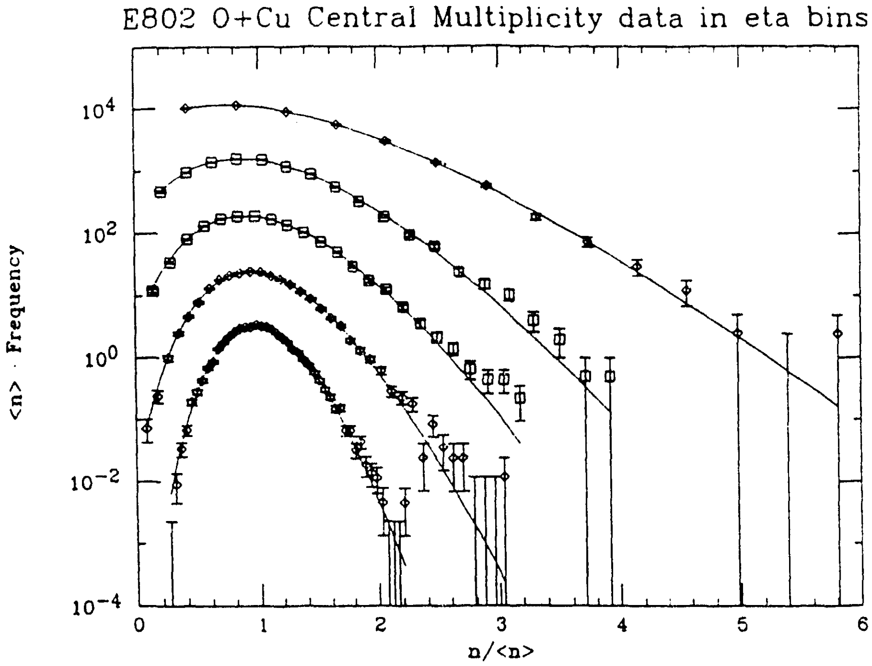

Fig. 3: E802 multiplicity distributions for O+Cu central collisions in individual $\delta\eta$ bins as indicated. The data are plotted scaled in multiplicity by the mean $< n >$ in each bin. The solid lines are NBD fits.

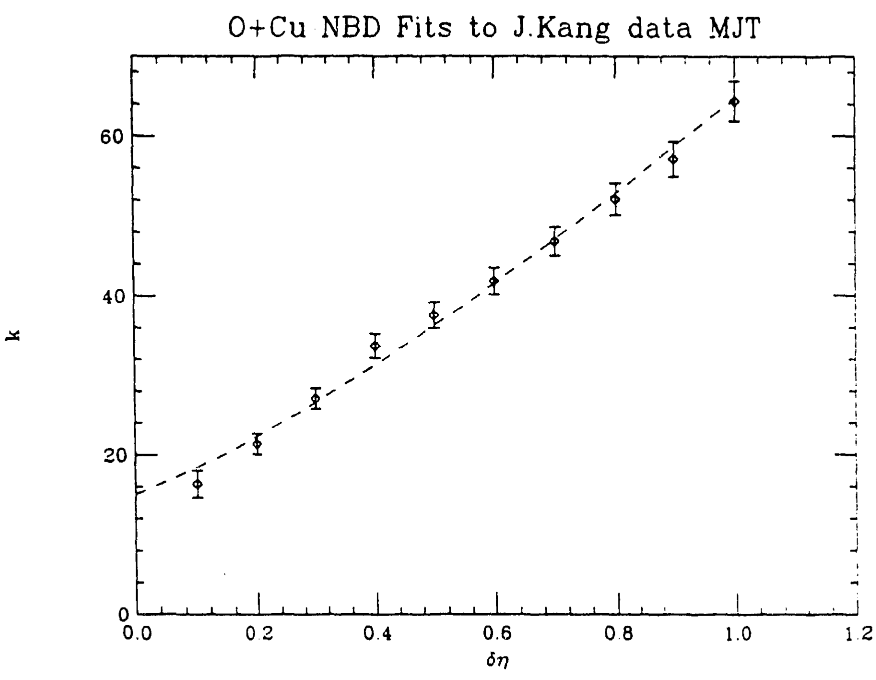

Fig. 4: The k parameter from fits to the data in Fig. 3 as a function of $\delta\eta$. While the data appear to indicate a strong linear relationship between k and $\delta\eta$ the dashed line is a more interesting representation indicating in mathematical form that if the multiplicity in adjacent $\delta\eta$ bins is correlated, the correlation length is $\xi \sim 0.12$.

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