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IMPULSIVE RESPONSE OF NONUNIFORM DENSITY LIQUID IN A LATERALLY EXCITED TANK

by

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ABSTRACT

A study on the impulsive component of the dynamic response of a liquid of nonuniform density in a tank undergoing lateral base excitations is presented. The system considered is a circular cylindrical tank containing an incompressible and inviscid liquid whose density increases with the liquid depth. The density distribution along the depth can be of any arbitrary continuous or discontinuous function. In the analysis, the liquid field is divided into n layers. The thickness of the liquid layers can be different, but the density of each liquid layer is considered to be uniform and is equal to the value of the original liquid density at the mid-height of that layer. The problem is solved by the eigenfunction expansion in conjunction with the transfer matrix technique. The effect of the nonuniform liquid density on the impulsive component of the dynamic response is illustrated in a numerical example in which the linear and cosine distributions of the liquid density are assumed. The response quantities examined include the impulsive pressure, base shear and moments. The results are presented in tabular and graphical forms. It is found that the impulsive pressure distribution along the tank wall is not sensitive to the detailed distribution function of the density, and the base shear and moments for the nonuniform liquid can be estimated by assuming an equivalent uniform liquid density that preserves the total liquid weight. The effect of tank flexibility is assessed by a simple approach in which the response quantities for flexible tanks are evaluated by simplified equations.

I. INTRODUCTION

A large number of high level waste (HLW) storage tanks at various U.S. Department of Energy (DOE) facilities contain liquids with nonuniform density. In order to evaluate the structural integrity of these HLW tanks under seismic events and to provide the necessary method of analysis for the future design of the HLW tanks, it is necessary to study the dynamic behavior of a liquid of nonuniform density subjected to ground excitations. To respond to this need comprehensive studies on a tank containing two liquids have been performed in the past two years by Tang and Chang at Argonne National Laboratory (ANL) [1,2,3] as the first step toward understanding the dynamic behavior of a liquid of nonuniform density in a tank. The ground excitations considered included both horizontal and rocking components of earthquake motions. Both rigid and flexible tanks were studied. Recently, a study on the sloshing response of a liquid of nonuniform density in a tank undergoing lateral base excitations has been performed by Tang and Chang [4]. In that study, it was shown that the sloshing response in a tank containing a liquid of nonuniform density is quite different from that of an identical tank containing uniform density liquid. Especially, the sloshing wave height may increase significantly in tanks containing a liquid of nonuniform density. Following that study and presented in this report is the impulsive component of the solution to a tank containing a liquid of nonuniform density.

The objectives of this study are: (1) to develop a method of analysis for the computation of the impulsive component of the dynamic response of a liquid of nonuniform density contained in a tank undergoing lateral base excitations; (2) to examine the results of analysis from which the effect of nonuniform density of the liquid on the dynamic response can be elucidated; and (3) to propose a simple approach with which this effect can be evaluated cost-effectively for the preliminary design. In the analysis, the liquid field is divided into n layers along its height. These liquid layers may have different thickness, but the liquid density of each layer is assumed to be uniform and its value is taken to be the value of the original density at the mid-height of that layer. The eigenfunction expansion in combination with the transfer matrix technique is employed to solve the problem. The response quantities examined include the impulsive hydrodynamic pressure, base shear and moments at sections immediately above and below the tank base plate. The tank wall is assumed to be rigid. The effect of the flexibility of the tank wall is assessed by a simple approached in which simplified equations for evaluating the dynamic response are proposed. For preliminary designs, this simple approach is very cost-effective.

II. SYSTEM DESCRIPTION

The tank-liquid system investigated is shown in Fig. 1. It is a ground-supported upright circular cylindrical tank of radius R filled with a nonuniform liquid to a height of H. The density of the liquid is assumed to have a minimum value, denoted by ρ_t, at the top of the liquid surface. The liquid density is assumed to increase monotonically with the increase of the liquid depth and reaches a maximum value, denoted by ρ_b, at the bottom of the liquid. The tank is assumed to be rigid and clamped to a rigid base. The liquid is considered to be incompressible and inviscid. The response of the liquid is assumed to be linear. The cylindrical coordinate system, r, θ, and z, is employed for the study with the origin defined at the center of the tank base where $\theta = 0$ is assumed to be the direction of the seismic excitation. The lateral excitation considered herein is denoted by $\ddot{x}(t)$. The temporal variation of $\ddot{x}(t)$ can be of any arbitrary function.

III. APPROACH AND SOLUTIONS

The liquid field is first divided into n layers as shown in Fig. 2. The thickness of the Layer j (j = 1, 2, 3...n) is denoted by H_j. The thickness of the liquid layers can be different. The liquid density for Layer j is taken to be the value of the original liquid density at mid-height of the layer, and it is denoted by ρ_j. It is assumed that the liquid is uniform in each layer. Thus, the mathematical model that represents the physical system depicted in Fig. 1 has n layers of liquids with different thickness and densities. For the convenience of derivation, a local cylindrical coordinate system, r, θ, z_j, is introduced for the Layer j where z_j is related to z by the equation

$$z = z_j + \sum_{k=1}^{j-1} H_k \text{ for } 0 \le z_j \le H_j \qquad (1)$$

Given the conditions that the liquids are incompressible and inviscid, the hydrodynamic pressure induced at Layer j, denoted by p_j, must satisfy the Laplace equation

$$\nabla^2 p_j = 0 \text{ for } j = 1, 2, 3 \ldots n \tag{2}$$

in the region $0 \leq r \leq R$, $0 \leq \theta \leq 2\pi$, and $0 \leq z_j \leq H_j$. The liquid acceleration at an arbitrary point in Layer j along s-direction is related to p_j by

$$a_s = -\frac{1}{\rho_j}\frac{\partial p_j}{\partial s} \tag{3}$$

The boundary conditions are:

1. The vertical acceleration of liquid at the tank base must be zero; i.e.,

$$\left.\frac{\partial p_1}{\partial z_1}\right|_{z_1=0} = 0 \tag{4}$$

2. The radial acceleration of liquid adjacent to the tank wall must equal the acceleration of the tank wall, i.e.,

$$\left.-\frac{1}{\rho_j}\frac{\partial p_j}{\partial r}\right|_{r=R} = \ddot{x}(t)\cos\theta, \qquad j = 1, 2 \ldots n \tag{5}$$

3. At the free surface, the boundary condition for the impulsive pressure is

$$\left.p_n\right|_{z_n=H_n} = 0 \tag{6}$$

At the interface of Layers j and j+1, the boundary conditions are:

4. Continuity of the vertical acceleration, i.e.,

$$-\frac{1}{\rho_j}\frac{\partial p_j}{\partial z_j}\bigg|_{z_j=H_j} = -\frac{1}{\rho_{j+1}}\frac{\partial p_{j+1}}{\partial z_{j+1}}\bigg|_{z_{j+1}=0} \tag{7}$$

and

5. Continuity of the hydrodynamic pressure, i.e.,

$$p_j\big|_{z_j=H_j} = p_{j+1}\big|_{z_{j+1}=0} \tag{8}$$

The method of separation of variables is used to solve the differential equation (2). The boundary conditions, Eqs. (4) (6) (7) and (8), are used to determine the eigenfunctions and eigenvalues, and the boundary condition (5) is used to determine the integration constants.

A. Eigenfunctions

The eigenfunction, denoted by ϕ_{jk}, for Layer j corresponding to the kth eigenvalue, denoted by λ_k, is given by

$$\phi_{jk}(z) = A_{jk}\cos(\lambda_k z_j) + B_{jk}\sin(\lambda_k z_j) \tag{9}$$

where the proportional constants, A_{jk} and B_{jk}, are related to A_{j+1k} and B_{j+1k} by the equation given by

$$\begin{Bmatrix} A_{j+1k} \\ B_{j+1k} \end{Bmatrix} = \begin{bmatrix} \cos(\lambda_k H_j) & \sin(\lambda_k H_j) \\ -\frac{\rho_{j+1}}{\rho_j}\sin(\lambda_k H_j) & \frac{\rho_{j+1}}{\rho_j}\cos(\lambda_k H_j) \end{bmatrix} \begin{Bmatrix} A_{jk} \\ B_{jk} \end{Bmatrix} \qquad j = 1 \ldots n-1 \tag{10}$$

From Eq. (4), one obtains

$$B_{1k} = 0 \qquad \text{for all } k \tag{11}$$

Equation (10) defines a relation between the proportional constants of jth layer and those of the (j+1)th layer. By repeating application of Eq. (23) from $j = 1$ to $j = n-1$ and making use of Eq. (11), one obtains the equation that links A_{1k} with A_{nk} and B_{nk}. Symbolically, this equation can be written as

$$\begin{Bmatrix} A_{nk} \\ B_{nk} \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} A_{1k} \\ 0 \end{Bmatrix} \tag{12}$$

where the 2×2 matrix in Eq. (12) is the transfer matrix, and T_{11}, T_{12}, T_{21} and T_{22} are its elements. It can be shown that the orthogonal condition for the kth and ℓth eigenfunctions is given by

$$\sum_{j=1}^{n} \frac{1}{\rho_j} \int_0^{H_j} \phi_{jk}(z_j)\, \phi_{j\ell}(z_j)\, dz_j = 0 \qquad \text{for } k \neq \ell \tag{13}$$

B. <u>Eigenvalues</u>

Equation (6) requires that

$$A_{nk}\cos(\lambda_k H_n) + B_{nk}\sin(\lambda_k H_n) = 0 \tag{14}$$

Substituting the constants A_{nk} and B_{nk} in Eq. (14) with those obtained from of Eq. (12), one obtains

$$T_{11} \cos(\lambda_k H_n) + T_{21} \sin(\lambda_k H_n) = 0 \tag{15}$$

which is the characteristic equation, and its roots are the eigenvalues.

For n = 2, it can be shown that Eq. (15) yields

$$\cos(\lambda H_1) \cos(\lambda H_2) - \frac{\rho_2}{\rho_1} \sin(\lambda H_1) \sin(\lambda H_2) = 0 \tag{16}$$

which is the characteristic equation for tanks containing two liquids [5].

Then, it can be shown that the impulsive pressure, p_j, that satisfies Eq. (2) takes the form

$$p_j(r,\theta,z_j) = \sum_{k=1}^{\infty} C_k \, I_1(\lambda_k r) \, \phi_{jk}(z_j) \cos\theta \tag{17}$$

where the integration constant C_k can be determined from the boundary condition defined by Eq. (5) by making use of the eigenfunction expansion and the orthogonal condition defined by Eq. (13). The resultant can be expressed as

$$p_j(r,\theta,z_j) = C_{0j}(r,z_j) \left(\rho_b \ddot{x}(t) R \cos\theta\right) \qquad j = 1...n \tag{18}$$

where the function $C_{0j}(r,z_j)$ is given by

$$C_{0j}(r,z_j) = -\frac{H}{R} \sum_{k=1}^{\infty} \frac{1}{\lambda_k} \frac{\Delta_k}{||\phi_k||} \frac{I_1(\lambda_k r)}{I'_1(\lambda_k R)} \phi_{jk}(z_j) \tag{19}$$

in which I_1 is the modified Bessel function of the first kind and I'_1 is its derivative; function Δ_k is given by

$$\Delta_k = \sum_{j=1}^{n} \int_0^{H_j} \phi_{jk}(z_j)\,dz_j \tag{20}$$

and $||\phi_k||$ is given by

$$||\phi_k|| = \sum_{j=1}^{n} \frac{\rho_b}{\rho_j} \int_0^{H_j} \phi_{jk}^2(z_j)\,dz_j \tag{21}$$

1. <u>Base Shear</u>

After the impulsive pressure is obtained, the base shear, denoted by Q(t), is computed from the following equation

$$Q(t) = \sum_{j=1}^{n} \int_0^{2\pi} \int_{0_{H_j}} p_j\Big|_{r=R} R\cos\theta\,dz_j\,d\theta \tag{22}$$

Substituting Eq. (18) into Eq. (21) and performing the integration, one obtains Q(t) which may be expressed as

$$Q(t) = r_s\, M_\ell^1\, \ddot{X}(t) \tag{23}$$

in which $M_\ell^1 = \pi\rho_b R^2 H$ = the total liquid mass if the tank is filled with a liquid having a density of ρ_b. Since Eq. (22) involves only simple integration of the trigonometry functions, the expression for r_s is not given herein.

2. Base Moments

The base moment at a section immediately above the tank base, M(t), is obtained by performing the following integration

$$M(t) = \sum_{j=1}^{n} \int_0^{2\pi} \int_0^{H_j} p_j \Big|_{r=R} Rz\cos\theta \, dz_j \, d\theta \tag{24}$$

and the result is cast into the form

$$M(t) = r_M \, M_t^1 \, H \, \ddot{x}(t) \tag{25}$$

The base moment induced by the pressure exerted on the tank base is denoted by ΔM(t) which is given by the following equation

$$\Delta M(t) = \int_0^{2\pi} \int_0^{R} p_1 \Big|_{z_1=0} r^2\cos\theta \, dr \, d\theta \tag{26}$$

and may be expressed in the form as

$$\Delta M(t) = \Delta r_M \, M_t^1 \, H \, \ddot{x}(t) \tag{27}$$

The base moment at the section immediately below the tank base, denoted by M′(t), is then given by the sum of Eqs. (25) and (27). The result is expressed as

$$M'(t) = r'_M \, M_t^1 \, H \, \ddot{x}(t) \tag{28}$$

in which

$$r'_M = r_M + \Delta r_M \tag{29}$$

IV. NUMERICAL RESULTS

Unlike the case of tanks containing uniform density liquid where H/R is the only parameter that controls the response, the parameters that control the response of tanks containing a liquid of nonuniform density are H/R, ρ_t/ρ_b, and the density variation along the liquid depth. For the numerical study presented herein, the variation of the liquid density between ρ_t and ρ_b is assumed to be either a linear or a cosine function. More precisely, if $\rho(z)$ represents the density at liquid depth of z, the linear function is given by

$$\rho(z) = \rho_t + \left(\frac{H - z}{H}\right)(\rho_b - \rho_t) \tag{30}$$

and the cosine function is given by

$$\rho(z) = \rho_t + \cos\left(\frac{\pi z}{2H}\right)(\rho_b - \rho_t) \tag{31}$$

It is important to determine the number of layers needed to approximate the liquid field in order to get accurate results. Therefore, the study for the convergence of the numerical results for base shear and base moments is performed first, the values of r_s, r_M and r'_M obtained by using different number of layers are presented in Table I for $\rho_t/\rho_b = 0.25$ and Table II for $\rho_t/\rho_b = 0.5$ assuming either linear or cosine variation for the liquid density. In these tables two values of H/R, 0.5 and 3, are considered and for simplicity the thickness of the layers are taken to be the same. As one can see from these tables that with n = 30, the results are quite accurate. Thus all solutions presented hereinafter are obtained by using n = 30. It should be mentioned herein that the approach presented in this report is implemented into a FORTRAN computer program that runs on PC 486 machine, and for n = 30 the CPU time is less than three minutes.

To study the effect of the nonuniform liquid density on the dynamic response, the response quantities for uniform, linear and cosine liquid density functions are computed and compared. In order to make the comparison meaningful, the total liquid mass in the tank is kept the same. If the total liquid mass in a tank for the linear variation case is chosen to be the base liquid mass for comparison, it can be shown that for the uniform density case the equivalent uniform liquid density, ρ^*, has to be

$$\rho* = \frac{1}{2}\left(1 + \frac{\rho_t}{\rho_b}\right)\rho_b \tag{32}$$

and for cosine variation case, the liquid density defined by Eq. (31) has to be multiplied by a coefficient given by

$$\left[\frac{1}{2}\left(1 + \frac{\rho_t}{\rho_b}\right)\right] \Big/ \left[\frac{2}{\pi} + \left(1 - \frac{2}{\pi}\right)\frac{\rho_t}{\rho_b}\right] \tag{33}$$

Presented in Tables III, IV and V are the comparisons of the coefficients for the base shear and moments for the linear, cosine and uniform variation cases. The coefficients defined in Eqs. (32) and (33) have been applied to the results for the uniform and cosine variation cases, respectively. Four values of H/R, 0.5, 1, 2 and 3, and three values of ρ_t/ρ_b, 0.25, 0.5 and 0.75, are considered in these tables. Examining these three tables, one notices that for the linear and cosine variation cases, the response quantities are very close. This indicates that the response is not sensitive to the detailed density variation along the depth. The difference, however, is observed between the results for the uniform case and those of the linear and cosine cases. The values for the base shear are larger for the uniform case than those for the linear and cosine cases for broad tanks ($H/R \leq 1$); however, for tall tanks ($H/R \geq 2$) the opposite trend is observed. The values for the base moments are larger for the uniform case for all the tanks considered in comparison with those for the linear and cosine cases. In order to gain more insight into the problem, the impulsive pressure exerted on the tank wall for tanks with H/R = 0.5, 1, 2 and 3 are plotted in Figs. 3, 4, 5 and 6 for the value of ρ_t/ρ_b = 0.25 and 0.5. It is clearly shown in these figures that the pressure for the linear and cosine cases are very close; however, the pressure for the uniform case is larger at the upper portion and smaller in the lower portion of the height. Therefore, the center of gravity for the enclosed area by the curve corresponding to the uniform case is higher than those of the linear and cosine variation cases. This explains why the moments are higher for the uniform variation case. Examining the data more critically, one may find that the results for the uniform case are about 15% higher than those of the other two cases. Realizing the conservatism required for design, one may conclude that the response for tanks filled with a liquid of nonuniform density may be evaluated, at least for the preliminary design, from the solutions for tanks filled with the equivalent uniform density liquid.

A. Effect of Tank Flexibility

The procedure proposed herein to assess the effect of tank wall flexibility is basically an extension of the simple procedure presented in Ref. 6 for tanks containing a uniform liquid. The procedure requires that the ground acceleration $\ddot{x}(t)$ in the rigid tank solution be replaced by the pseudoacceleration function $A_0(t)$ which is defined by

$$A_0(t) = \frac{\omega}{\sqrt{1 - \zeta^2}} \int_0^t \ddot{x}(\tau)\, \exp[-\zeta\omega(t-\tau)] \sin(\bar{\omega}(t-\tau))\, d\tau \tag{34}$$

where ω = the fundamental circular frequency of the tank-liquid system; ζ = the fraction of critical damping; and $\bar{\omega} = \omega\sqrt{1-\zeta^2}$. The resulting expressions for the response quantities from the simple procedure are given as follows.

The impulsive wall pressure is expressed as

$$p_j(\theta, z_j, t) = C_{0j}(R, z_j)\, A_0(t)\, \rho_b R \cos\theta \tag{35}$$

The base shear is expressed as

$$Q(t) = r_s \, M_t^1 \, A_0(t) \tag{36}$$

and the base moments are expressed as

$$M(t) = r_M \, M_t^1 \, H \, A_0(t) \tag{37}$$

and

$$M'(t) = r'_M \, M_t^1 \, H \, A_0(t) \tag{38}$$

The maximum values of these response quantities may be obtained by replacing $A_0(t)$ by its spectral value.

The most crucial part of this simple procedure is the evaluation of the fundamental natural frequency of the tank-liquid system. In this report, the simple equation presented in Ref. 3 for estimating the fundamental frequency of tank-liquid systems for tanks containing two liquids is extended herein to permit the consideration of the liquid density to be nonuniform. It has been shown [3] that this simple equation yields accurate results for tanks with values of H/R in the range between 0.25 and 1.0. This equation is based on two assumptions: (1) the effective mass of the tank-liquid system that participates in the first mode of vibration is equal to the impulsive component of the liquid added mass in the rigid tank, and (2) the mass of the tank wall itself is negligible in comparison to that of the liquid.

Let the fundamental frequency of the tank-liquid system be expressed as

$$f = \frac{1}{2\pi} \frac{c}{H} \sqrt{\frac{E}{\rho}} \tag{39}$$

in which E = Young's modulus of elasticity for the material of the tank wall; ρ = its mass density; and c = a dimensionless coefficient. Let the values of f and c for a tank filled with nonuniform liquid be designated by f^* and c^* and for the same tank filled with water be designated by f_w and c_w; then, c^* can be estimated from c_w by the equations given below.

For a rigid tank filled with water, the base shear, denoted by Q_w, can be expressed as [7]

$$Q_w(t) = \left(r_w \pi \rho_w R^2 H\right) \ddot{x}(t) \tag{40}$$

where r_w is a dimensionless coefficient depending on the value of H/R. The quantity enclosed by the parenthesis in Eq. (40) is the so-called liquid added mass. So, based on the assumptions mentioned above the stiffness of the tank, denoted by k_1, for the first mode of vibration is then given by

$$\begin{aligned} k_1 &= \left(r_w \pi \rho_w R^2 H\right) (2\pi f_w)^2 \\ &= \left(r_w \pi \rho_w R^2 H\right) \left(\frac{c_w}{H} \sqrt{\frac{E}{\rho}}\right)^2 \end{aligned} \tag{41}$$

For the same tank filled with a nonuniform liquid, the liquid added mass can be identified from Eq. (23). It is $r_s \rho_b \pi R^2 H$. So, the fundamental frequency, f^*, can be computed from the equation given by

$$f^* = \frac{1}{2\pi} \sqrt{\frac{k_1}{\left(r_s \rho_b \pi R^2 H\right)}} \tag{42}$$

Substituting Eq. (41) into Eq. (42), one obtains the equation that relates c* with c_w which is given by

$$c* = c_w \sqrt{\frac{r_w \rho_w}{r_s \rho_b}} \tag{43}$$

Extensive numerical data for the value of r_w and c_w for steel tanks filled with water have been reported in Refs. 7 and 8.

V. CONCLUSIONS

An analytical method for computing the dynamic response to earthquakes for tanks filled with a liquid of nonuniform density is presented. The response quantities examined include the impulsive component of the hydrodynamic pressure, base shear and moments. Unlike the cases of tanks containing a liquid of uniform density in which the response is controlled by one parameter, H/R, the response of tanks containing a liquid of nonuniform density is controlled by the parameters, H/R, ρ_t/ρ_b, and the density variation along the liquid depth. In the study presented herein, the variation functions are assumed to be either a linear or a cosine function. The results show that the response is not sensitive to the detailed density variation along the liquid depth. This study also shows that the response for tanks filled with a liquid of nonuniform density may be estimated by assuming that tank is filled with an equivalent uniform density liquid in which the equivalent uniform density is determined such that the total liquid mass is kept the same. Finally, a simple equation is derived for estimating the fundamental natural frequency of the tank-liquid system, and simple approach for computing the response quantities for flexible tanks is proposed.

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Table I. Convergence Table of Base Shear and Moments for $\rho_l/\rho_b = 0.25$

No. of Layers	r_s		r_m		r'_m	
	Linear	Cosine	Linear	Cosine	Linear	Cosine
	H/R = 0.5					
2	0.169	0.203	0.0625	0.0754	0.249	0.298
5	0.164	0.193	0.0585	0.0692	0.240	0.284
10	0.163	0.192	0.0577	0.0682	0.238	0.282
20	0.163	0.191	0.0577	0.0680	0.238	0.281
30	0.163	0.191	0.0577	0.0679	0.238	0.281
40	0.163	0.191	0.0577	0.679	0.238	0.281
	H/R = 3.0					
2	0.533	0.632	0.206	0.247	0.227	0.272
5	0.540	0.630	0.201	0.249	0.225	0.265
10	0.541	0.629	0.201	0.237	0.224	0.263
20	0.541	0.629	0.200	0.237	0.224	0.263
30	0.541	0.629	0.200	0.237	0.224	0.263
40	0.541	0.629	0.200	0.237	0.224	0.263

Table II. Convergence Table of Base Shear and Moments for $\rho_f/\rho_b = 0.5$

No. of Layers	r_s		r_m		r'_m	
	Linear	Cosine	Linear	Cosine	Linear	Cosine
	H/R = 0.5					
2	0.215	0.237	0.0823	0.0909	0.314	0.347
5	0.211	0.231	0.0799	0.0870	0.309	0.338
10	0.211	0.230	0.0795	0.0864	0.309	0.337
20	0.211	0.230	0.0794	0.0862	0.308	0.337
30	0.211	0.220	0.0794	0.0862	0.308	0.337
40	0.211	0.229	0.0794	0.0862	0.308	0.337
	H/R = 3.0					
2	0.641	0.706	0.262	0.290	0.286	0.316
5	0.645	0.705	0.260	0.285	0.284	0.311
10	0.645	0.705	0.259	0.284	0.284	0.310
20	0.645	0.704	0.259	0.284	0.284	0.310
30	0.645	0.704	0.259	0.284	0.284	0.310
40	0.645	0.704	0.259	0.284	0.284	0.310

Table III. Comparison of Base Shear Coefficient, r_s

	$\rho_f/\rho_b = 0.25$			$\rho_f/\rho_b = 0.5$		
H/R	Linear	Cosine	Uniform	Linear	Cosine	Uniform
0.5	0.163	0.164	0.188	0.211	0.210	0.225
1.0	0.317	0.319	0.342	0.399	0.399	0.411
2.0	0.480	0.480	0.477	0.580	0.580	0.572
3.0	0.541	0.540	0.526	0.645	0.646	0.631

Table IV. Comparison of Base Shear Coefficient, r_m

	$\rho_f/\rho_b = 0.25$			$\rho_f/\rho_b = 0.5$		
H/R	Linear	Cosine	Uniform	Linear	Cosine	Uniform
0.5	0.0577	0.0584	0.0749	0.0790	0.0790	0.0899
1.0	0.113	0.114	0.138	0.151	0.151	0.166
2.0	0.174	0.176	0.202	0.227	0.227	0.242
3.0	0.200	0.204	0.231	0.259	0.260	0.277

Table V. Comparison of Base Shear Coefficient, r'_m

	$\rho_f/\rho_b = 0.25$			$\rho_f/\rho_b = 0.5$		
H/R	Linear	Cosine	Uniform	Linear	Cosine	Uniform
0.5	0.238	0.241	0.275	0.308	0.308	0.330
1.0	0.228	0.228	0.247	0.288	0.286	0.296
2.0	0.221	0.222	0.239	0.278	0.277	0.286
3.0	0.224	0.226	0.248	0.284	0.284	0.298

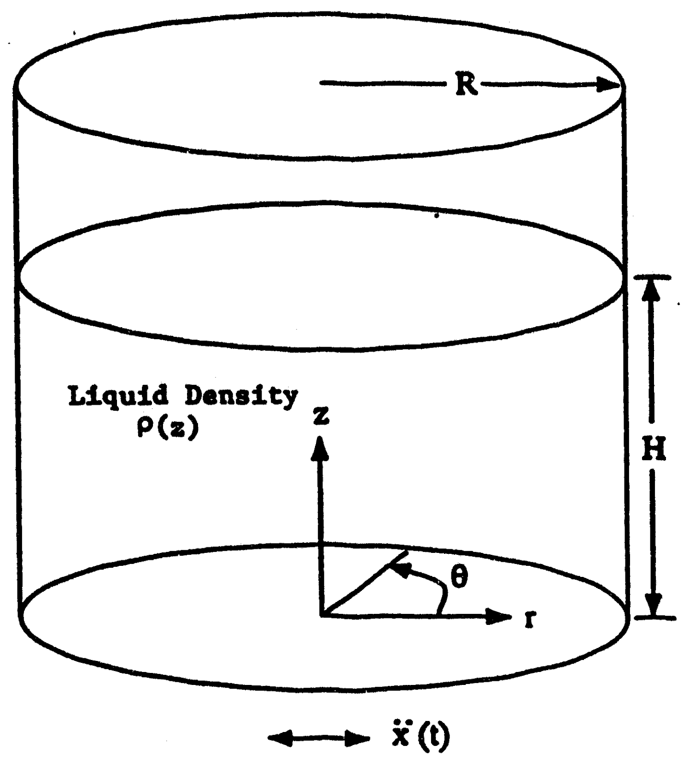

Fig. 1. System Considered

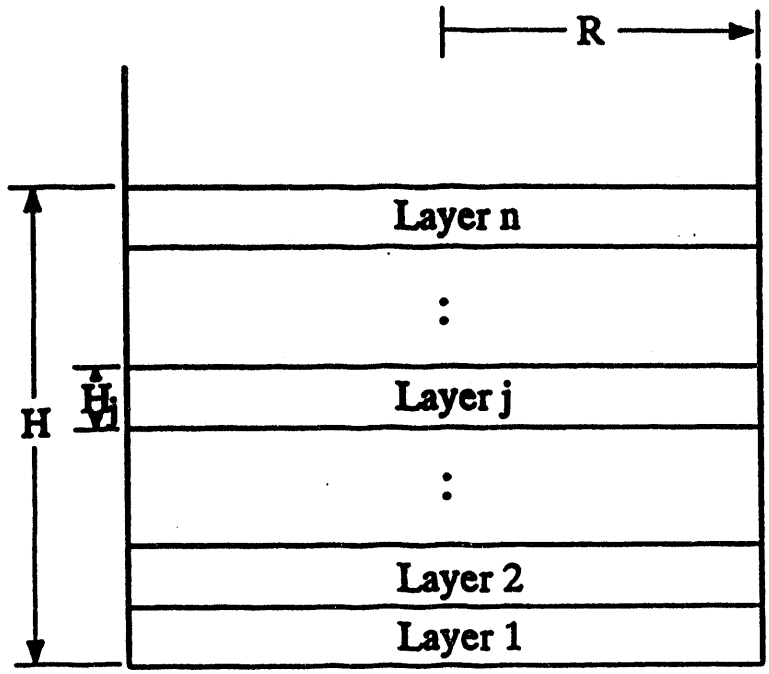

Fig. 2. Mathematical Model for the Liquid Field

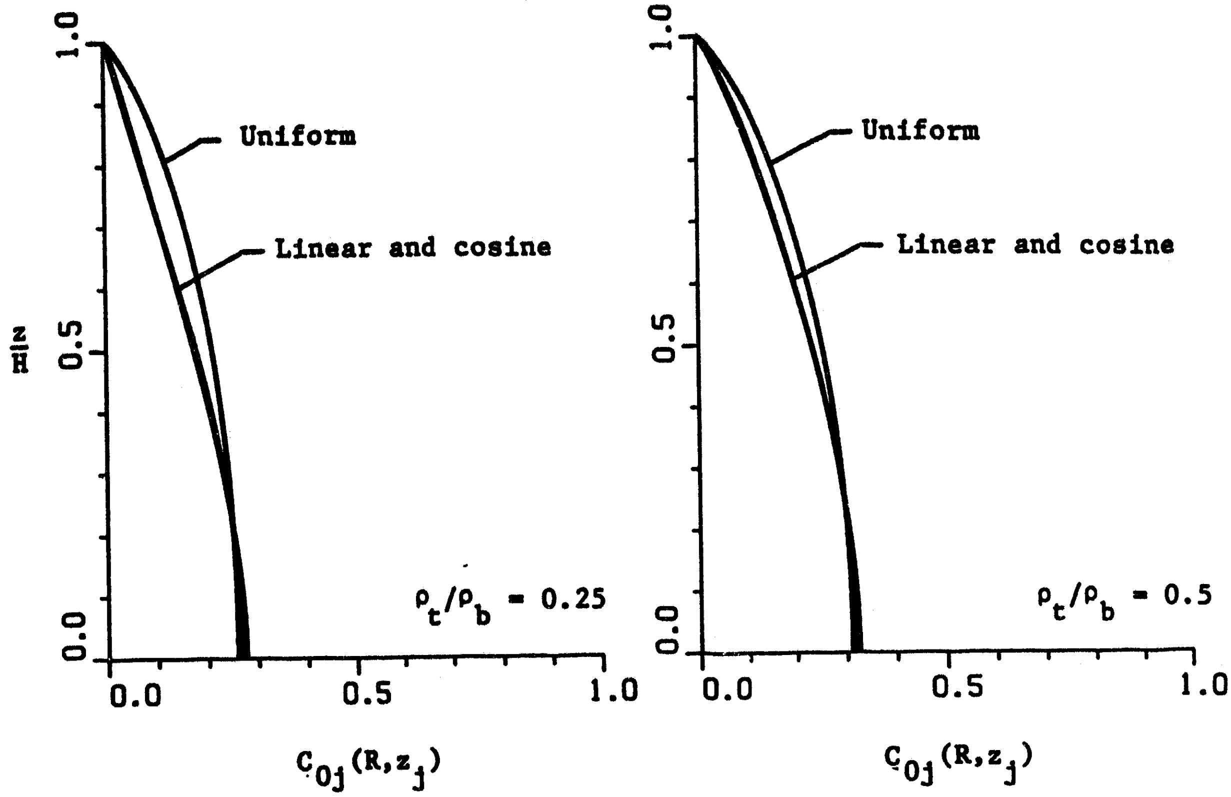

Fig. 3. Impulsive Pressure Exerted on Wall of Rigid Tank with H/R = 0.5

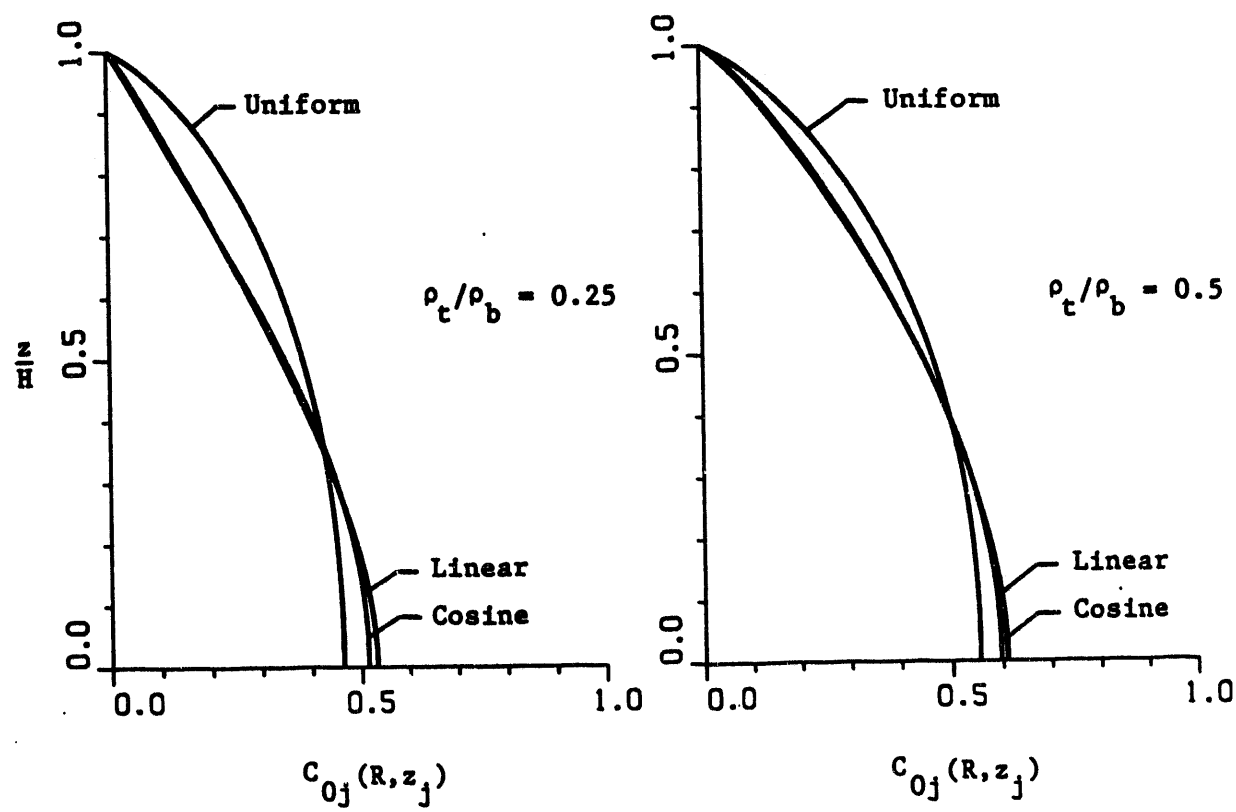

Fig. 4. Impulsive Pressure Exerted on Wall of Rigid Tank with H/R = 1

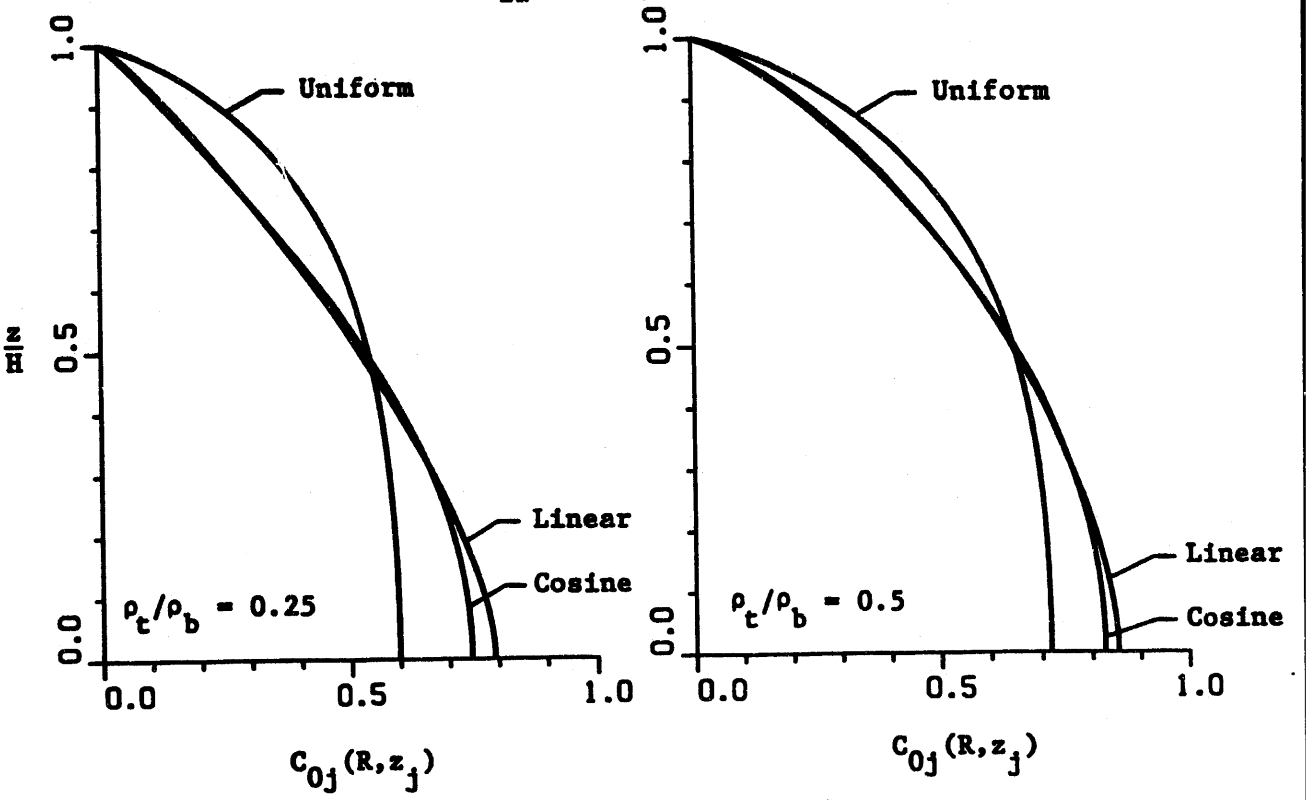

Fig. 5. Impulsive Pressure Exerted on Wall of Rigid Tank with H/R = 2

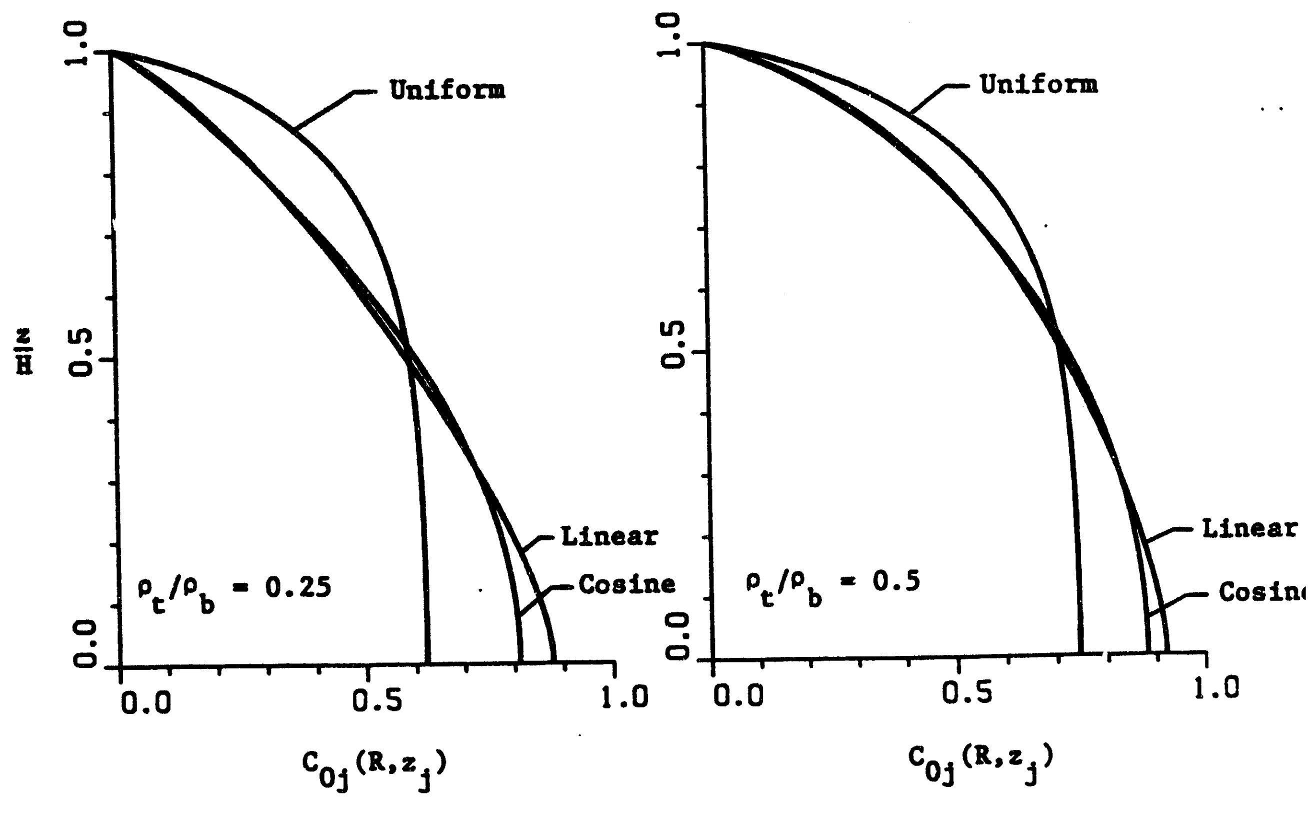

Fig. 6. Impulsive Pressure Exerted on Wall of Rigid Tank with H/R = 3

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