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# Decentralized Control of Uncertain Systems via Sensitivity Models*

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# DECENTRALIZED CONTROL OF UNCERTAIN SYSTEMS VIA SENSITIVITY MODELS

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**Abstract**

In this paper, we present a decentralized strategy for optimal control of interconnected systems exhibiting parametric uncertainty. First, we demonstrate that sensitivity models for linear interconnected systems can be generated at each subsystem using only locally available information. Second, we present an optimal control law that incorporates sensitivity functions in the feedback path. The control scheme is completely decentralized and is proposed as a means of making the closed loop system less sensitive to parameter deviations. Finally, we give an example of an interconnected system and show how this control strategy is implemented.

## I. Introduction

The use of sensitivity functions in control theory to make the closed loop system less susceptible to changes in plant parameters has been studied for several decades. In particular, methods of generating sensitivity functions such that they can be utilized on-line in a control system have been extensively researched. Sensitivity models are a means of generating these sensitivity functions from the nominal plant model. However, very little effort has been spent in generating sensitivity models for coupled subsystems in a decentralized manner.

As an additional tool for decentralized control, it is of interest to determine the feasibility of generating these models using only local signals for interconnected systems. That is, we wish to investigate the possibility that the $i$th subsystem's output sensitivity function can be generated using only plant signals from the $i$th subsystem. This is important if one wants to use sensitivity functions in a decentralized control environment. For instance, one could use decentralized sensitivity functions for self-tuning control (tuning the gains of a control law when some of the plant parameters are unknown) as is done in the work of Hung [2].

In this paper, decentralized sensitivity models are suggested for robust optimal control of certain classes of linear systems. Ultimately, the importance of this paper is to show that any control law that utilizes sensitivity functions (e.g. adaptive or optimal control laws) can be done in a decentralized framework. Thus, the results of this paper could be very practical provided one wishes to employ control laws that make use of sensitivity functions.

## II. Decentralized Sensitivity Models

Consider two interconnected MIMO linear systems. The outputs, $Y_i$, are of dimension $p_i$, $i = 1, 2$. The inputs, $U_i$, are of dimension $m_i$. The unknown parameter vectors, $\alpha_i$, $\beta_i$, $\gamma_i$, are of dimensions $n_i$, $r_i$, and $q_i$, respectively. The transfer matrices, $Q_i$, $W_i$, $W_{ij}$, are of compatible dimensions. All vectors and matrices are functions of the Laplace Transform complex variable, $s$. The transfer matrices, $Q_i$, represent dynamic feedback from the outputs to the inputs. The transfer matrices, $W_{ij}$, represent coupling terms between the subsystems. The transfer matrices, $W_i$, represent the primary dynamics between plant inputs $U_i$ and plant outputs $Y_i$.

The block transfer matrix of the entire system can be obtained by writing input-output relationships as follows

$$Y_1 = W_1(U_1 - Q_1Y_1) + W_{21}(U_2 - Q_2Y_2) \quad (1)$$
$$Y_2 = W_2(U_2 - Q_2Y_2) + W_{12}(U_1 - Q_1Y_1). \quad (2)$$

Letting

$$\Delta_1 = I + W_1Q_1 - W_{21}Q_2[I + W_2Q_2]^{-1}W_{12}Q_1 \quad (3)$$
$$\Delta_2 = I + W_2Q_2 - W_{12}Q_1[I + W_1Q_1]^{-1}W_{21}Q_2 \quad (4)$$

where $I$ is the identity matrix, we obtain the input-output description of the system (after some algebraic manipulation)

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad (5)$$

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with

$$F_{11} = \Delta_1^{-1}[W_1 - W_{21}Q_2(I+W_2Q_2)^{-1}W_{12}] \quad (6)$$
$$F_{12} = \Delta_1^{-1}[W_{21} - W_{21}Q_2(I+W_2Q_2)^{-1}W_2] \quad (7)$$
$$F_{21} = \Delta_2^{-1}[W_{12} - W_{12}Q_1(I+W_1Q_1)^{-1}W_1] \quad (8)$$
$$F_{22} = \Delta_2^{-1}[W_2 - W_{12}Q_1(I+W_1Q_1)^{-1}W_{21}]. \quad (9)$$

We are interested in output sensitivity vectors of the system with respect to the unknown parameter vectors, $\alpha_i$, $\beta_i$, and $\gamma_i$. It is sufficient to study sensitivity vectors of the first subsystem since the two subsystems are symmetric as is easily observed.

We proceed with the sensitivity of $Y_1$ with respect to $\alpha_1$ which begins by utilizing (1)

$$\frac{\partial Y_1}{\partial \alpha_{1i}} = \frac{\partial F_{11}}{\partial \alpha_{1i}}U_1 + \frac{\partial F_{12}}{\partial \alpha_{1i}}U_2 \quad (10)$$

where $\alpha_{1i}$ is the $i$th component of the unknown parameter vector. Continuing with the analysis, one obtains

$$\frac{\partial F_{12}}{\partial \alpha_{1i}} = -F_{11}\frac{\partial Q_1}{\partial \alpha_{1i}}F_{12} \quad (11)$$

which leads to

$$\frac{\partial Y_1}{\partial \alpha_{1i}} = -F_{11}\frac{\partial Q_1}{\partial \alpha_{1i}}(F_{11}U_1 + F_{12}U_2)\,. \quad (12)$$

Noting that the term in parentheses is merely (1), we have the result

$$\frac{\partial Y_1}{\partial \alpha_{1i}} = -F_{11}\frac{\partial Q_1}{\partial \alpha_{1i}}Y_1 \quad (13)$$

which is a completely decentralized result. The output sensitivity vector depends only on the output signal itself plus some transfer matrices. The remaining sensitivity functions can be obtained in a likewise manner, but the analysis is omitted here for brevity (see [4] for more details).

## III. Decentralized Optimal Control via Sensitivity Functions

It is now of interest to apply the above decentralized sensitivity models to performance issues associated with decentralized control. In particular, we concern ourselves with locally optimal control laws which include sensitivity functions in the feedback loop as a means of desensitizing the system from parameter variations. The framework pursued here is that of linear subsystems with linear couplings similar to that of [5] but utilizing sensitivity functions to address parametric uncertainty in an optimal manner. A version of the centralized case appears in [1]. The performance index consists of a sum of $N$ local cost criterions where $N$ is the number of subsystems.

Formally, we have

$$\dot{x}_i = A_i(\alpha_i)x_i + \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}x_j + B_iu_i\,, \quad i=1,\ldots,N \quad (14)$$

with $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, and $\alpha_i \in R_p$, where $\alpha_i$ are the vectors of unknown parameters. It is desired to minimize the quadratic cost criterion

$$J = \sum_{i=1}^{N}\int_0^\infty (x_i^TQ_ix_i + u_i^TR_iu_i \quad (15)$$
$$+ \sum_{j=1}^{N}\lambda_{ij}^TS_{ij}\lambda_{ij})dt \quad (16)$$

via local state feedback where $Q_i$,$S_{ij}$ are positive semidefinite matrices, $R_i$ are positive definite matrices, and $\lambda_{ij} = \frac{\partial x_i}{\partial \alpha_i}\,|_{\alpha_i=\alpha_i^n}$ are the sensitivity vectors associated with the $i$th subsystem with $\alpha_i^n$ the nominal parameter values. As was demonstrated in the previous section, the cross-sensitivity functions $\lambda_{ij}, j\neq i$ can be generated at the $i$th subsystem with only local information needed.

The sensitivity models are described by the following linear time-invariant differential equations

$$\dot{\lambda}_{ii} = \frac{\partial A_i}{\partial \alpha_i}x_i + A_i\lambda_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}\lambda_{ij} \quad (17)$$
$$\dot{\lambda}_{ij} = A_j\lambda_{ij} + \sum_{\substack{k=1\\ k\neq j}} A_{jk}\lambda_{ik} \quad (18)$$

which again demonstrate the decentralized nature of sensitivity models for the system (14). Define the augmented state vector

$$\tilde{x}_i = \left[x_i^T\ \lambda_{ii}^T\ \lambda_{ij}^T\right]^T\,, \quad (19)$$

which leads to the full augmented state space description

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{A}_c\tilde{x} + \tilde{B}u \quad (20)$$

where $\tilde{x} = \left[\tilde{x}_1^T \cdots \tilde{x}_N^T\right]^T$, $\tilde{A} = \text{block-diag}\left[\tilde{A}_1 \cdots \tilde{A}_N\right]$, $\tilde{A}_c = \left[\tilde{A}_{ij}\right]$, $j\neq i$, $\tilde{B} = \text{block-diag}\left[\tilde{B}_1\cdots\tilde{B}_N\right]$, and $u = \left[u_1^T\cdots u_N^T\right]$.

The performance criterion (16) can be re-expressed in these new coordinates as

$$J = \sum_{i=1}^{N}\int_0^\infty \left(\tilde{x}_i^T\tilde{Q}_i\tilde{x}_i + u_i^TR_iu_i\right)dt \quad (21)$$

where $\tilde{Q}_i = \text{block-diag}[Q_i\ S_{ii}\ S_{ij}]$. This is written in the full state space as

$$J = \int_0^\infty \left( \tilde{x}^T \tilde{Q} \tilde{x} + u^T R u \right) dt \tag{22}$$

where $\tilde{Q} = \text{block-diag}\left[\tilde{Q}_1 \cdots \tilde{Q}_N\right]$ and $R = \text{block-diag}\left[R_1 \cdots R_N\right]$. Note that $\tilde{Q}$ and $R$ will be positive semidefinite and positive definite, respectively, if $Q_i, S_{ij}$ and $R_i$ are positive semidefinite and positive definite, respectively.

It is assumed that the decoupled subsystems

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u \tag{23}$$

are controllable, i.e., $(\tilde{A}, \tilde{B})$ are a controllable pair. Thus, the cost criterion (22) can be minimized by solving the linear quadratic regulator separately for each subsystem. That is, let

$$u = -K\tilde{x} \tag{24}$$

where $K = \text{block-diag}\left[K_1 \cdots K_N\right]$. These $K_i$ are computed by solving the algebraic Riccati equation

$$\tilde{A}^T P + P\tilde{A} - P\tilde{B}R^{-1}\tilde{B}^T P + \tilde{Q} = 0 \tag{25}$$

for the unique positive definite matrix $P = \text{block-diag}\left[P_1 \cdots P_N\right]$. Because of the structure of $\tilde{A}, \tilde{B}, \tilde{Q}, R$, the solution $P$ will automatically be in this block-diagonal form.

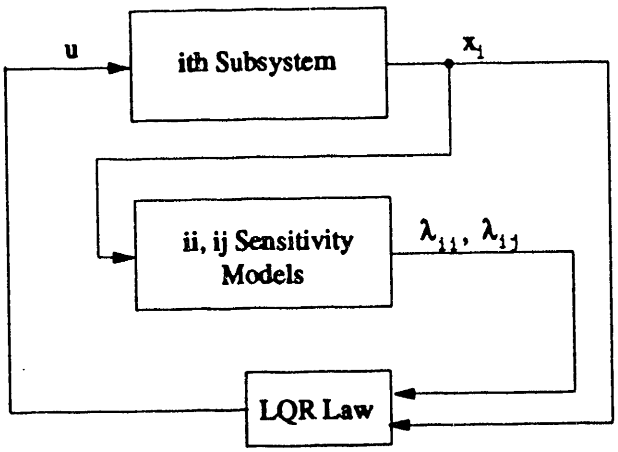


Figure 1: Decentralized optimal control with sensitivity models.

The control law (24) is completely decentralized, $u = -K_i\tilde{x}_i$, which means that each subsystem can be regulated with only locally available information even in the presence of parametric uncertainty. Indeed, it is the use of locally generated sensitivity functions that sets this strategy apart from others. Fig. 1 illustrates this method.

## IV. Example

We consider a system consisting of two inverted penduli coupled by a spring subject to two independent torque inputs as shown in Fig. 2. Physically, this system is analogous to two one-link manipulators joined together by a string, cable, or other spring-like medium. The deflections from vertical are assumed to be small enough such that the gravity term can be linearized. The equations of motion are [5]

$$\begin{aligned} m\ell_1^2\ddot{\theta}_1 &= mg\ell_1\theta_1 - ka^2(\theta_1 - \theta_2) + u_1 \\ m\ell_2^2\ddot{\theta}_2 &= mg\ell_2\theta_2 - ka^2(\theta_2 - \theta_1) + u_2 \end{aligned} \tag{26}$$

where all parameters are defined in Fig. 2 except for $g$ which is the gravitational constant. The state vector is chosen as $x_1 = (\theta_1, \dot{\theta}_1)^T$, the input vector is $u = (u_1, u_2)^T$, and the uncertain parameters are $\alpha_i = \frac{g}{\ell_i}$. These parameters physically represent the squares of the natural frequencies of oscillation of the decoupled penduli. We assume some uncertainty from their nominal values.

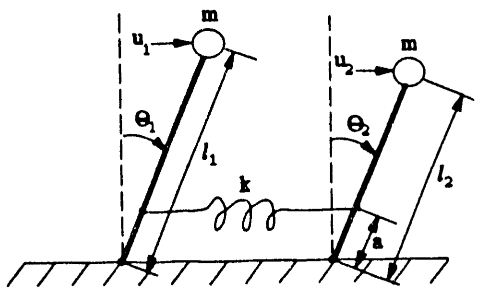


Figure 2: Inverted penduli coupled by a spring.

Utilizing (17)-(18), the sensitivity vectors

$$\begin{aligned}
\dot{\lambda}_{11} &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 & 1 \\ \alpha_1^n - \frac{ka^2}{m\ell_1^2} & 0 \end{bmatrix} \lambda_{11} \\
&+ \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{m\ell_1^2} & 0 \end{bmatrix} \lambda_{12} \\
\dot{\lambda}_{12} &= \begin{bmatrix} 0 & 1 \\ \alpha_2^n - \frac{ka^2}{m\ell_2^2} & 0 \end{bmatrix} \lambda_{12} + \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{m\ell_2^2} & 0 \end{bmatrix} \lambda_1 \quad (27) \\
\dot{\lambda}_{21} &= \begin{bmatrix} 0 & 1 \\ \alpha_1^n - \frac{ka^2}{m\ell_1^2} & 0 \end{bmatrix} \lambda_{21} + \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{m\ell_1^2} & 0 \end{bmatrix} \lambda_{22} \\
\dot{\lambda}_{22} &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 & 1 \\ \alpha_2^n - \frac{ka^2}{m\ell_2^2} & 0 \end{bmatrix} \lambda_{22} \\
&+ \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{m\ell_2^2} & 0 \end{bmatrix} \lambda_{21}
\end{aligned}$$

are generated. The augmented state vector $\tilde{x}$ is formed as follows

$$\tilde{x} = \left[x_1^T\ \lambda_{11}^T\ \lambda_{12}^T\ x_2^T\ \lambda_{22}^T\ \lambda_{21}^T\right]^T \tag{28}$$

which is 12th order. We choose $ka^2 = 1\text{N}\cdot\text{m}$, $m\ell_1^2 = 1\text{kg}\cdot\text{m}^2$, $m\ell_2^2 = 0.5\text{kg}\cdot\text{m}^2$, $\alpha_1^n = \frac{g}{\ell_1} = 1\frac{1}{s^2}$, and $\alpha_2^n = \frac{g}{\ell_2} = 2\frac{1}{s^2}$.

The quadratic cost criterion is chosen such that all states and sensitivity functions are weighted equally, i.e., $\bar{Q}$ is a 12x12 identity matrix. Likewise, $R$ is selected to be a 2x2 identity matrix. Analysis simulated on MATLAB solves the Riccati equation (25) and implements the decentralized control strategy of the last section. For comparison purposes, a decentralized design is carried out on the same system without using sensitivity models. That is, the nominal parameter values were taken as exact in the control design. The uncertainties tested were 10% and 20%, respectively, i.e., the true values were $\alpha_1 = 1.1$, $\alpha_2 = 2.2$, and $\alpha_1 = 1.2$, $\alpha_2 = 2.4$ for the two simulation runs. For both designs, the controllability assumption is satisfied.

Table 1: Closed loop poles of simulation runs with no uncertainty.

| | No sensitivity model | Sensitivity model |
|---|---|---|
| No interconnections | -0.866±j0.5<br>-1.414<br>-1.414 | -0.233±j1.255<br>-0.924±j0.582<br>-1.376, -1.058<br>-0.23±j1.206<br>-1.196±j0.41<br>-1.79, -1.036 |
| Interconnections | -1.116±j1.207<br>-2.329<br>0.0 | -2.957±j1.52<br>-0.53±j1.724<br>-0.136±j1.259<br>-0.182±j1.04<br>-0.22±j0.301<br>-1.189±j0.003 |

Table 2: Closed loop poles of simulation runs with 20% uncertainty.

| | No sensitivity model | Sensitivity model |
|---|---|---|
| No interconnections | 0.109<br>-1.84<br>0.135<br>-2.963 | -0.24±j1.459<br>-0.173±j0.841<br>-2.736, -3.707<br>-0.281±j1.563<br>-0.113±j0.908<br>-1.186, -1.187 |
| Interconnections | -3.114<br>0.601<br>-0.871<br>-1.177 | -3.45±j0.959<br>-0.183±j1.784<br>-0.168±j1.341<br>-0.063±j0.979<br>-0.163±j0.669<br>-1.188±j0.001 |

The results are summarized in Tables 1-2. The first column of numbers of each table represent the closed loop poles for the 4th order decentralized design without sensitivity models. The second column of numbers of each table represent the closed loop poles for the 12th order decentralized design with sensitivity models. The first row of each table corresponds to the case of ignoring the interconnection terms whereas the second row includes these terms in the closed loop analysis. The results show that the lower order design without sensitivity models fails to stabilize the closed loop system once inaccuracies in the parameters are introduced. In fact, even with no uncertainty in the parameters, one of the closed loop poles is at the origin once the coupling matrix is included. As the uncertainty is increased this pole moves further into the right half plane. But for both cases of uncertainty (0, 20%), the higher order design with sensitivity models maintains closed loop stability. The price paid is a higher order system, but the gain is a significant amount of stability robustness with respect to parametric uncertainty. Of course, it must be noted that the true optimal cost $J^*$ will be infinite for all three cases for the lower order design whereas it remains finite for the higher order design.

Finally, some comments are in order concerning the relationship of the above problem to the question of transient stability in power systems. The above system is mathematically similar (though not exact) to a two-machine swing equation model of a power system. The equations for a single machine are quite similar to that of a simple pendulum as noted in [3]. Adding a second machine results in coupling close to the spring connection in the above example. The forces exerted on the penduli correspond to the electrical power delivered by the machines. Thus, some transient stability results can be studied from this example, however in a realistic setting, constraints would have to be imposed on the inputs to the machines.

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