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MECHANICAL MODELS FOR TANKS CONTAINING TWO LIQUIDS

Y. Tang[1]

Abstract

The well-known Housner's mechanical model for laterally excited rigid tanks that contain one liquid is generalized to permit consideration of tanks that contain two liquids under the horizontal and rocking base motions. Two mechanical models are developed herein; one is for rigid tanks and the other for flexible tanks. The model for rigid tanks has a rigidly attached mass and infinite number of elastically supported masses. The rigid attached mass which possesses a mass moment of inertia represents the impulsive component, whereas the elastically supported masses which do not possess mass moment of inertia represent the convective component of the response. These masses and their heights are chosen such that, under the same base motions, the base shear and base moments of the model match those of the original liquid-tank system. The spring stiffness constants for the elastically supported masses in the model are determined from the sloshing frequencies of the liquid-tank system. The model for flexible tanks, however, only represents the impulsive action of the hydrodynamic response. It has an elastically supported mass that does not possess mass moment of inertia and a member that has no mass but possesses a mass moment of inertia. This latter model is proposed for the study of the effect of the soil-structure interaction.

Introduction

Liquid storage tanks are important elements of nuclear power plants, water storage facilities, LNG facilities, and petroleum facilities; therefore, when located in earthquake prone regions, these tanks should be designed to withstand the earthquakes to which they may be subjected. In the past thirty years, the focus of the studies on the dynamic response of liquid storage tanks has been on the responses of the tanks that contain only one liquid. To facilitate

[1]Structural Engineer, Reactor Engineering Division, Argonne National Laboratory, Argonne, IL 60439.

the computation of the dynamic response of the liquid in a rigid tank, Housner (1957) introduced a mechanical model which has been widely accepted in practice and has been adopted in many design guidelines. Following the same idea, a mechanical model based on an approximate analysis was proposed by Haroun and Ellaithy (1985) for flexible tanks subjected to base lateral and rocking motions, and a mechanical model based on the exact solutions was proposed by Veletsos and Tang (1987) for rigid tanks under lateral and rocking base excitations.

Recently, the papers by Burris, et al. (1987) and Bandyopadhyay (1991) indicated that there is a need to study the dynamic response of tanks that contain two liquids with different densities. To respond to this need, Tang (1993a, 1993b, 1993c) and Tang and Chang (1993) have studied the dynamic responses of both rigid and flexible tanks containing two liquids subjected to lateral and rocking base motions. The exact solutions have been presented, and the dynamic behavior of a tank containing two liquids under base excitations are now well-understood. Hence, the mechanical models capable of duplicating the response quantities of tanks containing two liquids are now ready to be developed. Based on the above mentioned studies by Tang and Chang, two mechanical models, one for rigid tanks and the other for flexible tanks, are developed and presented in this paper.

The objectives of this paper are to: (1) develop a mechanical model that permits consideration of tanks containing two liquids subjected to horizontal and rocking base motions; and (2) present numerical data with which the parameters for the mechanical model developed may be evaluated readily. The model for rigid tanks has a rigidly attached mass and infinite number of elastically supported masses. The rigid attached mass which possesses a mass moment of inertia represents the impulsive component, whereas the elastically supported masses which do not possess mass moment of inertia represent the convective component of the response. This model is further generalized to include the tank flexibility which may be used to evaluate, approximately, the critical response quantities for flexible tanks containing two liquids for tanks with a height-to-radius ratio between 0.3 and 1.0 (Tang, 1993c). This feature may also be used for the soil-structure interaction analysis (Tang, 1993d).

In the presentation of the paper, the information needed for developing the mechanical models are briefly reviewed first, followed by the development of the mechanical model. For more details about the background material, the reader is referred to the studies by Tang (1993a, 1993b, 1993c) and Tang and Chang (1993).

System Considered, Assumptions and Approach

The tank-liquid system considered is shown in Figure 1. It is a ground-supported upright circular cylindrical tank of radius R which is filled with two liquids to a total height of H. The lower portion liquid, identified as Liquid I, has heavier mass density, ρ_1, and the upper portion liquid, identified as Liquid II, has lighter mass density, ρ_2. The heights of Liquids I and II are H_1 and H_2, respectively. The tank wall is assumed to be uniform thickness and clamped to a rigid base. Both liquids are considered to be incompressible and inviscid. The response of the liquids is assumed to be linear. Let r, θ, z_1 denote the radial, circumferential, and vertical axial coordinates of a point in the Liquid I, and let r, θ, and z_2 be the corresponding coordinates for a point in Liquid II as shown in Figure 1. The origins of the two coordinate systems are at the central axis of the cylindrical tank. The base motions experienced by the tank are a horizontal acceleration, denoted by $\ddot{x}(t)$, acting in the direction along the $\theta = 0$ axis, and an angular acceleration, denoted by $\ddot{\theta}_b(t)$, acting in the direction along the $\theta = 90°$ coordinate axis. The temporal variations of $\ddot{x}(t)$ and $\ddot{\theta}_b(t)$ can be arbitrary.

Given the conditions that the liquids are incompressible and inviscid, the hydrodynamic pressures induced at Liquids I and II must satisfy the Laplace equations and the appropriate boundary conditions. The method of separation of variables was employed to solve the problems, and the exact solutions, in terms of the impulsive and convective components, were presented in Tang (1993a, 1993b). It has been found that the dynamic response of a rigid tank containing two liquids under base excitations is controlled by three parameters, H/R, H_2/H_1 and α where α is defined by

$$\alpha = \rho_2/\rho_1 \tag{1}$$

Response of Liquids to Lateral Excitations

For a tank containing two liquids under lateral base excitation, it has been shown (Tang, 1993a) that the base shear, Q(t), may be computed from the equation given by

$$Q(t) = m_0\,\ddot{x}(t) + \sum_{n=1}^{\infty}\sum_{k=1}^{2} m_{nk}A_{nk}(t) \tag{2}$$

The first term in Equation 2 represents the impulsive component, and the double series term represents the convective component of the solution; the functions $A_{nk}(t)$, k = 1, 2, are the pseudoacceleration functions for the nth sloshing mode of vibration, and are defined by

$$A_{nk}(t) = \omega_{nk}\int_0^t \ddot{x}(t)\sin\big(\omega_{nk}(t-\tau)\big)d\tau \tag{3}$$

in which ω_{nk} = the natural circular frequency associated with the nth sloshing mode of vibration. The characteristic equation for computing ω_{nk} can be found in Tang (1993a). Note that for each sloshing mode of vibration, n, there are two natural frequencies, k = 1 and 2, which is in sharp contrast to the case for tanks containing only one liquid where there is only one frequency associated with each sloshing mode. It is noted that this phenomenon is caused by the gravitational effect at the interface of the two liquids (Tang, 1993a).

The base moment, M(t), at a section immediately above the tank base is given by

$$M(t) = m_0 h_0 \ddot{x}(t) + \sum_{n=1}^{\infty} \sum_{k=1}^{2} m_{nk} h_{nk} A_{nk}(t) \tag{4}$$

The moment induced by the pressure exerted on the tank base is denoted by $\Delta M(t)$ which may be computed from the equation

$$\Delta M(t) = m_0 \Delta h_0 \ddot{x}(t) + \sum_{n=1}^{\infty} \sum_{k=1}^{2} m_{nk} \Delta h_{nk} A_{nk}(t) \tag{5}$$

The foundation moment, $M'(t)$, at a section immediately below the tank base is then given by the sum of Equations 4 and 5 as

$$M'(t) = m_0 h'_0 \ddot{x}(t) + \sum_{n=1}^{\infty} \sum_{k=1}^{2} m_{nk} h'_{nk} A_{nk}(t) \tag{6}$$

in which $h'_o = h_o + \Delta h_o$ and $h'_{nk} = h_{nk} + \Delta h_{nk}$.

Numerical values for m_0, m_{nk}, h_0, Δh_0 are available in Tang (1993a).

Response of Liquids to Rocking Excitations

For tanks containing two liquids undergoing rocking excitation, the base shear, $Q^r(t)$, is expressed in the form (Tang, 1993b)

$$Q^r(t) = m_0^r \ddot{x}_T(t) + \sum_{n=1}^{\infty} \sum_{k=1}^{2} m_{nk}^r A_{nk}^r(t) \tag{7}$$

in which the superscript r is used as a reminder for rocking, and $\ddot{x}_T(t) = H\, \ddot{\theta}_b(t)$ = the horizontal acceleration of the tank wall at the level of the still liquid surface, and the functions $A_{nk}^r(t)$, k = 1 and 2, are the pseudoacceleration functions for the nth sloshing mode of vibration, and are defined by the Equation 3 with the replacement of $\ddot{x}(t)$ by $\ddot{x}_T(t)$.

The corresponding base moment and foundation moment are given by

$$M^r(t) = i_0 m_\ell^1 H \ddot{x}_T(t) + \sum_{n=1}^{\infty} \sum_{k=1}^{2} m_{nk}^r h_{nk}^r A_{nk}^r(t) \tag{8}$$

$$M'^r(t) = i'_0 m_\ell^1 H \ddot{x}_T(t) + \sum_{n=1}^{\infty} \sum_{k=1}^{2} m_{nk}^r h'^r_{nk} A_{nk}^r(t) \tag{9}$$

in which $m_\ell^1 = \pi\rho_1 R^2 H$.

It is shown in Tang (1993b) that the response quantities of the tank containing two liquids subjected to rocking and lateral base motions are related to each other by Betti's principle. As a result, several components of the response for the tank in rocking may be determined from the corresponding response quantities for an identical tank under a horizontal base motion. The interrelationship between two response quantities may be quantified by the equations given below.

$$m_0^r = m_0 \left(\frac{h'_0}{H} \right) \tag{10}$$

$$m_{nk}^r = m_{nk} \left(\frac{h'_{nk}}{H} \right) \tag{11}$$

$$h_{nk}^r = h_{nk} \tag{12}$$

$$h'^r_{nk} = h'_{nk} \tag{13}$$

and

$$\Delta h_{nk}^r = \Delta h_{nk} \tag{14}$$

The background reasoning for Equations 10 to 14 can be found in Tang (1993b).

Mechanical Model for Rigid Tanks

For a tank containing two liquids excited laterally at the base, it has been shown (Tang, 1993a) that the base shear and base moment are the same as those induced in the model shown in Fig. 2, which is a generalization of the Housner's model (Housner, 1957). In Figure 2 the rigidly attached mass, m_0, is located at a height, h_0, from the tank bottom representing the impulsive component of the response, and the nkth elastically supported mass, m_{nk}, is located at a height, h_{nk}, from the tank base representing the convective component of the response. The heights, h_0 and h_{nk}, are used to evaluate the base moment at a section immediately above the tank base. By changing h_0 to h'_0 and h_{nk} to h'_{nk}, the model may also be used to evaluate the foundation moment. The numerical data for the parameters of the model, m_0, m_{11} and m_{12} are listed in columns 1, 2 and 3 of Table 1 and h_0, h'_0, h_{11}, h_{12}, h'_{11} and h'_{12} are tabulated in Table 2 for $H_2/H_1 = 1$, $\alpha = 0.5$ and values of H/R in the range between 0.3 and 1.

With the same adjustments as those presented in Veletsos and Tang (1987) for tanks containing only one liquid, the same model shown in Figure 2 can be used for evaluating the base shear and moments for tanks under rocking. These adjustments are reiterated herein:

1. For calculating the impulsive components of the response, the angular base acceleration $\ddot{\theta}_b(t)$ should be taken as $(h'_0/h_0)\ \ddot{\theta}_b(t)$, and for calculating the nkth mode of the convective components, it should be taken as $(h'_{nk}/h_{nk})\ \ddot{\theta}_b(t)$.

2. The rigidly attached mass must be considered to possess a mass moment of inertia about a normal centroidal axis of either $m_0 r_0^2$ or $m_0(r'_0)^2$, depending on whether the base moment or the foundation moment is to be evaluated. The corresponding mass moments of inertia about the axis of rotation of the tank are then $m_0\left(h_0^2 + r_0^2\right)$ and $m_0\left[(h'_0)^2 + (r'_0)^2\right]$, and they are related to the inertia coefficients i_0 and i'_0 as follows:

$$h_0^2 + r_0^2 = i_0 \frac{m_l^1 h_0}{m_0 h'_0} H^2 \tag{15}$$

and

$$(h'_0)^2 + (r'_0)^2 \frac{h'_0}{h_0} = i'_0 \frac{m_l^1}{m_0} H^2 \tag{16}$$

It can be shown easily that the model with the adjustments does yield the correct base shear and moments. Numerical values of i_0 and i'_0 are listed in columns 4 and 5 of Table 1.

Mechanical Model for Flexible Tanks

It is well-known that the effect of tank flexibility on the convective component of the response is small and can be neglected. Hence, in the computation of the response of a flexible tank, the convective component can be computed by considering the tank to be rigid, and only the effect of tank flexibility on the impulsive component needs to be considered. It has been shown in Tang and Chang (1993) that for thin tanks with values of H/R between 0.3 and 1, the response quantities of a flexible tank under lateral excitation can be obtained, with good accuracy, from the corresponding rigid tank solutions by replacing the peak ground acceleration, $\ddot{x}(t)$, with the pseudoacceleration function, $A_1(t)$, corresponding to the fundamental mode of vibration of the tank-liquid system. Based on the study presented in Tang (1993b), it is believed that this simple procedure can also be applied to tanks under base rocking as well. So, for tanks subjected to lateral and rocking base motions simultaneously, the base shear and moments obtained from this simple procedure are given as follows.

$$Q(t) = m_0 A_1(t) \tag{17}$$

$$M(t) = m_0 h_0 A_i(t) \tag{18}$$

$$M'(t) = m_0 h'_0 A_i(t) + I_b \ddot{\theta}_b(t) \tag{19}$$

in which I_b = the mass moment of inertia about a horizontal centroidal axis for the part of the liquid that may be considered to move in unison with the rocking base. The normalized values of this quantity are listed in column (6) of Table 1. The function $A_i(t)$ is defined by

$$A_i(t) = \frac{\omega}{\sqrt{1-\zeta^2}} \int_0^t \left[\ddot{x}(\tau) + h'_0 \ddot{\theta}_b(t)\right] \exp\left[-\zeta\omega(t-\tau)\right] \sin\left(\bar{\omega}(t-\tau)\right) d\tau \tag{20}$$

in which ω = the fundamental circular frequency of the tank-liquid system; ζ = the fraction of critical damping; and $\bar{\omega} = \omega\sqrt{1-\zeta^2}$.

The impulsive action of the tank-liquid system can then be represented by the model shown in Figure 3. The mass m_0 in this model is supported by a pair of springs and viscous dampers, and is located at a distance of h_0 from the base. The stiffness of the spring, k_0, and the constant of the viscous damper, c_0, are defined as follows.

$$k_0 = \omega^2 m_0 \tag{21}$$

$$c_0 = 2 m_0 \omega \zeta \tag{22}$$

In addition, at the base of the model, there is a member which has no mass but possesses a mass moment of inertia I_b about a centroidal axis normal to the plane of the paper. It is easy to verify that the base shear and moment of the model are indeed defined by Equations 17 and 18. Again, by changing h_0 to h'_0, the model may be used to compute the foundation moment defined by Equation 19. It should be noted herein that this simple proposed model for tank-liquid system is now being employed by Tang (1993d) in a study on the soil-structure interaction analysis of tanks containing two liquids.

To evaluate $A_i(t)$, one must first compute the fundamental natural frequency of the tank-liquid system. A simple approach for estimating this fundamental frequency has been proposed in Tang (1993c) for thin tanks with values of H/R between 0.3 and 1. With this simple approach, the spectral value of $A_i(t)$ can be evaluated readily.

Conclusions

An equivalent mechanical model is developed in this paper for rigid tanks containing two liquids subjected to lateral and rocking base motions. The model can be used to evaluate the critical response quantities of the liquids such as the base shear, base moment and foundation moment. In addition, in this paper, it is shown that the impulsive action of the flexible tank containing two liquids can be represented by a simple mechanical model as well. It should be noted that this simple model can be used in the soil-structure interaction analysis for tanks supported on flexible soil.

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Table 1. Masses and mass moments of inertia in mechanical model for tank-liquid system, $H_2/H_1 = 1$, $\alpha = 0.5$.

$\frac{H}{R}$	$\frac{m_0}{m_\ell^1}$ (1)	$\frac{m_{11}}{m_\ell^1}$ (2)	$\frac{m_{12}}{m_\ell^1}$ (3)	$\frac{i_0}{m_\ell^1 H}$ (4)	$\frac{i'_0}{m_\ell^1 H}$ (5)	$\frac{I_b}{m_\ell^1 R^2}$ (6)
0.3	0.117	0.565	0.018	0.095	2.124	0.122
0.4	0.158	0.535	0.019	0.096	1.216	0.095
0.5	0.200	0.500	0.019	0.096	0.709	0.082
0.6	0.241	0.464	0.020	0.096	0.572	0.071
0.7	0.281	0.428	0.020	0.097	0.437	0.062
0.8	0.318	0.393	0.021	0.098	0.351	0.055
0.9	0.353	0.359	0.022	0.099	0.293	0.050
1.0	0.385	0.329	0.022	0.100	0.252	0.045

Table 2. Heights for the masses in mechanical model for tank-liquid system, $H_2/H_1 = 1$, $\alpha = 0.5$.

$\frac{H}{R}$	$\frac{h_0}{H}$	$\frac{h'_0}{H}$	$\frac{h_{11}}{H}$	$\frac{h'_{11}}{H}$	$\frac{h_{12}}{H}$	$\frac{h'_{12}}{H}$
0.3	0.365	2.589	0.465	4.185	-0.919	20.774
0.4	0.365	1.884	0.471	2.513	-0.891	11.026
0.5	0.365	1.471	0.479	1.745	-0.857	6.549
0.6	0.365	1.203	0.488	1.334	-0.818	4.149
0.7	0.365	1.020	0.499	1.092	-0.775	2.732
0.8	0.365	0.890	0.511	0.942	-0.730	1.840
0.9	0.366	0.793	0.523	0.845	-0.684	1.253
1.0	0.366	0.721	0.537	0.781	-0.637	0.855

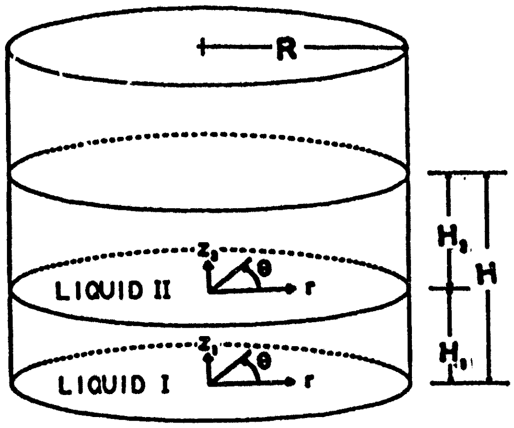

Figure 1. System considered.

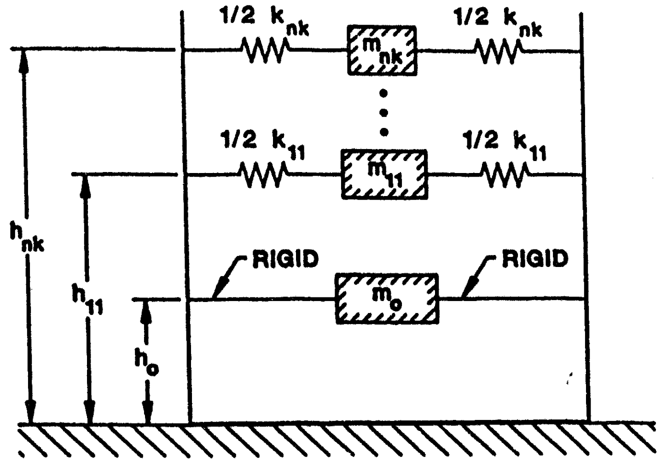

Figure 2. Mechanical model for rigid tanks.

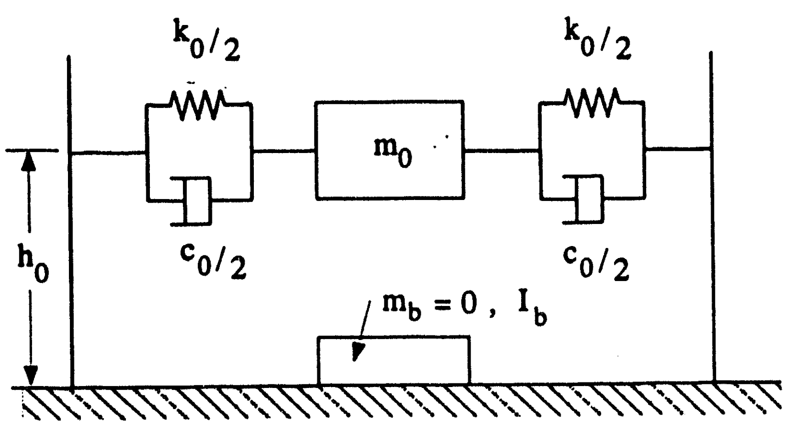

Figure 3. Mechanical model for flexible tanks.

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