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CALCULATIONS OF BUILDUP OF PLUTONIUM ISOTOPES AND BURNOUT OF U^{235} IN 1.44% U^{235} ENRICHED URANIUM

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CAICULATIONS OF BUILDUP OF PLUTONIUM ISOTOPES AND BURNOUT OF U^{235} IN 1.44% U^{235} ENRICHED URANIUM

W. E. NIEMUTH

CALCULATIONS OF BUILDUP OF PLUTONIUM ISOTOPES AND BURNOUT OF U^{235} IN 1.44% U^{235} ENRICHED URANIUM

INTRODUCTION

In order to investigate the rupture stability of uranium elements irradiated at power generations per unit length larger than those encountered in natural uranium fuel elements, three partial columns of 1.44% U^{235} enriched, internally-externally cooled, uranium elements were irradiated in PT-IP-1-A(1). After discharge and examination of the fuel elements 25 pieces (182.5 pounds) of this metal were available for special analysis of U^{235} burnout and plutonium content. The average exposure of these pieces was 2187 ± 6% MWD/Ton.

The purpose of this document is to summarize some calculations of buildup of plutonium isotopes and burnout of U^{235} in an attempt to correlate calculations with the results from experimental analysis.

SUMMARY

The results of the experimental analysis of the 1.44% E metal have been reported previously(2). The results are as follows:

Table I

Analysis of 1.44% U^{235} Enriched Uranium After Irradiation

Enrichment	1.4411%
U^{235} content after irradiation	1.191%
Grams Pu/Ton U	1050 ± 17
Plutonium Data	
Pu^{238}	0.04%
Pu^{239}	89.31%
Pu^{240}	9.43%
Pu^{241}	1.16%
Pu^{242}	0.06%
Measured Exposure (Flow and Δ T)	2187 ± 10% MWD/Ton(3)

Calculations of the buildup of plutonium isotopes, burnout of U^{235} and the accumulation of fissions were made to obtain agreement with the measured values listed in Table I. The equations used in the calculations are listed in Appendix I. This set of equations yield the nuclei per cc of the isotopes Pu^{239}, Pu^{240}, Pu^{241}, and U^{235}, or in the case of fissions, the fissions per cc as a function of the parameter ϕt (local flux x time). The U^{238} concentration was assumed constant, and

SUMMARY (CONT'D)

Westcott cross sections were employed in the calculations. The results of the calculations indicated that the neutron temperature during the irradiation was approximately 380 - 390° C. Graphite temperatures during the test ranged from 500 - 525° C. The results of the calculations for various conditions during the irradiation are shown in Figures 1 and 2.

DISCUSSION

Twenty-five pieces of internally-externally cooled 1.44% E metal which had been irradiated in C-reactor were specially dissolved and analyzed for plutonium composition and burnout of U^{235}. The discussion of the analysis can be found in HW-57277. The results are shown in Table I of this report.

Calculations were made attempting to obtain the results of the laboratory analysis of the 1.44% E metal. The equations used in the calculations are listed in Appendix I along with values of the cross sections. The calculations were made in terms of the flux-time parameter ϕt in which ϕt ranged from 0 - 3.5×10^{20} cm^{-2} in intervals of 0.7×10^{20}. The stepwise calculation was used in order to allow for the change in the Pu^{240} cross section due to self shielding of the 1.055 ev resonance as the concentration of Pu^{240} increases (Appendix II). The Westcott[4] cross section formulation was employed in the calculations. The Westcott formulation includes the resonance portion of the cross section in an effective cross section for the reactor spectrum defined as follows:

$$\sigma = \sigma_0 (g + rs)$$

where g and s indicate respectively the departure of the cross section from a 1/v law in the thermal and resonance portions of the reactor spectrum, σ_0 is the cross section for 2220 meter per second neutrons, and r indicates the fraction of resonance neutrons in the spectrum. For non 1/v absorbers g and s are functions of neutron temperature, and are listed in CRRP-787. The Westcott cross section formulation eliminates the use of resonance escape probabilities for several isotopes in the system of different equations for product buildup, and consequently the differential equations can be solved as a linear set. The assumption is adequate as long as there is no gross depletion in the resonance spectrum of neutrons at a particular energy such as 1.055 ev the Pu^{240} principle resonance energy. Since the U^{238} is clumped, it is necessary to employ an artificial value of s for U^{238}; this value can be determined from knowledge of the conversion ratio in a natural uranium element of the same size. Although this problem was not recognized at the onset of these calculations, the need for this artificial s_{28} soon became apparent. The determination of s_{28} is shown in Appendix II.

Having accepted the Westcott formulation, the problem was reduced to one of determining the proper values of neutron temperature and r to use in computing the cross sections. To get a feel of the situation a calculation was made using a calculated resonance integral for U^{238} (10.1533 barns)[5] and an r equal to 0.08751 measured from the cadmium ratio in a natural uranium slug. The neutron temperature was assumed to be 260° C. A step-wise calculation was made in which the Pu^{240} cross section was changed as the concentration of Pu^{240} increased (Appendix II).

DISCUSSION (CONT'D)

The results of this calculation are shown in Figure I. Since this calculation yielded values which were inadequate a series of calculations were performed, the results of which are listed below along with those from the above calculation.

Table II

Neutron Temperature	260° C	260° C	300° C	380° C
Spectral Index, r	0.08751	0.10624	0.10624	0.10624
Grams Pu/Ton*	930	972	992	1008
% Pu^{240}*	7.85	7.83	8.25	9.07
% Pu^{241}*	0.86	0.92	1.05	1.20

* Values determined at ϕt for which % U^{235} remaining equals measured value; U^{235} burnout is not very sensitive to either temperature or spectral index.

From the results of the above calculations, the following general statements can be made. (a) % 240 is dependent mainly upon the neutron temperature assumed for the calculation.

(b) % 241 is dependent upon both temperature (Pu^{240} content) and upon spectral index (cross section of Pu^{240})

(c) The burnout of U^{235} is almost independent of either temperature on spectral index i.e. U^{235} is very close to a 1/v absorber in absorption characteristics.

(d) Buildup of total plutonium is dependent upon the product rs_{28}.

It is apparent from the results listed in Table II that the assumption of the spectral index and neutron temperature were inadequate. But both further increases in neutron temperature and spectral index which are indicated to increase the total plutonium content and the % Pu^{240} would yield more Pu^{241} than indicated from the measurements. To attain better agreement, a method for calculating the spectral index was derived (Appendix IV). The spectral index in the uranium is given by

$$r_u = \frac{\beta_u}{1 + b\beta_u}$$

where

$$\beta = \frac{\bar{\Sigma}_{au} V_u L_\tau}{f_u \left(\sum_i \Sigma_{ri} V_i \xi_i \right)}$$

and $b = \frac{4}{\sqrt{\pi}\bar{\Sigma}_{au}}$, μkT specifies the energy at which the Maxwellian and slowing down flux join. $\bar{\Sigma}_{au}$ is the maxwellian averaged uranium cross section at the neutron temperature

DISCUSSION (CONT'D)

T, Vu is the volume of the uranium, fu is the thermal utilization of the cell, and $\Sigma s_i f_i V_i$ is the total slowing down power of the components of the lattice cell. In order to avoid a complicated calculation of the thermal leakage from the 1.44% E metal cell surrounded by natural uranium an effective boundary was calculated at which the gradient of the thermal flux between the adjacent cells equals zero. For the 1.44% E metal adjacent to natural uranium, this is an increase in the cell radius from 12.00 cm to 12.66 cm. The value of r can then be calculated stepwise throughout the irradiation as the macroscopic absorption cross section changes.

The value of s for a particular isotope is given by

$$\sqrt{\frac{4T}{\pi T_o}}\left[\frac{\Sigma}{\sigma_o} - g\sqrt{\frac{4\ E_o}{E_{cd}}}\right]$$

where T is the neutron temperature, T_o = 293.6° K, Σ is the resonance integral above a stated cadmium cutoff energy, E_{cd}, including the 1/v contribution, and σ_o is the cross section for 2200 m/sec neutrons. Since $\bar{\Sigma}_{au}$ in the computation of β is given as

$$\bar{\Sigma}_{au} = N_u \sigma_{2200}\, g \sqrt{\frac{\pi T_o}{4T}},$$

if g does not vary rapidly with temperature, the product rs is essentially independent of neutron temperature. The total plutonium produced is thus dependent upon the value of the spectral index, and essentially independent of temperature.

The value of s_{28} was determined from the relationship

$$\frac{\sigma_{28} N_{28}}{\sigma_{25} N_{25}} = \frac{\bar{\sigma}_{28} N_{28}}{\bar{\sigma}_{25} N_{25}} + \eta_{25} \epsilon (1-p) e^{-\beta^2 \tau}$$

Where σ_{28} and σ_{25} are the Westcott absorption cross sections for U^{238} and U^{235}, respectively, $\bar{\sigma}_{28}$ and $\bar{\sigma}_{25}$ are the Maxwellian averaged absorption cross sections for the same isotopes.

A calculation of the buildup of plutonium isotopes, burnout of U^{235} and the accumulation of fissions was made using 380° C neutron temperature and r values which are listed below:

ϕt	$\bar{\Sigma}_a$*	r
0	0.34278	0.08387
0.7×10^{20}	0.34780	0.08477
1.4×10^{20}	0.35125	0.08537
2.1×10^{20}	0.35355	0.08577
2.8×10^{20}	0.35489	0.08600

*Σ_a does not include fission products

DISCUSSION (CONT'D)

The results of this calculation are shown in Figure II. Rather good agreement is obtained between calculation and experimental results. Approximately 10° C increase in the neutron temperature would be required to bring the burnout of U^{235}, grams Pu/Ton U and % Pu^{240} into exact agreement as well as bring the % Pu^{241} into closer agreement with the experimental results. The calculated exposure is approximately 2100 MWD/Ton, which is about 4% less than the measured 2187 MWD/Ton and well within the uncertainty of the latter value.

Although the 17 pieces of E metal were charged into C reactor with a normal length downstream dummy pattern, no averaging over the expected cosine distribution of flux has been made. Examination of Figure II shows that the buildup and burnout deviates from linear only slightly over the course of this irradiation, consequently the error incurred by not averaging over the flux distribution is negligible.

It is interesting to note that the effective neutron temperature which pertained during the irradiation as indicated by the calculations was about 390° C. Similar comparisons made with natural uranium fuel elements have indicated neutron temperatur approximately 300 - 320° C for graphite temperatures in this range. Certainly the 1.44% E metal selectively hardens the neutron spectrum to a larger extent than does natural uranium, but the data at present are insufficient to determine hardening for a given absorption and slowing down characteristics.

The calculations of product buildup and U^{235} burnout indicate that sufficiently accurate values of the spectral index can be calculated by the methods in Appendix IV This parameter is required for general utility of the Westcott formulation for other reactor lattices.

W. E. Niemuth (Rm)

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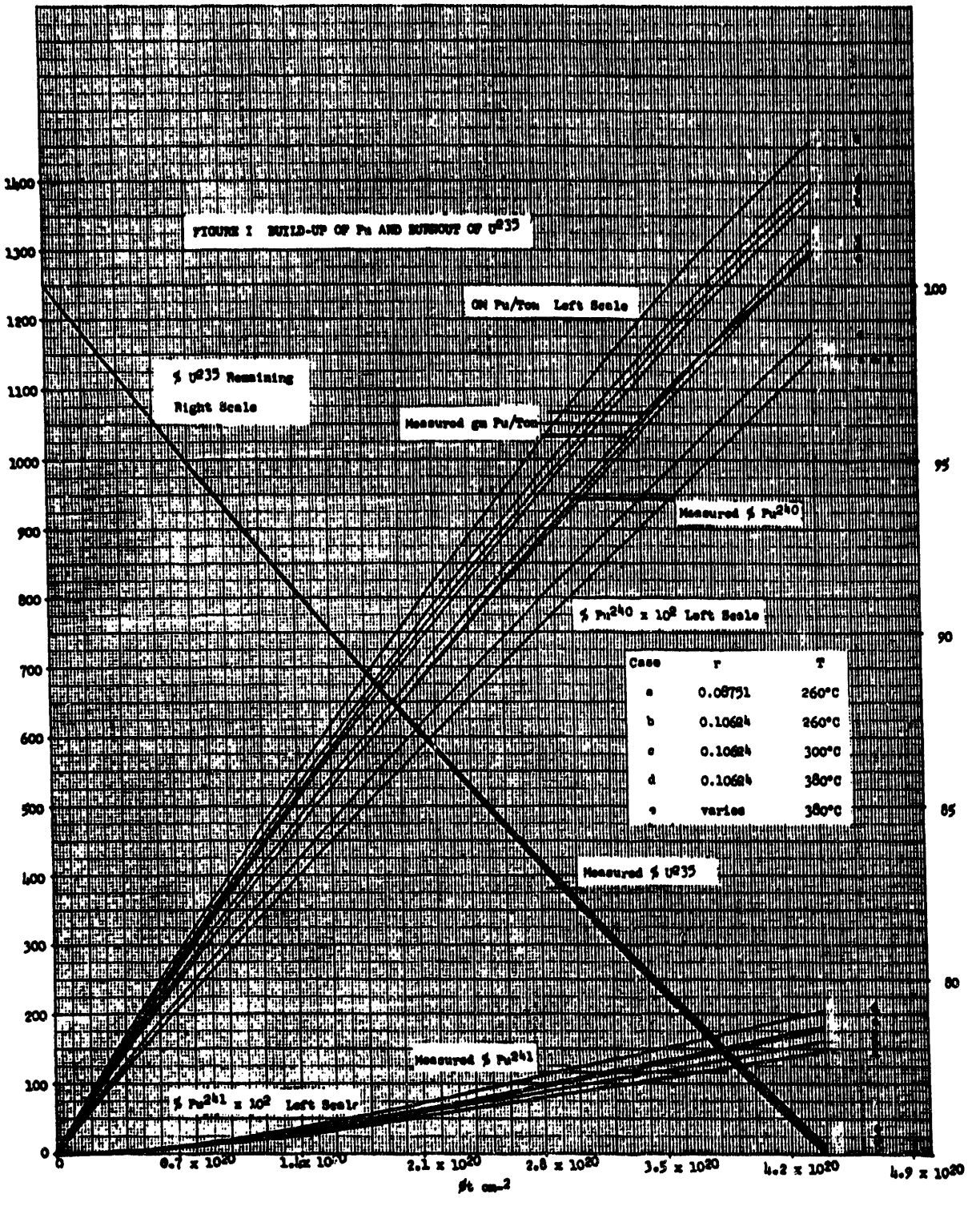

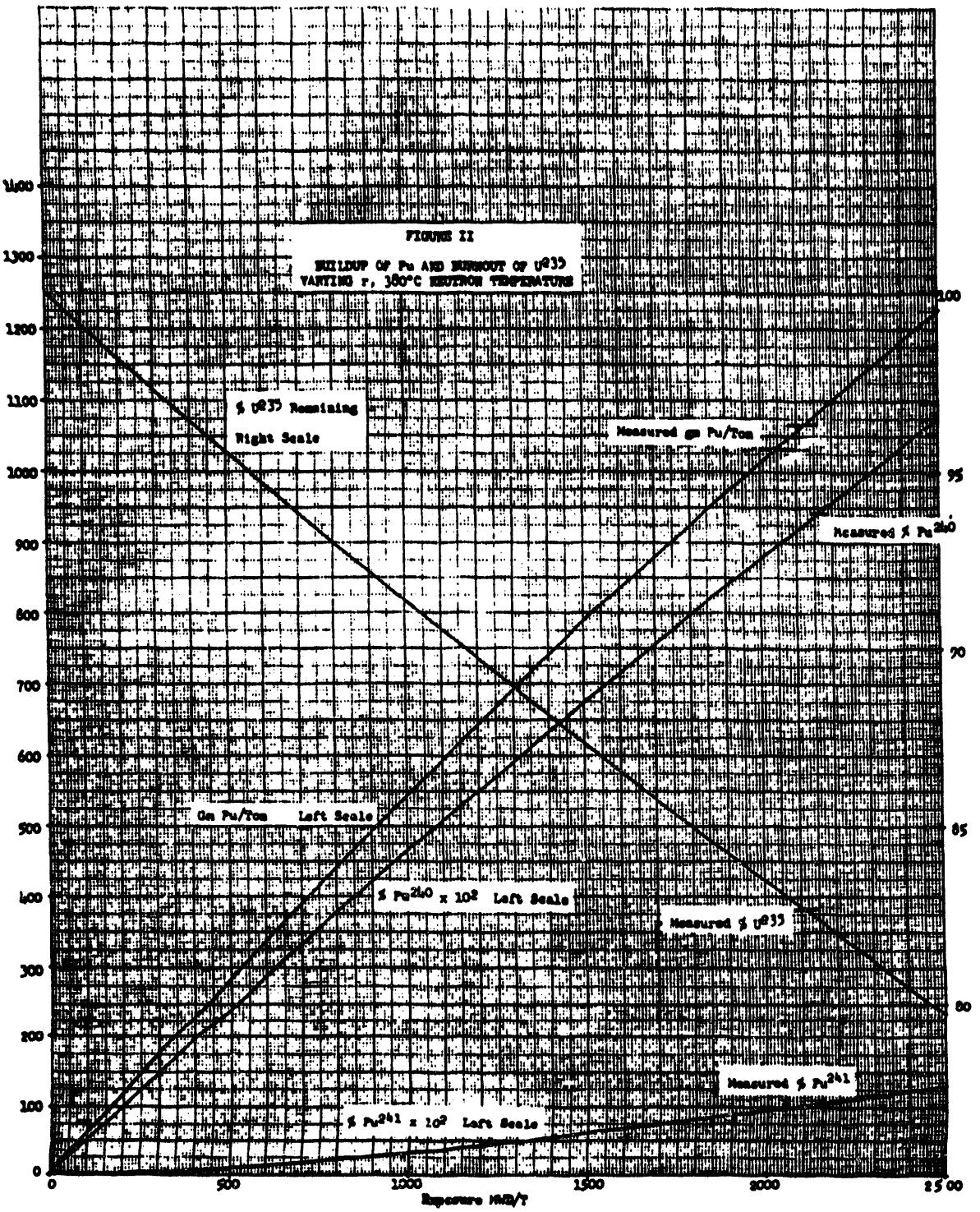

APPENDIX I - BUILDUP EQUATIONS(6)

(1) $$N_{25}(\phi t) = N_{25}^{\circ} e^{-\sigma_{25}\phi t}$$

(2) $$N_{49}(\phi t) = \frac{N_{28}^{\circ}\sigma_{28}}{\sigma_{49}}\left(1 - e^{-\sigma_{49}\phi t}\right) + N_{49}^{\circ} e^{-\sigma_{49}\phi t}$$

(3) $$N_{40}(\phi t) = N_{40}^{\circ} e^{-\sigma_{40}\phi t} + \left(\frac{\sigma_{49}-\sigma_{f49}}{\sigma_{49}-\sigma_{40}}\right) N_{49}^{\circ}\left[e^{-\sigma_{40}\phi t} - e^{-\sigma_{49}\phi t}\right] +$$

$$\frac{N_{28}^{\circ}\sigma_{28}}{\sigma_{49}}\left(\frac{\sigma_{49}-\sigma_{f49}}{\sigma_{40}}\right)\left[1 + \frac{\sigma_{40}}{\sigma_{49}-\sigma_{40}} e^{-\sigma_{49}\phi t} - \frac{\sigma_{49}}{\sigma_{49}-\sigma_{40}} e^{-\sigma_{40}\phi t}\right]$$

(4) $$N_{41}(\phi t) = N_{41}^{\circ} e^{-K_{41}\phi t} + \left(\frac{\sigma_{49}-\sigma_{f49}}{K_{41}}\right) N_{49}^{\circ}\left[\left(\frac{\sigma_{40}}{\sigma_{49}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{40}}\right) e^{-\sigma_{40}\phi t} +\right.$$

$$\left.\left(\frac{\sigma_{40}}{K_{41}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{49}}\right) e^{-K_{41}\phi t} - \left(\frac{\sigma_{40}}{\sigma_{49}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{49}}\right) e^{-\sigma_{49}\phi t}\right] +$$

$$\frac{N_{28}\sigma_{28}}{\sigma_{49}}\left(\frac{\sigma_{49}-\sigma_{f49}}{K_{41}}\right)\left[1 + \left(\frac{\sigma_{40}}{\sigma_{49}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{49}}\right) e^{-\sigma_{49}\phi t} - \left(\frac{\sigma_{49}}{\sigma_{49}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{40}}\right) e^{-\sigma_{40}\phi t} -\right.$$

$$\left.\left(\frac{\sigma_{40}}{K_{41}-\sigma_{40}}\right)\left(\frac{\sigma_{49}}{K_{41}-\sigma_{49}}\right) e^{-K_{41}\phi t}\right] + \left(\frac{\sigma_{40}}{K_{41}-\sigma_{40}}\right) N_{40}^{\circ}\left[e^{-\sigma_{40}\phi t} - e^{-K_{41}\phi t}\right]$$

(5) $$N_F(\phi t) = (1+\delta)\left\{\frac{\sigma_{f49}}{\sigma_{49}} N_{49}^{\circ}\left(1 - e^{-\sigma_{49}\phi t}\right) + \frac{N_{28}^{\circ}\sigma_{28}}{\sigma_{49}}\left(\frac{\sigma_{f49}}{\sigma_{49}}\right)\left[\sigma_{49}\phi t - \left(1 - e^{-\sigma_{49}\phi t}\right)\right] +\right.$$

$$\frac{\sigma_{f41}}{K_{41}} N_{41}^{\circ}\left(1 - e^{-K_{41}\phi t}\right) + \frac{\sigma_{f25}}{\sigma_{25}} N_{25}^{\circ}\left(1 - e^{-\sigma_{25}\phi t}\right) + \frac{\sigma_{f41}}{K_{41}} N_{40}^{\circ}\left[1 +\right.$$

$$\left.\left(\frac{\sigma_{40}}{K_{41}-\sigma_{40}}\right) e^{-K_{41}\phi t} - \left(\frac{K_{41}}{K_{41}-\sigma_{40}}\right) e^{-\sigma_{40}\phi t}\right] + \frac{\sigma_{f41}}{K_{41}}\left(\frac{\sigma_{49}-\sigma_{f49}}{\sigma_{49}}\right) N_{49}^{\circ}\left[1 +\right.$$

$$\left.\left(\frac{\sigma_{40}}{\sigma_{49}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{49}}\right) e^{-\sigma_{49}\phi t} - \left(\frac{\sigma_{49}}{\sigma_{49}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{40}}\right) e^{-\sigma_{40}\phi t} - \left(\frac{\sigma_{40}}{K_{41}-\sigma_{40}}\right)\left(\frac{\sigma_{49}}{K_{41}-\sigma_{49}}\right) e^{-K_{41}\phi t}\right]$$

$$+\frac{\sigma_{f41}}{K_{41}}\frac{N_{28}\sigma_{28}}{\sigma_{49}}\left(\frac{\sigma_{49}-\sigma_{f49}}{K_{41}}\right)\left[K_{41}\phi t + \left(\frac{\sigma_{40}}{\sigma_{49}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{49}}\right)\left(\frac{K_{41}}{\sigma_{49}}\right)\left(1 - e^{-\sigma_{49}\phi t}\right) -\right.$$

$$\left.\left.\left(\frac{\sigma_{49}}{\sigma_{49}-\sigma_{40}}\right)\left(\frac{K_{41}}{K_{41}-\sigma_{40}}\right)\left(\frac{K_{41}}{\sigma_{40}}\right)\left(1 - e^{-\sigma_{40}\phi t}\right) - \left(\frac{\sigma_{40}}{K_{41}-\sigma_{40}}\right)\left(\frac{\sigma_{49}}{K_{41}-\sigma_{49}}\right)\left(1 - e^{-K_{41}\phi t}\right)\right]\right\}$$

In these equations 25, 49, 40, 41 and F represent U^{235}, Pu^{239}, Pu^{240}, Pu^{241}, and fissions, respectively, ϕt is the product of local flux and time, N represents nuclei/cc, N^0 the initial concentration for the interval in the calculations, σ without subscript represents absorption cross section, and σ_f is the fission cross section.

$K_{41} = \sigma_{41} + \frac{\lambda_{41}}{\phi}$ where λ_{41} is the decay constant for Pu^{241}.

Sample Cross Sections:

The cross sections were calculated using the parameters from CRRP 787. A slight adjustment was made in the cross section of isotopes of Pu^{239}, Pu^{240}, and Pu^{241}, in order to allow for the preferential buildup of the isotopes near the surfaces of the fuel element. The results of this preferential buildup near the surfaces is that these isotopes "see" greater flux than either of the uniformily dispersed uranium isotopes. Conversely, the spectrum is not hardened as much near the surface as for the average over the slug, and r decreases. The corrections were determined for the buildup of plutonium isotopes in natural uranium, and are thus only approximate. Furthermore the corrections are really flux corrections, but the normalization of buildup of one isotope compared to another is quite complicated if the flux time parameter is different for each. The following corrections were applied to the cross sections for this effect.

Isotope	Flux Correction	r correction
U^{235}	1.000	1.000
Pu^{239}	1.014	0.986
Pu^{240}	1.024	0.977
Pu^{241}	1.034	0.968

The cross sections in barns for r = 0.08377 and a neutron temperature of 380° C are:

$$\sigma_{25} = 693.52\left[0.9197 + 0.08387\ (0.1631)\right] = 647.32$$

$$\sigma_{f25} = 582.78\left[0.9168 + 0.08387\ (0.0456)\right] + 536.52$$

$$\sigma_{28} = 2.71\ \left[1.01025 + 0.08387\ (8.7314)\right] = 4.7223$$

$$\sigma_{49} = (1.014)\ (1031.10)\left[1.8280 + 0.08271\ (0.842)\right] = 1984.09$$

$$\sigma_{49} = (1.014)\ (747.73)\left[1.6194 + 0.08271\ (0.644)\right] = 1268.24$$

$$\sigma_{40} = 1.024\ (300)\ \left[1.145 + 0.08193\ (48.97)\right] = 1583.89$$

$$\sigma_{41} = (1.034)\ (1.3765)\ .1015.22)\left[1.386 + 0.08115\ (0.032)\right] = 2005.70$$

$$\sigma_{f41} = 1.034\ (1015.22)\left[1.386 + 0.08115\ (0.032)\right] = 1457.10$$

$$\frac{\lambda^{41}}{\phi} = \frac{1.6907 \times 10^{-9}\ sec^{-1}}{3 \times 10^{13}\ cm^{-2}\ sec^{-1}} = 56.358$$

APPENDIX II

Variation in Pu^{240} cross section as Pu^{240} is produced.

In the Westcott cross section, $\sigma = \sigma_0 (g + rs)$, the value of s may be represented as

$$s = \sqrt{\frac{4T}{\pi T_0}} \left[\frac{\Sigma}{\sigma_0} - g \sqrt{\frac{4E_0}{E_{cd}}} \right]$$

in which Σ is the resonance integral including the 1/v contribution above a cadmium cutoff energy E_{cd}. The value of Σ_{40} suggested in Westcott's paper is 8850 for infinite dilution. A value of the effective resonance integral may be calculated from[7]

$$\Sigma_{eff} = \sum_i \frac{I}{\Sigma} \frac{\sigma_i' \Gamma_{\gamma i}}{E_i'} \left(\frac{N \sigma_i'}{\Sigma_D} + 1 \right)^{-1/2}$$

where σ_i', $\Gamma_{\gamma 1}$, and E_1' are the peak cross section, radiation width and energy at the peak of the i th resonance. Σ_d is the macroscopic scattering cross section of the diluent, and N is the nuclei/cc of the isotope whose resonance integral is being determined. For infinite dilution, $N \sigma_i' / \Sigma_D = 0$. When the Pu^{240} is dispersed in uranium, the scattering cross section of the diluent is scattering cross section of uranium which was taken to be 8.3 barns. Thus as the Pu^{240} builds up, the resonance integral may be calculated for various concentrations. The assumption was made that all of the resonance absorption in Pu^{240} occurred in the 1.055 ev resonance and the fission contribution was negligible. With $\Gamma_{\gamma i}$ equal to 32 mv,

$$\sigma_0 = 8.850 \times 10^3 \left(\frac{2}{\pi} \right) \frac{1.055}{3.2 \times 10^{-2}} = 1.8575 \times 10^5 \text{ barns}$$

$$\frac{\sigma_0}{8.3} = 2.2379 \times 10^4$$

With these relationships and the assumption of constant concentration of U^{238} (47.116×10^{21}/cc), the cross sections of Pu_{240} for each of the flux-time intervals was calculated. The results are shown below for 380° C neutron temperature and varying r:

ϕt	N_{40}/N_{28}	Σ_{eff}	s_{40}	σ_{40}
0	0	8850	48.97	1583.89
0.7×10^{20}	0.07624×10^{-4}	8180	45.013	1496.47
1.4×10^{20}	0.20242×10^{-4}	6927	37.986	1324.59
2.1×10^{20}	0.59121×10^{-4}	5801	31.663	1166.44
2.8×10^{20}	0.99042×10^{-4}	4935	26.802	1043.19

APPENDIX III

Determination of s_{28}

The value of s for various isotopes are listed in CRRP-786 for thin samples of absorbing material and the self shielding of the resonances is neglected. When the material under consideration is clumped, such as the U^{238} in this experiment, the problem of what to use for s_{28} arises. Initial calculations assumed that the Westcott formula was applicable for clumped samples, and that the resonances integral to use in the s formulation could be calculated by the method of Hellstrand(8).

$$RI = 2.81 + 24.7\sqrt{\frac{s_{eff}}{M}}$$

where s_{eff} is the effective surface, and M is the mass.

However, it was impossible to correlate the data with such an assumption, and the value of s_{28} was determined by use of a conversion ratio formula which has been shown to give satisfactory results, namely.

$$C_o = \frac{N_{28}\bar{\sigma}_{28}}{N_{25}\bar{\sigma}_{25}} + \eta_{25}\epsilon(1-p)e^{-B^2\tau}$$

where ϵ is the fast effect, p is the resonance escape probability, $e^{-B^2\tau}$ is the fast leakage and η_{25} is the average number of neutrons per thermal neutron absorption in U^{235}. Equating the above formula to

$$\frac{N_{28}\sigma_{o28}(g_{28}+rs_{28})}{N_{25}\sigma_{o25}(g_{25}+rs_{25})}$$

and using a calculated value of r (Appendix IV) the value of s_{28} was determined for a natural uranium slug of the same size as the 1.44% E slugs. The value of s_{28} for natural uranium slugs was determined since the conversion ratio is approximately known in a symmetrical C-pile. Using the following information,

$$1 - p = 0.12299$$

$$\eta_{25} = 2.057$$

$$r_{nat} = 0.06357$$

$$\epsilon = 1.037$$

$$e^{-B^2\tau} = 0.98$$

s_{28} for 260° neutron temperature equaled 7.052, or the effective resonance integral is 12.562. The latter value is considerably larger than the 10.155 used in the calculation of p originally, however, the discrepancy was not investigated further.

APPENDIX IV

Calculations of the Spectral Index, r.

The flux spectrum in the reactor is assumed to be

$$\phi(E)\,dE = (n\bar{v})_M \left[\frac{E}{(kT)^2}\, e^{-E/kt} + \beta \frac{\Delta}{E}\right] dE$$

where $(n\bar{v})_M$ is the true Maxwellian flux and is a convenient normalizing factor, β is the height of the 1/E slowing down distribution at the energy at which the Maxwellian and 1/E distribution join, and Δ is a step function equal to 0 below the joining energy and 1 above*

At a given energy E, the slowing down rate will be equal to the total neutron absorption plus leakage rate below energy E; this preserves neutron balance. Since

$$q(E) = \phi(E) E \Sigma_s \xi V$$

we can write, for the energy at which the Maxwellian and 1/E tail join

$$\phi(E)\,(n\bar{v})_M\,\frac{\beta}{E}$$

where the neutrons in the Maxwellian above the joining energy are neglected. Thus,

$$q(E) = \beta \Sigma_s \xi (n\bar{v})_M V$$

The total reaction rate in the cell for a Maxwellian distribution is given by

$$R = \sum_i \int_0^\infty N_i V_i \sigma_i(E) (n\bar{v})_{Mi} \left\{\frac{E}{(kT_i)^2} e^{-E/kT_i}\right\} dE$$

which for a 1/v absorber reduces to

$$R = \sum_i \int_0^\infty N_i V_i \sigma_{0i} \left(\frac{kT_0}{E}\right)^{1/2} (n\bar{v})_{Mi} \left\{\frac{E}{(kT_i)^2} e^{-E/kT_i}\right\} dE$$

Upon integration this becomes

$$R = \sum_i N_i V_i \sigma_{0i} (n\bar{v})_{Mi} \sqrt{\frac{\pi T_0}{4T}} = \sum_i V_i \bar{\Sigma}_i (n\bar{v})_{Mi}$$

*This is not a serious error, since a transfer function can be formulated which will reduce to the step function. When a resonance appears near the joining energy i.e. Pu^{239} the error may not be negligible, however.

To preserve neutron balance

$$\sum_i \rho_i \Sigma_{si} \xi_i (n\bar{\nu})_{Mi} V_i = \sum_i V_i \bar{\Sigma}_i (n\bar{\nu})_{Mi} \ell_T$$

where ℓ_T is the thermal leakage.

From the equation for the flux distribution, for an energy at which the Maxwellian is negligible.

$$\phi(E)_1 = (n\bar{\nu})_{Mi} \frac{\beta_i}{E}$$

and if the fine structure variation of the resonance flux is neglected

$$(n\bar{\nu})_{Mu} \cdot \frac{\beta_u}{E} = (n\bar{\nu})_{Mi} \frac{\beta_i}{E}$$

where the u subscript designates uranium.

By definition, the thermal utilization for a reactor cell is given by

$$f_u = \frac{V_u \bar{\Sigma}_{au} (n\bar{\nu})_{Mu}}{\sum_i V_i \bar{\Sigma}_{ai} (n\bar{\nu})_{Mi}}$$

and thus

$$\beta_u = \frac{V_u \bar{\Sigma}_{au} \ell_T}{f_u \sum_i \Sigma_{si} \xi_i V_i}$$

By definition,

$$r = \frac{\beta}{1 + b\beta} \qquad b = \frac{4}{\sqrt{\pi \mu}}$$

where kT is the energy at which the Maxwellian and slowing down distribution join. A value of μ equal to 4 was used in this set of calculations, however, it is not well known. Fortunately, the value of r is rather insensitive to μ.

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