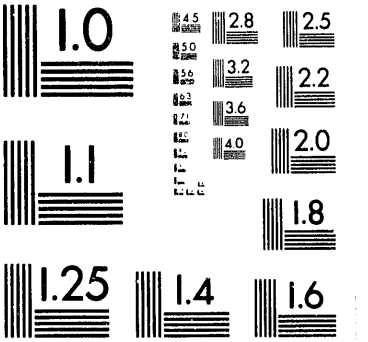
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DOE/ER/10713--T5

Final Report submitted to the Department of Energy for Grant No. AS05-80ER10713 Task A

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1. Introduction

This is the Final Report on our DOE supported research project on theoretical particle physics (grant # DE-AS05-80ER10713). Our project is entitled " Investigations in Gauge Theories, Topological Solitons and String Theories", termination date 1/31/93.

The high energy physics group at Virginia Tech has continued to be a strong, well-rounded research program. The theoretical component of this effort is an integral part of the total effort. After the retirement of the late Prof. R. Marshak , it has consisted of two faculty members currently at the rank of full professor, a post-doctoral research associate and two graduate students. The group interacts quite closely with the two experimental groups. The first is supported by the DOE under Task B of the above mentioned contract. These groups have been involved in the ZEUS and AMY collaborations respectively and will be involved in the SSC (one group is already part of the SDC collaboration) and B-factories. Greater details on these high energy experimental groups and other physics and mathematics groups at VPI with whom we interact were already given in our progress reports and continuation proposal.

The DOE grant provided support for a post-doct and a graduaite student. Our post-doct has been Dr. Gary Kleppe, a Ph. D. from the University of Florida, Gainsville. This fall , he will post-doctoral fellow at the University of Alabama. The graduate student assistantship had been held alternately by Marek Grabowski, working under C.H. Tze and Chopin Soo, working Laynam Chang. The fine students now have their Ph. D. degrees and we are glad to report that both have gotten post-doctoral positions. Grabowski is doing particle theory research as a post-doctoral fellow supported by the Canadian research Council at the Laval University in Quebec, Canada. Soo, a recipient of the prestigious Cunningham Dissertation year Fellowship in 91-92, will be joining the new Center for Quantum Gravity Research of Drs. Astekhar, Smolin et al at Penn State University, as a research associate, starting this fall.

The major theme of particle theory pursued under DOE support has been within the rubric of the standard model, particularly on the interplay between symmetries and dynamics. Thus the research has been carried out primarily in the context of gauge with or without chiral fermions and in effective chiral Lagrangian field theories. The topics studied include: the physical implications of abelian and non-abelian anomalies on the spectrum and possible dynamical symmetry breaking in a wide range of theories. A wide range of techniques of group theory , differential geometry and function theory have been applied to probe a) topological and conformal properties of quantum field theories in two and higher

dimensions, b) the breaking of global chiral symmetries by vector-like gauge theories such as QCD, c) the phenomenology of a possibly strongly interacting Higgs sector within the minimal standard model, and the relevance of solitonic ideas to non-perturbative phenomena at SSC energies.

The following summarizes what has been achieved by the members of our particle theory group during the period of the above DOE contract.

1. Report of Lay Nam Chang

Novel Facets of the Chiral Anomaly

There has been a recent upsurge in the application of chiral perturbation theory in analyzing low energy phenomena, such as rare meson decays and heavy quark decays. The starting point of such calculations is the non-linear sigma model. In order to take care of anomalies in the electroweak sector, the action must be augmented by the Wess-Zumino-Witten (WZW) term[1]. In the momentum power count of chiral perturbation theory, this term will become important at $O(p^4)$.

While the augmented non-sigma model action density can be expressed in unique form, up to chiral re-definitions of the Nambu-Goldstone fields, the corresponding expression in the presence of vector bosons is not so easy to write down. However, at least as far as the interactions due to the anomlies are concerned, the form is fixed by general topological arguments[1]. The resulting Wess-Zumino term determines the corresponding amplitudes near threshold.

It turns out that such arguments can be generalized to produce a constraint on all such amplitudes[2]. For example, in the absence of any gauge field couplings, the lowest meson amplitude receiving a contribution from the Wess-Zumino term is the five-point function. The coefficient of this amplitude at threshold is fixed by this term to be proportional to the number of colors of the underlying QCD interaction responsible for the breaking of the flavor chiral symmetry. When gauge field couplings are present, there will be other five-point amplitudes, wherein the external Nambu-Goldstone bosons are replaced successively by spin-1 fields. The coefficients of these amplitudes near threshold are fixed, once again, by the Wess-Zumino term involving these fields.

The sum of all such amplitudes defines the total five-point correlation function of axial currents. The constraint referred to above fixes the longitudinal part this correlation function to be equal to a universal constant multiplying a given kinematic factor, for *all* values of external momenta, and independent of the chain of symmetry breaking[2]. Similar constraints exist for higher point amplitudes. Since this constraint holds for all values of external momenta, it provides a relationship for both the real and imaginary parts for these amplitudes away from threshold, thereby furnishing a check on higher order calculations of chiral perturbation theory.

There is another aspect of these arguments which is worth emphasizing. In $1+1$ dimensions, the non-linear sigma model with Wess-Zumino term furnishes a concrete realization of the idea of non-abelian bosonization[3]. For example, the local current algebra derived from the effective action is identical in structure to that defined by the underlying fermions, including the structure of the extension terms in the time-time commutators. To what extent does this idea carry over to $3+1$ dimensions? What can be demonstrated on the basis of general topological arguments is that there is a similar constraint in $3+1$ dimensions, but it refers to the triple commutator of charge densities, rather than to the simple commutator, as in $1+1$ dimensions[4],[5]. However, the coeeficients in the two constraints are similiar, and both are proportional to the number of colors carried by the underlying fermions.

The surprising thing about the constraint in $3+1$ dimensions is that it tells that the simple commutator between two charge densities must have an extension term, which is of the form

$$i\, d^{ab}{}_{c} \nabla \times \vec{\mathcal{A}}^{c} \cdot \nabla\, \delta^{3}\{\vec{x}-\vec{y}\}$$

Here d^{abc} is the symmetric coefficient of the Lie group, here assumed to be the chiral $SU(N_f)$ appropriate for N_f species of chiral fermions, which is defined by the trace of three λ matrices. $\vec{\mathcal{A}}^{a}$ is a set of spin-1 fields transforming as the adjoint under the group, whose commutator with the charge density contains a Schwinger term proportional to $\nabla\delta^{3}\{\vec{x}-\vec{y}\}$. The constraint on the triple commutator then requires that the product of the coefficient of this Schwinger term and the coefficient of the extension term given above equals a universal number, which is proportional to the number of colors of the underlying fermions.

The existence of such extensions modifies several existing sum rules, many of which were derived using naive commutation relations. For example, the spin-flip Compton scattering amplitude is expected to satisfy a superconvergence relation, primarily because the naive commutation relation between two electric charge densities is thought to be zero[6]. If there is an extension, however, this sum rule will be modified, because the extension itself implies that there will be fixed pole at $J=1$, whose residue is simply proportional to the matrix element $\vec{\mathcal{A}}^{a}$ between the target state[7].

In [8], the phenomenological consequence of such modifications is studied for nucleon Compton scattering. The coefficient of the extension term is determined under the approximation that the relevant algebra is closed with only pions and spin-1 bosons, which

are the only fields present in the non-linear sigma model with Wess-Zumino term. Data involving scattering off neutrons can now be combined with those for protons to provide a finer check on the sum rule, and it has been known for sometime that this particular sum rule doesn't seem to work at all[9]. What is shown in [8] is that, under the approximation stated above, the addition of the extension improves the situation substantially. In addition, the modified sum rule now joins smoothly onto the Bjorken sum rule for polarized deep inelastic scattering.

Classical and Quantum Gravity

Unlike the Yang-Mills system, general relativity is defined entirely through a series of constraints. This has made the quantization of general relativity somewhat more complicated than that of gauge theories. Over the last several years, however, the set of variables introduced by Ashtekar[10] appears to be very useful for studying quantum gravity.

The basic idea is that the constraint algebra of general relativity can be realized rather simply in terms of a set of $SO(3)$ Yang-Mills fields[1]. General relativity is then expressed in terms of these gauge fields and the set of bein fields which describe the metric, and these are related canonically.

What kinds of Yang-Mills fields correspond to specific solutions to Einstein equations? It turns out that every Einstein manifold corresponds to an anti-instanton configuration in the $SO(3)$ gauge field[11]. Now, quite a bit is known about such configurations[12]. For example, the moduli space of these configurations contains singularites, which correspond to anti-instantons which are abelian in character. In [11], the configurations for several well-known Einstein manifolds are identified, and included among these are the singular abelian anti-instantons. In addition, a procedure for obtaining new solutions to Einstein's equations based on such a relationship is also spelled out, and several explicit examples are worked out.

The construction is particularly well-suited for realizing the topological structures of given Einstein manifolds. Indeed, the procedure spelled out in [11] incorporates the Hitchin bound automatically[13].

In general relativity, solutions to Einstein equations are classified according to the scheme of Petrov. In the framework under discussion, this classification is expressed simply

[1] For simplicity, we will be concentrating on Riemannian manifolds. For pseudo-Riemannian manifolds, the corresponding gauge fields belong to the complex $SU(2)$ group, and these are required to satisfy some reality conditions[10].

in terms of a 3×3 matrix which relates the set of canonically conjugate variables of gauge fields and bein fields on shell. For the case of Einstein manifolds described by abelian anti-instantons, this 3×3 matrix takes on the structure of a Higgs-like field, whose on-shell values break the local $SO(3)$ symmetry down to the abelian sub-group[11]. It appears therefore that general relativity can exist in different phases, and that such phases are perhaps characterized in terms of the Ashtekar variables.

The existence of different phases of general relativity has already been suggested by Witten[14]. Conventional counting in phase space tells us that general relativity can accomodate up to two local degrees of freedom, which may be thought of as the two helicity states of the massless graviton. Witten suggests that both general relativity and the Yang-Mills system may have a phase which has no local degrees of freedom whatsoever, wherein correlations are expressed entirely in terms of global invariants. This is referred as an example of topological quantum field theory (TQFT)[15].

In [16], we study this question by examining a gauge-fixed Ashtekar action under BRST transformations. Several sequences of BRST invariants are identified through the method of descent equations. Generically, all such sequences are independent of each other, and the usual counting in canonical phase space indeed indicates that there two local degrees of freedom. However, under appropriate circumstances, these sequences become degenerate[16]. When this happens, the same counting says that there are no local degrees of freedom. What is rather interesting about this circumstance is that the resultant form of the action becomes identical to the TQFT action used by Witten in his characterization of the the Donaldson map[17]. The Ashtekar variables therefore provide a convenient basis for studies of the properties of the TQFT phase of general relativity.

All such results allow for the presence of a cosmological constant. Indeed, in some instances, the nature of the solutions changes drastically as the cosmological constant approaches zero. An example lies in the abelian anti-instanton sector, where the associated Einstein manifold becomes singular in this limit. Whatever the total phase structure of general relativity may be, it appears that it is critically dependent upon the value of this constant. However, the nature of any possible transitions among these different phases remains an open question at this point.

Chiral Symmetry Breaking in QCD at Zero and Finite Temperatures

It is generally believed that, in the absence of electrweak interactions, QCD is responsible for the breaking of the global $SU_L(N_f) \times SU_R(N_f)$ chiral symmetry of massless quarks[18]. Here N_f is the number of flavors, and the global symmetry is exact in the absence of Yukawa and electroweak gauge couplings. There are no scalar fields in the standard model which carry indices relevant to this group, and the dogma of the standard model is that the color gauge interactions alone are responsible for the spontaneous breaking of this symmetry. The associated Nambu-Goldstone bosons, which acquire masses as a result of the electroweak and Higgs interactions, are identified with the observed pseudo-scalar bosons. The interactions of these bosons are dictated in effect by the spontaneous breaking of this global symmetry.

Lattice calculations, especially within the quenched fermion approximation, generally confirm that this actually takes place[19]. The conclusion is strengthened by more recent studies using dynamical fermions[20].

If indeed such spontaneous chiral symmetry breaking,XSB, does take place, one may inquire how this phenomenon behaves under renormalization group, RG, transformations. In [21], the form of the fermion 2-point function is analyzed using RG techniques, and the solution exhibiting XSB identified. In particular, it can be used to relate the chiral symmetry order parameter $\langle\psi\bar{\psi}\rangle$ to the RG invariant QCD cut-off parameter Λ_c, thereby providing quantitative means of studying XSB in QCD.

A similar analysis has also been carried out at finite temperatures[22]. The result turns out to be rather surprising in that XSB appears to take place for all temperatures. Specifically, we find that both the chiral-flip part of the fermion 2-point function and the appearance of a massless pole in the pseudo-scalar channel persist for arbitrarily high temperatures.

In [23], an effective potential for the scalar and pseudo-scalar excitations at finite temperature is constructed, with all parameters determined by the underlying parameters of QCD. As a result, the mass of the scalar excitation is fixed relative to the fermion mass. The mass of the pseudo-scalar pion, however, remains zero for all temperatures.

In the analysis, crucial use is made of the renormalization group properties of QCD. This property enables us to sum over higher order loops so that the signatures for chiral symmetry breaking are given by RG invariant quantities. A system such as that described

by the Nambu-Jona-Lasinio equations is not renormalizable, and so the chiral symmetry breaking parameters will be dependent explicitly on a cut-off. These parameters, determined from summing over diagrams of a certain order in loops, are reliable only for energies much less than this cut-off. With this caveat, one can look for the possibility of a critical temperture above which the symmetry is restored. And indeed, for the Nambu-Jona-Lasinio system, a finite critical temperature can be shown to be present. The situation for a renormalizable theory such as QCD is apparently quite different.

How these results are affected by the presence of electroweak interactions remain an open question at this time.

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2. Report of C.H. Tze

The focus of my research has been on the physics and mathematics of non-perturbative phenomena in particle physics. I have undertaken a combined algebraic, group theoretic and functional approach to study strong interactions, various global and solvable aspects of quantum field theories as well as possible minimal extensions of the Standard Model. The specific problems under study belong to four connected frontier areas. They are:

1) D≥4 dimensional topological solitons:

a) To seek realistic and explicit examples of D≥3 fermi-bose transmutation by way of the representation theory of infinite dimensional Kac-Moody typed groups.
b) The construction of B >1 Skyrmion ansatzes à la Atiyah-Manton , their subsequent applications to low energy spectra and scattering of extended objects in effective lagrangians
of QCD or the standard electroweak model and minimal extensions

2) Self-duality, conformal structures and covariant quantization of field theories in various dimensions:

a)Harmonic superspace extension and applications of hypercomplex analysis to D≥ 4 analogs of D=2 conformal field theories and to D≥ 4 topological field theories.
b) Exceptional algebraic and group theoretical aspects of superstrings and supermembranes.
c) Applications of non-commutative geometry to understand the standard model and what may lay behond it.

3) Topological field theories in D≥3 dimensions, pure or with sources : Aharonov-Bohm-Berry phase phenomena in field theories, anyonic properties of relativistic membranes and the possibility of exotic spins and statistics in higher dimensions. Various applications to Kaluza-Klein scenarios and to condensed matter physics. Their equivalences to field theory of points, strings and membranes, their geometric quantization via the Kirillov-Kostant method of group orbits.

4) Classical and quantum integrable systems in D≥ 2 dimensions: in particular, the search for explicit solutions of simplex equations and generalized (Yang-Baxter-Zamolodchikov) Lawrence N-algebras in topological quantum field theories. Their dimensional reductions to known integrable systems, the reductions of their special D ≤ 8 calibrated geometries, their associated division,Jordan and related algebraic properties.

The mathematical tools being jointly used are varied and somewhat state of the art. They include division algebras, superalgebras, index theory, theories of harmonic mappings, extended Füeter function theory over a quaternionic variable, twistors techniques, the group orbit method, the representation theories of D≥ 2 current and diffeomorphism algebras, of links and knots, differential geometry of exceptional manifolds and non-commutative geometry.

To put in context the problems under study and the progress to be reported, pertinent past results by Tze (and his collaborators) will be highlighted. They are natural stepping stones to evaluate our results obtained in the past year.

In Section 1 we recall the Skyrmion approach to Tev strong interactions of the standard model's Higgs sector, we then report on our progress on problems there. In Section 2 we assess our ongoing program to formulate and solve higher dimensional analogs of D=2 conformal field theories. In Section 3 we discuss our work with our student, Marek Grabowski on two topics: 1) D≥4 self-dual gauge fields, self-dual membranes and their relations to the large N limit of SU(N) gauge theory 2) a new division algebra formulation and supersymmetric generalization of the Kustannheimo-Stiefel maps with diverse applications to quantum mechanics and quantum field theories.

I. Solitons and Representations of Current Algebras

While still untractable in its strong coupling regime, QCD remains the uncontested theory of strong interactions. Yet the continued lack of analytic techniques to unravel its expected confining infrared behavior has prevented confrontation with an ever more accurate massive body of data on hadrons.

At a more qualitative, phenomenological level, there does exist a tractable, albeit only semi-qualitative and limited analytic method of attacking the strong interaction problem. We are referring to the Skyrmion approach [1]. In the past decade, this soliton method spearheaded by T.R. Skyrme [2] and revived by N.K. Pak and Tze [3] in the context of QCD, has rekindled much activity on effective chiral lagrangians and current algebras. Key contributions first came from two groups, that of Balanchandran [1] at Syracuse and of Witten [4]. It has blossomed into a cottage industry among nuclear theorists and has produced a semi-quantitative, conceptually unified picture of hadrons and many aspects of their low energy interactions [5]. .

For particle physics, the attraction of this method lies elsewhere. Beside and behond the QCD interactions at Gev energies, there may well be detectable manifestation of new strong forces. Thus while the Standard Model has withstood every empirical test, it is nevertheless

effective and incomplete. First there is the arbitrariness in the mass of the Higgs particle; $M_H = [8\pi a_H \sqrt{2} G_F]^{1/2}$; with a_H the quartic scalar coupling being unknown. Should the physical Higgs mass exceed 1 Tev (i.e. $a_H \sim 1$), a likely experimental situation, standard perturbative analysis will fail in the then strongly coupled Higgs sector. The W_L's, the W and Z bosons' longitudinal field components (made up by the 3 components of teh Higgs doublet), will interact strongly among themselves and with the quarks and leptons, as was already clear from Veltman's pioneering analysis [6]. These interactions closely parallel the familiar chiral dynamics of the SU(2) sigma model of low energy hadronic interactions as they reveal an additional strong "Higgs force", determined within the standard model, and apparently not accounted by a gauge principle. There would thus be a new "Yukawa sector" potentially as rich if not richer in structure as the quark or lepton systems.

Gipson and Tze [7] first recognized that the effective, 1-loop finite gauged sigma model [8] used to model heavy Higgs effects to have precisely all the global and dynamical features of the Skyrme's action and hence that it too should admit Skyrmions. All the physical implications of the Skyrme model could then naively be carried over to the strongly coupled Higgs regime. One expects a rich, calculable isobaric-liked spectrum of fermionic resonances to show up and the estimated the bounds for the lowest soliton mass are $2.5 \leq M_{Soliton} \leq 4.0$ Tevs,

Such a soliton scheme is at least economical by providing a "minimal extension" of the standard model. This is so since no specific assumption whatsoever is made about the compositeness of the Higgs particle(s). These "weak skyrmions" couple to leptons on which they induce exotic attributes such as a leptonic charge L equal to their baryonic charge B. Similar approaches to the Higgs sector have been proposed in recent years by several groups[9]. Not long ago Tze and P.Q. Hung [10] further applied this approach to the electroweak sector, using the Skyrme's fermionization mechanism to generate new leptons within the Standard Model, without the need of accompanying new quarks (to cancel anomalies) and without any assumption on the compositeness of the Higgs bosons.

The weakness of the Skyrme approach has been a lack of new tools to raise above current qualitative semi-classical level of description, to systematically extract more quantitative results. Thus we could use a formalism involving a sequence of approximations describing one, two and many quantum Skyrmions along with their mutual interactions. Admittedly it is an ambitious problem. Our approach has been to find some kind of quantum Backlund transformation e.g. a D=4 counterpart of Mandelstam's 1- soliton operator bridging the fermion of the massive Thirring model to the quantum soliton in the Sine-Gordon model [11]. The latter intertwining operator emerges naturally from the representation theory of D= 2 Kac-Moody algebra. However about the latter $D \geq 3$ counterpart, the mapping group $\{ S^3 \rightarrow G \}$ with its noncentral Abelian Q-number extension, very little is known. Some mathematical

progress has been made by Rajeev and Mickelsson [11,12,13,14]. The motivations for our program were surveyed in my Voralberg Symposium talk [15]. During the past year, we have actually taken a two prong approach. On the one hand, we tried to combine the methods of representation theory and the dimension specific tools of quaternionic analyticity (to be discussed below) to construct an explicit algebraic construction of at the 1- or better yet a multi-Skyrmion creation operator in the representation theory of D≥3 current (and diffeomorphism) algebras. On the other hand, we have taken a less ambitious, less frontal , semi-classical approach. It consists in first mastering the techniques of Atiyah and Manton [16]which ingeniously generate a classical multi-Skyrmion configuration from a corresponding multi-instanton solution.

Concrete success will clearly be of great relevance to the phenomenological parametrization of the infrared behavior of QCD and a possible strongly coupled Higgs sector. In the past year, we have been mostly tooling up to undertake this second approach by learning the algebraic geometry techniques developed by Atiyah, Hitchin [17] for BPS monopole scattering and the Atiyah and Manton formalism [16] in order to implement the Manton's approach to the low energy interactions of extended hadrons. The next stage will be done in collaboration of Dr. P.Q. Hung of UVA to identify specific phenomenological situations where such techniques may be justifiably applied, preferably in the context of hadrons with at least one heavy quark systems, to extensions of the standard model and to colliding cosmic strings.

2. D≥ 4 Critical Phenomena: Self-Duality, Infinite Dimensional Symmetries and Hypercomplex Analyticity.

With the advent of supergravity, superstrings and supermembranes, there has been a growing appreciation of the importance of mathematical techniques specially tailored for the investigation of field theories in the so called critical (e.g. D= 3, 4, 6 and 10) spacetime dimensions.

That two dimensional conformal field theories are solved by powerful and elegant complex analytic techniques is well known. Nevertheless few know of the existence of (much less have exploited the potentials of) a function theory over a quaternion (more generally over a Clifford algebra) and an octonion variable. In a series of papers (see attached Vita of H.C.T) and through the works of Gürsey et al [18], the role of such function theories over a hypercomplex variable have been amply demonstrated in self-dual Yang Mills theory, in sigma models, and in higher dimensional strings and membrane theories [18,20,21,22]. In Ref[21] it was shown that a conformally Kahlerian structure of the euclidean Schwarzschild Ricci flat

geometry leads to its form invariance under the quaternionic analytic Fueter transformations. The latter form a remarkable infinite subgroup of the general covariance group. Such hypercomplex analyses constitute particular realizations of Penrose's twistor program [23,24] for solving nonlinear equations in classical and quantum field theories.

Recently significant progress[25,26,27] were made on the above Fueter transformations. There is the O(4) (and hence Lorentz) covariantization of these transformations, the grafting on the latter the essential self-(antiself-dual) structure, then the elucidation of various forms of quaternion analyticity and the latter's relation to (super)harmonic space analyticity. The resulting infinite group now has as a subgroup, the D=4 conformal group. When combined with the construction of self and antiself dual quaternionic functions, this improved Fueter formalism gives a D=4 analog of infinite dimensional D=2 Virasoro conformal group. Sothe door may be finally opened for the formulation and for solving for D=4 counterparts of 2-dimensional conformal field theories. These developments haveinduced us to revive our old program for quaternionic analyticity [18]. In an increasing order of priority,our immediate goals have been

a) to see how the new results [25,26,27] may help unlocking the problem of getting a) *explicit*forms for D≥ 4 multi-gauge and gravitational instantons and the connection to the hierachy of ansatzes of Atiyah, Ward [28] , generalizing that of t'Hooft's b) solutions to higher dimensional self-dual integrable systems.

b) to develop and exploit their connections to twistorial approaches, such as to the work of Nair et al [29], to understand different types of quaternionic holomorphic structures from the standpoints of harmonic spinors and superharmonic spaces[30] and of the representation theory of D=4 diffeomorphism groups . Here we may hope to make contact with the investigation of Laynam Chang and his student Soo (for more details see Chang's report) on aspects of D=4 quantum gravity in ashtekar's variables.

c) to find applications to D=4 critical or topological field theories .

In a larger perspective, an overview of the scope of applicability to particle theory of division and related algebras and their associated structures has been gained by Tze over the past four years in the writing of a comprehensive book with the late F. Gürsey . Entitled " On the Role of Division, Jordan and Related Algebras in Particle Physics" this volume will be published this winter by World Scientific. The work of indexing and inserting references in the book is being completed. A table of content is enclosed.

We are continuing to work hard particularly on topics b and c.in collaboration with Marek Grabowski and Peter Orland of Baruch College of the CUNY . Some results achieved in the

areas of classical self-dual systems will be reported below.

3. Division algebras, generalizations of the KS transformations, D≥3 Self dual Yang-Mills fields and membranes, Large N limits.

Our own interest on anyons began with the striking paper of Polyakov [31]. Our goal to understand the physical roles of various anomalies, of added topological action to the standard actions has since then been centered on probing the spin and statistics connection for extended objects in higher dimensional spacetimes. By exploiting a deep connection between the topology of knots and links in specific spacetime dimensions and the uniqueness of four division algebras, our papers [32] showed precisely how anyonic solitons could exist not only in two spatial dimensions but that, surprisingly enough, also in some very specific higher spatial dimensions, their charged membrane counterparts could also exist and seem on topological grounds to be similarly endowed with nonstandard spin and exchange properties. Such is the case for certain membrane solitons in D=3,7 and 15 spacetime dimensions . These solitons are realizations of the so called essential Hopf mappings[33] of ubiquutous presence in many areas of mathematical and condensed matter physics.

This past year, as a natural extension of our work on quantum field theory applications of the four sequences of Hopf maps or fibrations $S^{2n-1} \dashrightarrow S^n$ (n=1, 2,4,8) associated to the four division algebras, we became interested in doing the same for their generalizations called the Levi-Civita transformation $R^2 \dashrightarrow R^2$ and the Hurwitz transformations $R^{2m} \dashrightarrow R^{2m-n}$ where (2m,2m-n) = (2,1), (4,3) and (8,5) [34]. These remarkable transformations have turned up in many areas of physics ranging from celestial mechanics to atomic physics and moleculatr physics . Through them, the Schrödinger problem of a particle in a three dimensional and a five dimensional coulomb field gets mapped onto one of a harmonic oscillator in four and eight dimensions respectively. Our goal is to apply them to probe nonperturbative structure in linear sigma models in quantum field theory.

We have thus far found a very compact and suggestive reformulation of the $R^4 \dashrightarrow R^3$ and $R^8 \dashrightarrow R^5$ maps using quaternions and octonions respectively.This result will be published as the first part of a larger work [35] which will include a) the corresponding supersymmetric generalizations describing a supersymmetric particle in external monopole fields b) connections to N=2 and N=4 superconformal supersymmetries c) applications to linear sigma models and the like in quantum field theory. These topics are being worked out by Tze and Grabowski.

Next we wish to report on progress done in the area of higher dimensional self-dual Yang Mills field and membranes as well as their connection to the large N limit of classical gauge

field theory . A short account of our results is to be found in the enclosed preprint VPI-IHEP-92-9 (July 92) , it will be published in the Proceedings of the Symposium on Symmetries in Science VI: From Rotation Group to Quantum Algebras" held at Bregenz, Austria, August 2-7,1992 and to be published by Plenum Press, B. Gruber, Editor. Two longer papers [36] with further results detailed below are in preparation.

The notion of self-duality has come to occupy a central position at the frontier of mathematical physics and pure mathematics. Self-duality has provided an avenue to probe the vacuum structure of nonlinear field theories such as Yang-Mills theory and Einstein gravity through study instanton solutions. In mathematics, it has opened up new vistas and was instrumental in Donaldson's work bringing about new insights on the topology of four-manifolds. Of special relevance to physics is the following realization about the self-dual Yang-Mills (SDYM) system. A natural object from the viewpoint of topological field theory, the SDYM theory sits at the apex of a hierarchy of solvable dynamical models since, through its various dimensional reductions emerges an unified approach to all integrable systems (e.g. sigma models, KdV, Toda systems) in low dimensions[37].

Furthermore recent results [38]in string and membrane theories leave no doubt about the ubiquitous presence of SDYM solutions and their indirect physical relevance at least as building blocks of nonperturbative string solutions (in particular of string vacua). Thus, on the one hand, solutions of the d = 8 linear self-duality may be used in the construction of the extended soliton solutions of the low-energy heterotic field theory equations of motion. On the other hand each instanton of a d=8 generalized Yang-Mills action give a corresponding heterotic string solution in the low energy theory of the heterotic five-branes.

All the above connections are then motivations enough to investigate self-duality in higher dimensional gauge theories and theories of extended objects.

In four dimensions, an important formulation of self-duality is given by the Nahm equations which generalize the Euler top, the integrable model par excellence. There is for instance a general equivalence holding for any Lie (gauge) group G between the solution of the Nahm equations obeying appropriate boundary conditions, contrained algebraic curves in $TP_1(C)$ and SU(2) monopoles.

Self-duality can be extended to theories of points,namely field theories to theories of extended objects. By analogy to gauge theories, where the self-duality can be naturally and uniquely formulated only for d = 4, one expects that self-dual p-branes in d-dimensional spacetime can exist but again only for some particular sets of p and d. In one approach one may try to get to the self-duality equation for p-branes by exploiting the analogy with gauge

theories. In our work with Grabowski, work constituting part of his completed thesis [39], this has been done for membranes(p=2). To undertake this program, we had to first understand the precise meaning of the gauge-membrane correspondence. We did so successfully by way of the coadjoint orbit method pioneered by Kirillov, Kostant, and Souriau.

From the physicists' viewpoint, the importance of the "coadjoint orbit method" rests not only in the fact that it gives a geometric formulation of the classical dynamics and field theory in a illuminating group-theoretical setting, but mainly that it sets the proper stage for the geometrical quantization. For instance the orbit method has been used to quantize the Toda lattice and the symmetric top. Among the various quantization schemes, geometric quantization occupies a special position. Exploiting the natural symplectic geometry of dynamical systems it provides an elegant, coordinate independent generalization of the hamiltonian dynamics in which quantum systems are realized as irreducible unitary representations of the relevant groups. In practice however great technical difficulties have been met in trying to implement this scheme for more complicated systems, e.g. gauge theories or theories of extended objects. These difficulties are due to the fact that these systems are tightly constrained by their gauge symmetry. There the phase space is effectively reduced from a linear space to a symplectic submanifold, the hypersurface defined by the gauge constraints.

So while the orbit method (and hence its attendent method of geometric quantization) is still unable at present to fully implement the quantization of d= 4 YM theories and theories of extended objects, they have given many insights amongs which the gauge-membrane correspondence. We had showed in a previous paper[40] the classical gauge theories can be formulated as natural theories on the co-adjoint orbits of an infinite group, a semidirect product of the group of gauge transformations and the Heisenberg-Weyl group. Thus the SU(N) classical gauge mechanics has as its unreduced phase space the coadjoint orbit of the semidirect produc of SU(N) and the Heisenberg-Weyl group. Since the two inequivalent limits of the su(N) algebra can be identified with the algebra of (local) area preserving diffeomorphisms of a torus or a 2-sphere (which is in fact the symmetry group of membrane in the light cone gauge) we suspected and indeed checked that the classical SU(∞) gauge mechanics is equivalent to the membrane theory. We have also tried to prove that the large N limit of quantum Yang-Mills mechanics should contain the membrane theory. This task is technically harder and little progress has been made thus far. We have also used the gauge-membrane correspondence to translate the Yang-Mills self- duality in d = 4 (and its generalization for d = 8) into the membrane language. We obtained as results self-dual membranes in 5 and 9 dimensions. Such an approach is however limited to p = 2 only.

Now we recall a general approach to self-dual membrane initiated by us [40] , it is based on the construction of the dual momenta, using the notion of the existence of p-fold vector

products on specific spaces. We showed that remarkably such generalized self-duality can be formulated for 4 classes classified by values of (p, d) according to the existence of the p-fold vector product in the d dimensional vector space. The underlying reason for these 4 classes is the existence of four division algebras of the reals : **R**, **C**, **H** and **Ω**. We have found several classes of solutions in each of the 4 cases; some of them have already been investigated in the literature in a different context. Also the self-dual membranes in 5 and 9 dimensions gotten earlier by way of the membrane-gauge correspondence, can be obtained by a simple transformation from particular cases of this generalized self-duality. Of particular interest are two "exceptional" solutions, given by the octonionic calibrated geometries of Harvey and Lawson: a 2-membrane in 7 dimensions and a 3-brane in 8 dimensions. The self-duality condition in the former case is identical to the octonionic Nahm equation in Poisson gauge theory and can be derived from the vanishing of an octonionic moment map. We see this undertaking as a search for notable analytic structures in higher dimensions,structures which may ultimately some bearing on nonperturbative physics.

4. Other Non-Perturbative Approaches

Most of our research themes fit under the category of non-perturbative methods. We worked on dual string models, mechanism for quark confinement, topological classification and dynamics of monopoles, vortices, instantons and solitons . In last twenty years, powerful algebraic tools have been developed and tailored to solve whole families of integrable systems, classical as well as quantum, primarily in two spacetime dimensions. They are the method of the (quantum) inverse transform , of the generalized Bethe ansatz [40], " vertex operators" or representation theory of Kac-Moody, and more recently the representation theory of Hopf algebras and quantum groups. The burning questions concern the extensions of such methods to higher dimensions.

Aiming to solve for higher dimensional classical and quantum integrable systems, we have been seeking explicit realizations of $D\geq 3$ counterparts of the Yang-Baxter-equations, called simplex equations [42] . We also hope to connecting them to structures already familiar to us, namely higher dimensional topological quantum field theory such as the Donalson theory in four dimensions and its $D>4$ counterpart. The difficulties we have encountered thus far are a) our inability to solve for delta function interactions in two spatial dimensions. The same problem also exists for anyons b) to the fact that Zamodlochikov's equations are not the most general $D=3$ Yang-Baxter typed equations as evidenced by the work of Ruth Lawrence on N-algebras [43]. These issues of higher dimensional integrability are also intimately connected to higher dimensional analogs of braids groups, to the representation theory $D\geq 3$ infinite generalized Kac-Moody and Virasoro groups, as well as to the quaternionic analyticity approach to $D\geq 2$ analogs of $D=2$ conformal field theories.

Our progress has been slow and has yet to hit new fruitful ground. The nature of the problem requires persistence, patience, technical preparedness in the event of breakthroughs by other workers in the field. Here our ideas have much overlap with those of Dr. Peter Orland of Baruch College of the CUNY [44]. Our ongoing discussions with him on these matters have led to a tentative long distance collaboration with mutual visits during the course of this next year.

Publications (91-92)

Marek Grabovski and H.C. Tze, "" **Classical Gauge Theories on the Coadjoint Orbits of Infinite Groups** " Phys. Lett. 258B (1991) 145-150

M. Grabowski and H.C. Tze " **On the Octonionic Nahm Equations and Self-dual Membranes in 9 Dimensions**" VPI & SU Preprint VPI-IHEP-92-9, to appear in the Proceedings of the Symposium " Symmetries in Science VI : From Rotation Group to Quantum Algebras". B. Gruber, Editor (Plenum Press 1993)

Marek Grabovski and H.C. Tze " **Division Algebras, Supersymmetry and Generalizations of the Kustaanheimo-Stiefel Transformation**", preprint in preparation

Marek Grabowski and H.C. Tze " **Self-Dual membranes and the large N limit of gauge theories** " preprint in preparation

Marek Grabowski and H.C. Tze " **Self-Dual membranes and Polyakov Self-intersecting Strings** " preprint in preparation

F. Gürsey and H.C. Tze, "**On Division, Jordan and Related Algebras in Particle Physics**", book to be published this winter or spring by World Scientific, Singapore. Circa 450 printed pages.

Trip activities:

This past year, my seminar and conference activities have been uncharacteristically at low level. This was due to a combination of a protracted, painful viral infection, terminal illnesses of a close friend, mentor and collaborator, F. Gürsey and of my father in law with several connected trips to Chicago and New Haven, CT as well as scheduling conflicts with teaching .

Though I was invited and had agreed to give a series of lectures on algebraic methods in particle physics a) at the XXVIII Karpacz Winter School of Theoretical Physics in Poland 2/ (17-29) /1992 -b) at the 1992 Scottish Workshop on High Energy Physics at the University of Durham, England, (August 23-Sept 11) , I was finally unable for the reasons cited above to attend these meetings. I also had to postpone visits to the Center for Fundamental Theory, U. of Florida, Gainsville, to Baruch College, New York City, till later this fall and winter . By attending the Texas/Pascos'92 Meeting in Berkeley (Dec. 13-18) to be followed by seminar trips to UNC, Chapel Hill and U. of Florida, Gainsville, I will thus initiate a more active trip season for the coming year.

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3. Report of Gary Kleppe

My work in the earlier part of this year involved the study of a technique known as nonlocal regularization, which I had developed earlier in collaboration with Prof. R. P. Woodard of the University of Florida. This technique is intended to be useful as a regularization for cases where previously known methods fail to be applicable. It may also give theories which are valid in their own right, since the technique does not sacrifice unitarity of the S-matrix.

This year I made progress in this area. In collaboration with Prof. Woodard I finished giving the general prescription for regularization of any theory in this manner, and carried out detailed calculations at two loops to show that the method actually worked in practice. I also worked on two applications of this technique. One was $D = 3$ topologically massive gravity, which has been conjectured but not proved by Deser & Yang to be renormalizable. In collaboration with B. Keszthelyi of the University of Florida, I made progress toward proving this conjecture, but we were unable to obtain a complete proof. The second application I worked on was quadratic divergence cancellation in the standard model, which has already been studied by Al-sarhi, Jack, and Jones. I showed that their results were strongly dependent on a particular regulator, and hence unphysical.

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