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PLASTIC BUCKLING OF CYLINDRICAL SHELLS[1]

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ABSTRACT

Cylindrical shells exhibit buckling under axial loads at stresses much less than the respective theoretical critical stresses. This is due primarily to the presence of geometrical imperfections even though such imperfections could be very small (e.g., comparable to thickness). Under internal pressure, the shell regains some of its buckling strength. For a relatively large radius-to-thickness ratio and low internal pressure, the effect can be reasonably estimated by an elastic analysis. However, for low radius-to-thickness ratios and greater pressures, the elastic-plastic collapse controls the failure load. In order to quantify the elastic-plastic buckling capacity of cylindrical shells, an analysis program was carried out by use of the computer code BOSOR5 developed by Bushnell of Lockheed Missiles and Space Company. The analysis was performed for various radius-to-thickness ratios and imperfection amplitudes. The analysis results are presented in this paper.

INTRODUCTION

Thin cylindrical shells buckle under axial loads and the corresponding theoretical critical stress can be calculated by using the classical, elastic shell theory. However, experimental results and field observations have demonstrated that in reality these shells exhibit buckling at a much lower compressive stress than the classical critical stress based on the elastic theory. A close examination has attributed the causes of this reduction of the buckling strength to the inevitable and inherent construction and field conditions that are not reflected in the assumptions for the classical elastic buckling theory. The major contributors are the geometric imperfections, nonuniform membrane stress distribution, and unaccounted boundary conditions and residual stresses. This paper focusses on the effect of geometric imperfections which are manifested as local deformed shapes.

On the other hand, it has been observed that the elastic buckling strength of a thin cylindrical shell is greater if the shell is subjected to internal pressure in addition to the axial compressive load. This is due to reduction of the geometric imperfection by the internal pressure load. However, when the pressure load becomes too large the hoop stress becomes significant and the effect of biaxiality comes into play, which in turn reduces the gain that has been obtained due to presence of the internal pressure.

An ASME Code Case (Miller, 1991) provides formulas to determine the capacity reduction factors (i.e., the ratio of the allowable buckling strength to the theoretical critical buckling capacity based on the classical elastic shell theory). These formulas are based on experimental results for practical geometric imperfections but do not include the beneficial effects of the internal pressure. The code case recognizes this beneficial effect but does not quantify it. On the other hand, a NASA test program (1968) considered the effect of the internal pressure but did not necessarily reflect the geometric imperfections expected in the field. Extensive studies including experiments and theoretical analyses were performed

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in Europe, Australia, and New Zealand to address both the detrimental effect of the imperfection and the beneficial effect of the internal pressure (Vandepette, 1980; Saal, 1977; ECCS, 1988; Rotter, 1985; Rotter, 1989, Priestly, 1986).

The purpose of a current analytical program at Brookhaven National Laboratory (BNL) was to compute the buckling strength of underground cylindrical tanks, that are used for storage of nuclear wastes, for realistic geometric imperfections and internal pressure loads. Elastic analyses were performed for various geometric imperfection shapes and magnitudes by use of BOSOR4 developed by Bushnell and the results were published (Bandyopadhyay, 1993). Subsequently, elastic-plastic analyses have been performed for a particular geometric imperfection shape. This paper presents the results of these elastic-plastic analyses and compares them with other available information for various pressure loads.

COMPUTER MODEL

Similar to the elastic analysis model (Bandyopadhyay, 1993), a 40-foot high circular cylinder was considered in the analysis. The top concrete or steel enclosure of an underground tank was represented in the mathematical model by a rigid diaphragm. The bottom of the cylinder was assumed open and the wall was modeled with a hinge connection along the bottom periphery. The analyses were performed for various radius-to-thickness ratios (R/t) by maintaining a constant value of the radius (R = 40 feet) and varying the thickness. As a result of the earlier studies, a most reasonable shape of an imperfection was represented by an axisymmetric inward bulge of shape ($1-\cos\theta$) located at the bottom of the wall (Figure 1). The length of the bulge, L, was selected to match the natural buckling shape as follows:

$$L = 3.5\sqrt{Rt} \qquad (1)$$

The magnitude of the imperfection in the radial direction, e, was assumed for normal quality construction as follows (ECCS, 1988; Rotter, 1985):

$$\frac{e}{t} = \frac{1}{16.5}\sqrt{R/t} \qquad (2)$$

The computer program BOSOR5 developed by Bushnell (1974) was used for the analysis. An axisymmetric uniform compressive load was applied on top of the wall.

ANALYSIS RESULTS

The results of the analysis are presented in Figure 2 for R/t values of 400, 600, 900, 1200, and 1500. The buckling strength is plotted against the hoop stress so that the benefit of the internal pressure as well as the biaxiality effect is incorporated. Nondimensional quantities are used with the following notations:

σ_{axial} = Axial compressive stress at failure obtained from BOSOR5, corresponding to the governing failure mode

σ_{cr} = Theoretical critical buckling stress based on the classical, elastic shell theory for a perfect cylinder

$$= \frac{E}{[3(1-\nu^2)]^{1/2}}\,\frac{t}{R}$$

E = Modulus of elasticity = 28.3×10^6 psi (assume)

ν = Poisson's ratio = 0.3 (assume)

F_y = Yield strength = 30×10^3 psi (assume)

p = Internal pressure

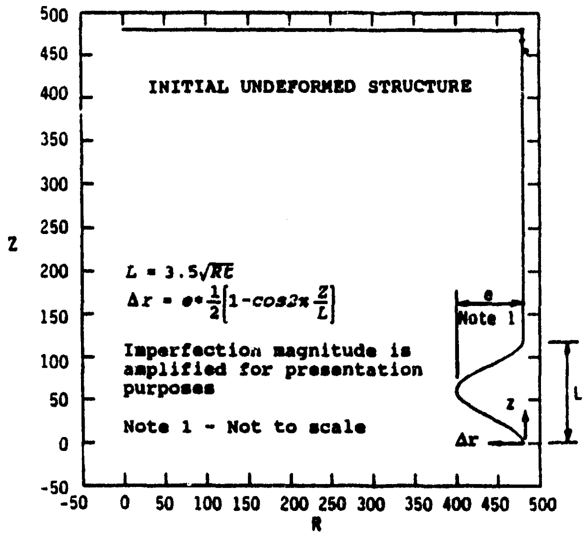

Figure 1: Modeled Imperfection Shape

The major observation is that the shell failure is controlled by elastic buckling until the hoop stress reaches about 25%-50% of the material yield strength. Beyond that the elastic-plastic collapse governs the failure mode. In the literature, the first failure mode (i.e., elastic buckling) is also termed as "diamond buckling." Similarly, the second failure mode (i.e., elastic-plastic collapse) is also called "local bending" or "elephant foot buckling."

Subsequently, in Figures 3 through 7, the results are compared with those obtained by using the formulas provided in the ECCS (1988) and New Zealand (Priestly, 1986) codes and the ASME Code Case N284 (Miller, 1991). It is worth noting that the presentations of the design formulas are different in these codes. The ECCS code formulas incorporate the biaxial effect (meridional and hoop), and require an iterative procedure to incorporate the effect of independent internal pressure. The New Zealand code formulas are decoupled and do not require iteration. The ASME Code Case formula is very simple and independent of the internal pressure. The ECCS code results are obtained with the "additional partial factor of safety" of 1.33 specified in this code, which was introduced to envelope certain test data. This factor of safety gradually reduces to 1.0 for short and thick shells. The other results were calculated without introduction of any intentional conservatisms. The ASME Code Case formulas do not apply for R/t greater than 1000. The following are major observations:

- The BNL results compare well with both the ECCS and New Zealand code results in the elastic region.

- In the inelastic region, the BNL buckling results are slightly higher than the New Zealand results, and both these results are larger than the ECCS results. This is apparently because the EECS code capacity was reduced to include certain test data by introducing the factor of safety of 1.33. The BNL and New Zealand results match better with the ECCS results in this region if the factor of safety is assumed to be 1.0 as shown in Figure 8.

- A significant improvement in the buckling capacity in the presence of internal pressure occurs over the ASME Code Case when the hoop stress is in the range of 20%-80% of the yield strength. The difference is more pronounced for thinner shells.

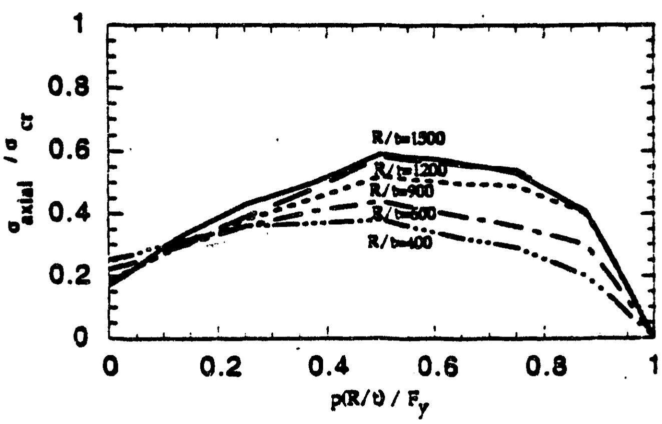

Figure 2: Influence of Pressure on Axial Compressive Strength

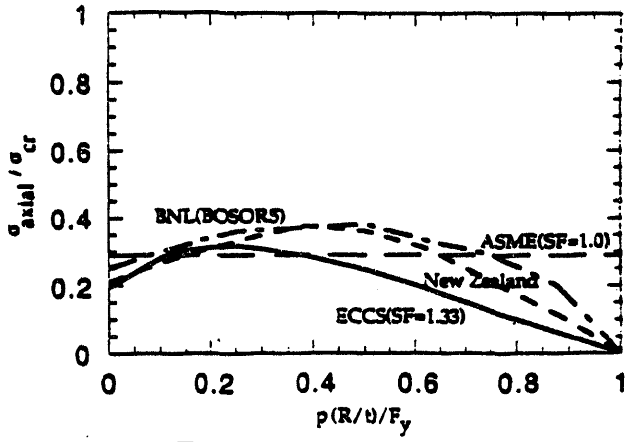

Figure 3: Comparisons of Results for R/t = 400

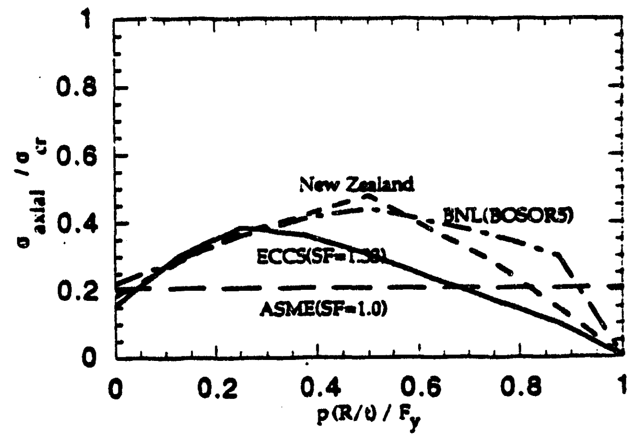

Figure 4: Comparison of Results for R/t = 600

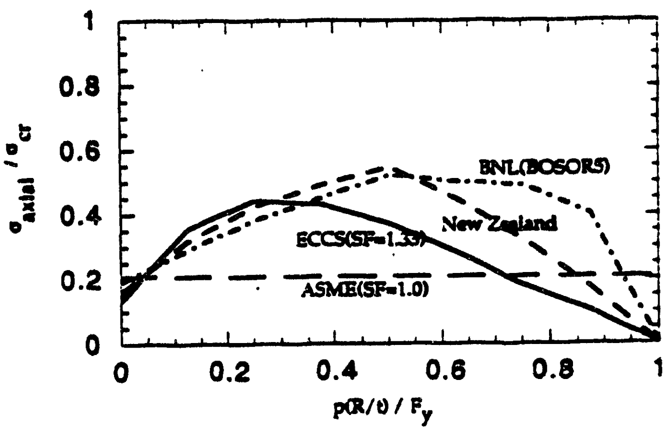

Figure 5: Comparison of Results for R/t = 900

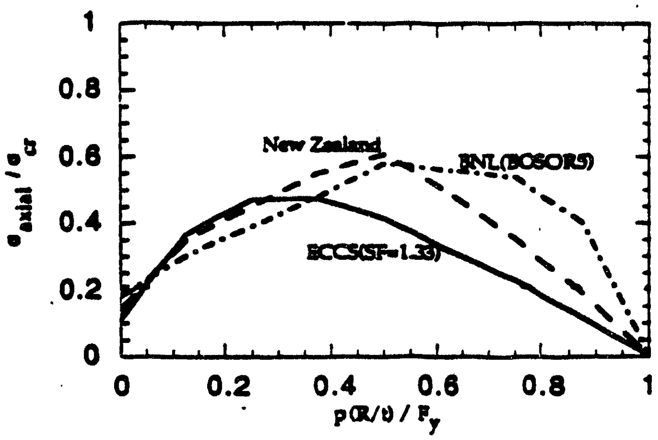

Figure 6: Comparison of Results for R/t = 1200

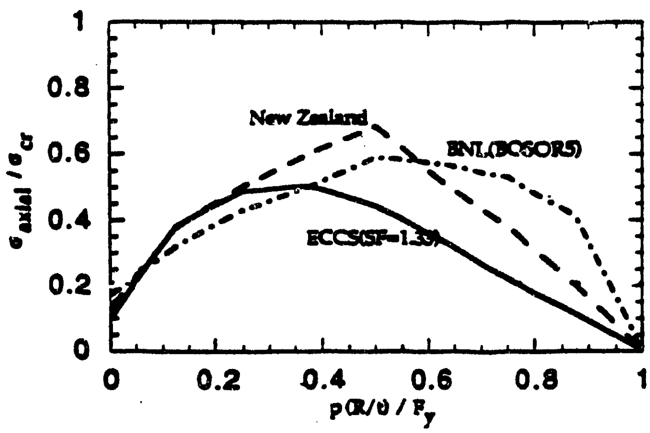

Figure 7: Comparison of Results for R/t = 1500

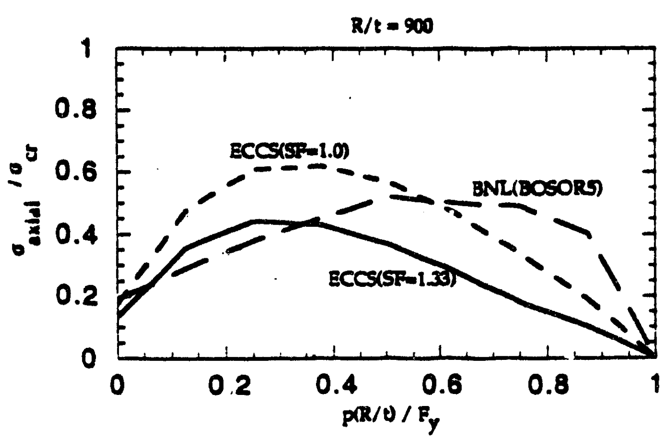

Figure 8: Comparison with ECCS Code Results for SF = 1.0 and 1.33

CONCLUSIONS

The BOSOR5 analysis results confirm the beneficial effect of internal pressure on the buckling capacity. The results also show how the gain becomes gradually reduced as the pressure increases beyond a certain value when the hoop stress starts playing a role. As such, the cylindrical shell should be designed for the minimum possible internal pressure in the elastic range and for the maximum possible internal pressure in the elastic-plastic range.

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REFERENCES

Bandyopadhyay, K., Xu, J., and Shteyngart, S., "Buckling Analysis of Cylindrical Waste Tank Shell," 4th DOE NPH Mitigation Conference, Atlanta, GA, October, 1993.

"Buckling of Thin-Walled Circular Cylinders," NASA SP-8007, National Aeronautics and Space Administration, August 1968.

Bushnell, D., "BOSOR4: Program for Stress, Buckling and Vibration of Complex Shells of Resolution," Lockheed Missiles & Space Co.

Bushnell, D. "BOSOR5: A Computer Program for Buckling of Elastic-Plastic Complex Shells of Revolution including Large Deflection and Creep," Lockheed Missiles & Space Co., December 1974.

ECCS, "Buckling of Steel Shells - European Recommendations," Fourth Edition, 1988.

Miller, C.D., "ASME Code Case N-284, Metal Containment Shell Buckling Design Methods," May 1991.

Priestly, M.J.N., et al., "Seismic Design of Storage Tanks - Recommendations of a Study Group of the New Zealand National Society for Earthquake Engineering," December 1986.

Rotter, J.M., "Buckling of Ground-Supported Cylindrical Steel Bins Under Vertical Compressive Wall Loads," Metal Structures Conferences, Melbourne, Australia, May 23-25, 1985.

Rotter, J.M., and J.G. Teng, "Elastic Stability of Cylindrical Shells with Weld Depressions," ASCE, Journal of Structural Engineering, Vol. 115, No. 5, May 1989.

Saal, H., "Buckling of Long Liquid-Filled Cylindrical Shells," Buckling of Shells, Ed. E. Ramn, Springer-Verlag.

Saal, H., "Buckling of Circular Cylindrical Shells under Combined Axial Compression and Internal Pressure," ECCS Second International Colloquium, Stability of Steel Structures, Liege 13-15, April 1977.

Vandepette, D. and J. Rathe, "Buckling of Circular Cylindrical Shells under Axial Load in the Elastic-Plastic Region," Der Stahlbau, Heft 12, 1980.