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NUMERICAL STUDY OF THE FLOW OF GRANULAR MATERIALS DOWN AN INCLINED PLANE

Project Director : K. R. Rajagopal
Department of Mechanical Engineering
University of Pittsburgh
Pittsburgh, PA 15261

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Student(s) and the degree for which they are registered : Mr. R. Gudhe, (Ph.D)

SUMMARY

In the previous report the governing equations for the flow of granular materials down an inclined plane, modeled by the consitutive theory proposed by Boyle and Massoudi (1990) were derived. These equations are solved numerically subject to the appropriate boundary conditions. The effect of various non-dimensional parameters on the volume fraction and velocity are presented in the form of graphs. In this report we derive the governing equations for the flow of granular materials down an inclined plane using the continuum model proposed by Rajagopal and Massoudi [cf. Goodman and Cowin (1971, 1972), Rajagopal and Massoudi (1990)]. Here, we study the effect of the various forms of the materials parameters (β's) as discussed in Case I, II, and III in the following sections. The numerical solutions for case I and case II were presented in the earlier reports and case III are presented in the form of graphs for various non-dimensional parameters. Also, the solutions from Case I, II, and III are compared to see how the structure of the material parameters affect the velocity, volume fraction and temperature fields.

INTRODUCTION

Granular materials have both the properties of a solid and a fluid, as they can take the shape of the vessel containing them, thereby exhibiting fluid like characteristics, or they can be heaped, thereby behaving like a solid. Also, granular materials can sustain shear stresses in the absence of any deformation, and the critcal stress at which shearing begins depends on the normal stress. Also it is very difficult to characterize bulk solids, which are composed of a variety of materials, *i.e.* mainly due to the fact that small variations in some of the primary properties of the bulk solids such as the size, shape, hardness particle density and surface roughness can result in very different behavior. Furthermore, secondary factors such as the presence or absence of moisture, the severity of prior compaction, the ambient temperature etc., which are not directly associated with the particles, can have significant effect on the behavior of the bulk solids.

Due to their complexity, the modeling of granular materials would require a fusion of the ideas from solid, fluid and soil mechanics. Also granular materials, like non-Newtonian fluids and non-linearly elastic solids exhibit normal-stress differences in simple shear flow. Thus modeling granular materials and slurries is very complex, and has to draw upon experiences from non-linear fluid and solid theories. One approach used in modeling the mechanics of granular materials is the **continuum approach** and the other is the **kinetic theory approach**. Several models have been proposed by various investigators using ideas of kinetic theory [cf. Ogawa et al. (1980), Jenkins and Savage (1983), Lun et al. (1984), Jenkins and Richman (1985), Boyle and Massoudi (1989)], similarly based on the continuum theory approach [cf. Goodman and Cowin (1971, 1972), Cowin (1974a, b), Massoudi and Boyle (1987), Passman et al. (1980), Ahmadi (1982a, b) and Rajagopal and Massoudi (1990)].

GOVERNING EQUATIONS

A constitutive model that predicts the possibilty of both of the normal stress-differences and that is properly frame invariant is given by [cf. Rajagopal and Massoudi (1990)]:

$$\mathbf{T} = \{ \beta_0(\nu) + \beta_1(\nu)\, \nabla\nu \cdot \nabla\nu + \beta_2(\nu)\, tr\mathbf{D} \}\, \mathbf{1} + \beta_4(\nu)\, \nabla\nu \otimes \nabla\nu + \beta_3(\nu)\, \mathbf{D}, \qquad (1)$$

where $\beta_0(\nu)$ is similar to pressure in a compressible fluid and is given by an equation of state, $\beta_2(\nu)$ is like the second coefficient of viscosity in a compressible fluid, $\beta_1(\nu)$ and $\beta_4(\nu)$ are the material parameters connected with the distribution of the granular materials and $\beta_3(\nu)$ is the viscosity of the granular material. The above model allows for normal-stress differences, a feature observed in granular materials. In general, the material properties β_0 through β_4 are functions of the density (*or* volume fraction ν), temperature, and the principal invariants of the stretching tensor **D**, given by

$$\mathbf{D} = \frac{1}{2}[\,(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\,],$$

where **u** is the velocity of the particles. In equation (1), **1** is the identity tensor, ∇ the gradient operator, $\otimes$ indicates the outer (dyadic) product of two vectors, and *tr* designates the trace of a tensor.

Consider the flow of granular materials down an inclined plane modeled by the continuum model proposed by Rajagopal and Massoudi (1990) (cf. Figure 1) due to the action of gravity. The flow of granular materials down an inclined plane has been studied by several authors [cf. Goodman & Cowin (1971), Savage(1979), Hutter, Szidarovszky, and Yakowitz (1986a, b), Johnson and Jackson (1987)]. In this problem, we consider steady one-dimensional flow of incompressible granular materials (i.e., γ = constant) down an inclined plane, where the angle of inclination is α. Here, we assume the flow is a fully developed steady flow. Let us further suppose that the inclined plane is maintained at a constant temperature Θ_w, which is at a higher temperature than the temperature of the surrounding environment Θ_∞, and, as a result, there is transfer of heat. The heat flux vector **q** satisfies Fourier's law with constant thermal conductivity, i.e.,

$$\mathbf{q} = -K\,\nabla\Theta, \tag{2}$$

where Θ is the temperature and K is the thermal conductivity, which in general is a function of volume fraction and Θ. At this juncture it would be appropriate to point out that in theories for granular materials based on a kinetic theory approach, the fluctuations in the velocity field give rise to the notion of granular temperature. The convective heat transport, within the context of such theories, is determined by the fluctuations in the velocity field. It is also the conventional wisdom that this mechanism is important for the heat transfer process. In this approach, we have ignored the fluctuations in the velocity field as the theory does not allow for such velocity fluctuations, and moreover within the context of the continuum theory, the phenomena of heat transfer is incorporated in the energy equation. To include in addition to the energy equation, the notion of granular temperature would be inconsistent with this approach. We feel that this approach is applicable when the packing of the material is reasonably compact and the fluctuation from the mean are not significant. For a fully developed flow, within the context of a continuum theory, wherein the flow is unidirectional as in this case, fluctuations of the velocity normal to the flow direction cannot be incorporated. While this may be a shortcoming of this approach we see that even with the neglect of such fluctuations, heat transfer within the context of the continuum model has a pronounced effect on the nature of the solution [cf. Schlünder (1980, 1982), Wunschmann and Schlünder (1980), Buggisch and Löffelmann (1989)].

For the problem under consideration, the following assumptions are made:

- Steady motion
- Incompressible granular materials, i.e., γ = constant
- Negligible radiant heating, i.e., $r = 0$
- The constitutive equation for the stress tensor is given by equation (1), and density, velocity, and temperature fields are assumed to be of the form

$$\begin{aligned} \nu &= \nu(y) \\ u &= U(y) \\ \Theta &= \Theta(y) \end{aligned} \tag{3}$$

We shall consider three cases. The first is the case that β_1 through β_4 and the thermal conductivity K are assumed to be constants. In the second case, it was assumed that β_1 and β_4 to be constant, β_2 and β_3 have a quadratic variation in volume fraction and the thermal conductivity K to be linear in volume fration. In the third case, we consider purely a mechanical problem and assume β_1 through β_4 have a quadratic variation in volume fraction.

Case I: β_1 Through β_4 and Thermal Conductivity K are Constant

Here, we assume β_1 through β_4 and the thermal conductivity K to be constant, with β_0 given by equation (4).

$$\beta_0 = k\,\nu \tag{4}$$

With the above assumptions, the conservation of mass is identically satisfied and the balance of linear momentum and energy in the non-dimensionalized form reduces to

$$R_1 \frac{d\nu}{d\bar{y}} + R_2 \frac{d\nu}{d\bar{y}} \frac{d^2\nu}{d\bar{y}^2} = \nu \cos\alpha \tag{5}$$

$$R_3 \frac{d^2\bar{U}}{d\bar{y}^2} = -\nu \sin\alpha \tag{6}$$

$$\frac{d^2\bar{\Theta}}{d\bar{y}^2} = -R_4 \left\{ \frac{d\bar{U}}{d\bar{y}} \right\}^2, \tag{7}$$

and the boundary conditions become

$$\bar{U} = 0$$

$$\bar{\Theta} = 1 \qquad at\ \bar{y} = 0\ (on\ the\ inclined\ plane) \tag{8}$$

$$N = \int_0^1 \nu \, d\bar{y} \tag{9}$$

and,

$$\frac{d\bar{U}}{d\bar{y}} = 0$$

$$R_1 \nu + \frac{R_2}{2} \left\{ \frac{d\nu}{d\bar{y}} \right\}^2 = 0 \qquad at\ \bar{y} = 1\ (at\ the\ free\ surface) \tag{10}$$

$$\bar{\Theta} = 0$$

with,

$$\bar{y} = \frac{y}{h}; \qquad \bar{u} = \frac{U}{u_0}; \qquad \bar{\Theta} = \frac{\Theta - \Theta_\infty}{\Theta_w - \Theta_\infty}, \tag{11}$$

where h is a characteristic length and u_0 is a reference velocity. The non-dimensional parameters R_1, R_2, R_3, and R_4 are given by

$$R_1 = \frac{k}{h\gamma g}; \qquad R_2 = \frac{2(\beta_1 + \beta_4)}{h^3 \gamma g}$$

$$R_3 = \frac{\beta_3 u_0}{2 h^2 \gamma g}; \qquad R_4 = \frac{\beta_3 u_0^2}{2K(\Theta_w - \Theta_\infty)} \tag{12}$$

These dimensionless parameters have the following physical interpretations. R_1 could be thought of as the ratio of the pressure force to the gravity force. R_2 is the ratio of volume distribution force to the gravity force. R_3 is the ratio of the viscous force to the gravity force (related to Reynolds number) and R_4 is the product of the Prandtl number and the Eckert number. Since $k < 0$, R_1 can only have negative values, and since $\beta_3 > 0$, R_3 and R_4 are only given positive values.

Numerical Results

The system of equations (5), (6), and (7) with the boundary conditions (8), (9), and (10) and subject to the restriction $k < 0$ are solved numerically using a collocation code COLSYS [cf. Ascher et al. (1981)]. The numerical results for the above case was presented in the earlier reports.

Case II: β_3 Varies Quadractically in ν and Thermal Conductivity K Varies Linearly in ν with β_1, β_2 and β_4 Being Constant

Here the viscosity of the granular material and its thermal conductivity is allowed to vary with the volume fraction in a manner that is consistent with the physics of the problem. The equations for the conservation of mass, momentum, and energy are solved numerically and explicitly demonstrate the possibility of non-unique solutions, corresponding to the same flow rate of the granular materials. Further, it is assumed that β_1, β_2 and β_4 are constant. However, the viscosity β_3 is assumed to be of the form [cf. Johnson et al. (1991a, b)]:

$$\beta_3 = \hat{\beta}_3 (\nu + \nu^2), \qquad \textit{where, } \hat{\beta}_3 \textit{ is a constant} \tag{13}$$

The numerical simulations of Walton and Braun (1986) suggest a quadratic variation in volume fraction. However, their analysis allows for the viscosity to vary with the shear rate, a feature that is not present in this work. Even so, at fixed shear rate, their simulation suggests a quadratic variation in the volume fraction. For K, it is assumed as [cf. Bashir and Goddard (1990) and Batchelor and O'Brien (1977)]:

$$K = K_m (1 + 3\,\zeta\,\nu), \tag{14}$$

where,

$$\zeta = \frac{(\psi_1 - 1)}{(\psi_1 + 2)} \tag{15}$$

Here, ψ_1 = ratio of conductivity of the particle to that of the matrix, and K_m = conductivity of the matrix.

With the above assumptions and the flow field given by equation (3), the conservation of mass is identically satisfied and the balance of linear momentum and energy in the non-dimensional form reduces to

$$R_1 \frac{d\nu}{d\bar{y}} + R_2 \frac{d\nu}{d\bar{y}} \frac{d^2\nu}{d\bar{y}^2} = \nu \cos\alpha \tag{16}$$

$$A_3\, \nu\, (1+\nu) \frac{d^2\bar{U}}{d\bar{y}^2} + A_3\, (1+2\,\nu) \frac{d\nu}{d\bar{y}} \frac{d\bar{U}}{d\bar{y}} = -\,\nu \sin\alpha \tag{17}$$

$$(1+3\,\zeta\,\nu) \frac{d^2\bar{\Theta}}{d\bar{y}^2} + 3\,\zeta \frac{d\nu}{d\bar{y}} \frac{d\bar{\Theta}}{d\bar{y}} = -\,A_4\, \nu\, (1+\nu) \left\{ \frac{d\bar{U}}{d\bar{y}} \right\}^2, \tag{18}$$

and the boundary conditions become

$$\bar{U} = 0 \qquad at\ \bar{y} = 0\ (on\ the\ inclined\ plane) \tag{19}$$

$$\bar{\Theta} = 1 \qquad at\ \bar{y} = 0\ (on\ the\ inclined\ plane) \tag{20}$$

$$N = \int_0^1 \nu\, d\bar{y} \tag{21}$$

and,

$$\frac{d\bar{U}}{d\bar{y}} = 0$$

$$R_1\, \nu + \frac{R_2}{2} \left\{ \frac{d\nu}{d\bar{y}} \right\}^2 = 0 \qquad at\ \bar{y} = 1\ (at\ the\ free\ surface) \tag{22}$$

$$\bar{\Theta} = 0$$

and,

$$\frac{d\bar{\Theta}}{d\bar{y}} = -\frac{A_5}{(1 + 3\,\zeta\,\nu)} \qquad at\ \bar{y} = 1\ (at\ the\ free\ surface) \qquad (23)$$

The non-dimensional parameters A_3, A_4, and A_5 are given by

$$A_3 = \frac{\hat{\beta}_3\, u_0}{2\, h^2 \gamma\, g}; \qquad A_4 = \frac{\hat{\beta}_3\, u_0^2}{2\, K_m\, (\Theta_w - \Theta_\infty)} \qquad A_5 = \frac{q_f\, h}{K_m\, (\Theta_w - \Theta_\infty)} \qquad (24)$$

These dimensionless parameters have the following physical interpretations: A_3 is the ratio of the viscous forces to gravity and A_4 is the product of the Prandtl number and the Eckert number.

Numerical Results

The system of equations (16), (17), and (18) with the boundary conditions (19), (20), (21), $(22)_{1\text{-}2}$ and (22_3) or (23) and subject to the restriction $k < 0$ are solved numerically using a collocation code COLSYS [cf. Ascher et al. (1981)]. The numerical results for the above governing equations was presented in the earlier reports.

Case III: β_1 Through β_4 Vary Quadratically in ν

It is assumed that β_0 and β_3 are given by equations (4) and (13) with β_1, β_2 and β_4 to be quadratic in volume fraction are given by

$$\beta_1 = \hat{\beta}_1\,(1 + \nu + \nu^2), \qquad where,\ \hat{\beta}_1\ is\ a\ constant \qquad (25)$$

$$\beta_2 = \hat{\beta}_2\,(1 + \nu + \nu^2), \qquad where,\ \hat{\beta}_2\ is\ a\ constant$$

$$\beta_4 = \hat{\beta}_4\,(1 + \nu + \nu^2), \qquad where,\ \hat{\beta}_4\ is\ a\ constant \qquad (26)$$

In this case, we consider a purely mechanical problem. With the above assumptions and the flow field given by $(3)_{1,2}$, the conservation of mass is identically satisfied and the balance of linear momentum reduces to

$$k\frac{dv}{dy} + 2\,(\hat{\beta}_1 + \hat{\beta}_4)\,(1 + v + v^2)\,\frac{dv}{dy}\frac{d^2v}{dy^2} + (\hat{\beta}_1 + \hat{\beta}_4)\,(1 + 2\,v)\left\{\frac{dv}{dy}\right\}^3 = \gamma\, g\, v\, cos\alpha, \tag{27}$$

$$\hat{\beta}_3\,(v + v^2)\,\frac{d^2U}{dy^2} + \hat{\beta}_3\,(1{+}2\,v)\,\frac{dv}{dy}\frac{dU}{dy} = -2\,\gamma\, g\, v\, sin\alpha, \tag{28}$$

The equations (27) and (28) are to be solved subject to the following boundary conditions

$$U = 0 \qquad \textit{at } y = 0 \textit{ (on the inclined plane)} \tag{29}$$

and,

$$\frac{dU}{dy} = 0$$

$$k\,v + (\hat{\beta}_1 + \hat{\beta}_4)\left\{\frac{dv}{dy}\right\}^2 = 0 \qquad \textit{at } y = h \textit{ (at the free surface)} \tag{30}$$

and the constraint that

$$Q_0 = \int_0^h v\, dy, \qquad Q_0 \textit{ being given.} \tag{31}$$

Also, notice that equations $(30)_{1,2}$ are the stress free conditions, equation $(29)_1$ indicates the no-slip condition (rough wall) assumption. Now, the system of equations (27) and (28) subject to the boundary conditions (29), (30), and (31) are non-dimensionalized using equation (11). Now, the above system of equations reduces to

$$R_1\frac{dv}{d\bar{y}} + A_2\,(1 + v + v^2)\,\frac{dv}{d\bar{y}}\frac{d^2v}{d\bar{y}^2} + \frac{A_2}{2}\,(1 + 2\,v)\left\{\frac{dv}{d\bar{y}}\right\}^3 = v\, cos\alpha \tag{32}$$

$$A_3\, v\,(1{+}v)\,\frac{d^2\bar{U}}{d\bar{y}^2} + A_3\,(1{+}2\,v)\,\frac{dv}{d\bar{y}}\frac{d\bar{U}}{d\bar{y}} = -\,v\, sin\alpha \tag{33}$$

and the boundary conditions become

$$\bar{U} = 0 \qquad \textit{at } \bar{y} = 0 \textit{ (on the inclined plane)} \tag{34}$$

$$N = \int_0^1 v\, d\bar{y} \tag{35}$$

and,

$$\frac{d\bar{U}}{d\bar{y}} = 0$$

$$R_1 \nu + \frac{A_2}{2}\{\frac{d\nu}{d\bar{y}}\}^2 = 0 \qquad at\ \bar{y} = 1\ (at\ the\ free\ surface) \qquad (36)$$

The non-dimensional parameter A_2 is given by

$$A_2 = \frac{2\,(\hat{\beta}_1 + \hat{\beta}_4)}{h^3\,\gamma\,g} \qquad (37)$$

The dimensionless parameter has the following physical interpretations, A_2 is the ratio of forces developed in the material due to the distribution of the voids to the force of gravity.

Numerical Results

The system of equations (32) and (33) with the boundary conditions (34), (35) and (36) and subject to the restriction $k < 0$ are solved numerically using a collocation code COLSYS [cf. Ascher et al. (1981)]. It follows from Rajagopal and Massoudi (1990) that R_1 must always be less than zero for the solution to exist and all the other non-dimensional parameters, *i.e.* A_2, and A_3 must be greater than zero. A parametric study of the equations is carried out to see how the various non-dimensional paramters affect the volume fraction and velocity profiles.

The manner in which the volume fraction and velocity profiles change with R_1 is shown in Figures 2 and 3. respectively. Notice, that the volume fraction profile decreases from the surface of the plane to the free surface, which is to be expected. Increasing the magnitude of R_1, with the other constants being held fixed, results in a decrease of velocity. Increasing values of A_2 results in a decrease of volume fraction and increase in velocity (cf. Figures 4 and 5). Figure 6 shows the effect of A_3 on velocity profile.

Comparision of Case I, II and III Results

The volume fraction,velocity and temperature profiles are compared for case I, II and III. In Figure 7, the volume fraction profiles are shown for case I and III for the same values of non-dimensional paramters and for case II it is same as Case I. The velocity profiles are shown for Case I, II and III in Figure 8. Notice, that the velocity depends upon the form assumed for β_1 through β_4. Finally, in Figure 9 the temperature profiles are shown for case I and II. We notice that significant changes in the temperature profile can be effected depending upon the form assumed for the thermal conductivity of the particles.

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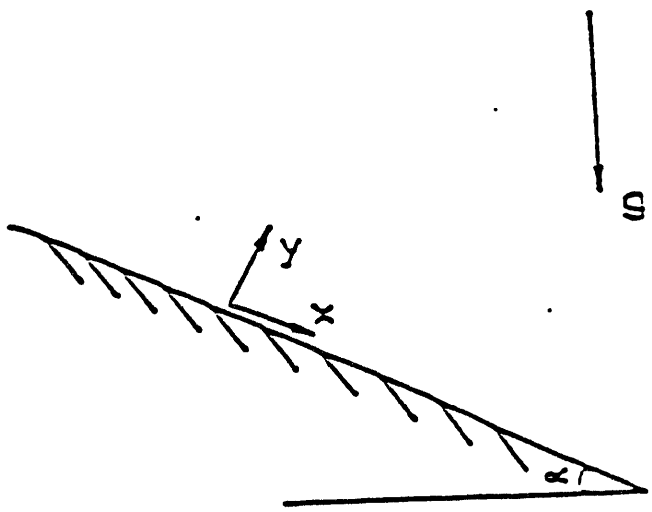

Figure 1. Flow Down An Inclined Plane

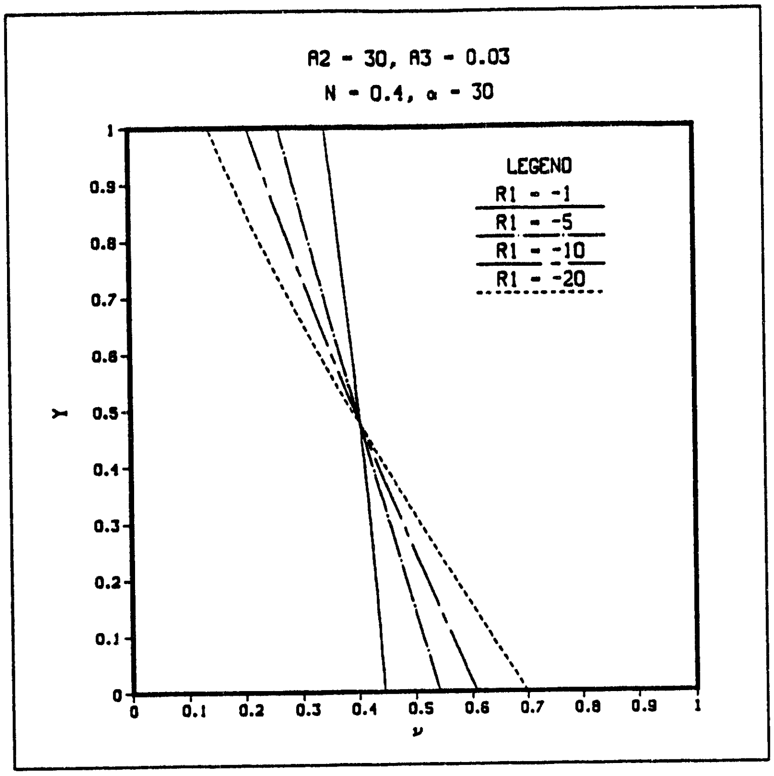

Figure 2. Effect of R_1 on the Volume Fraction

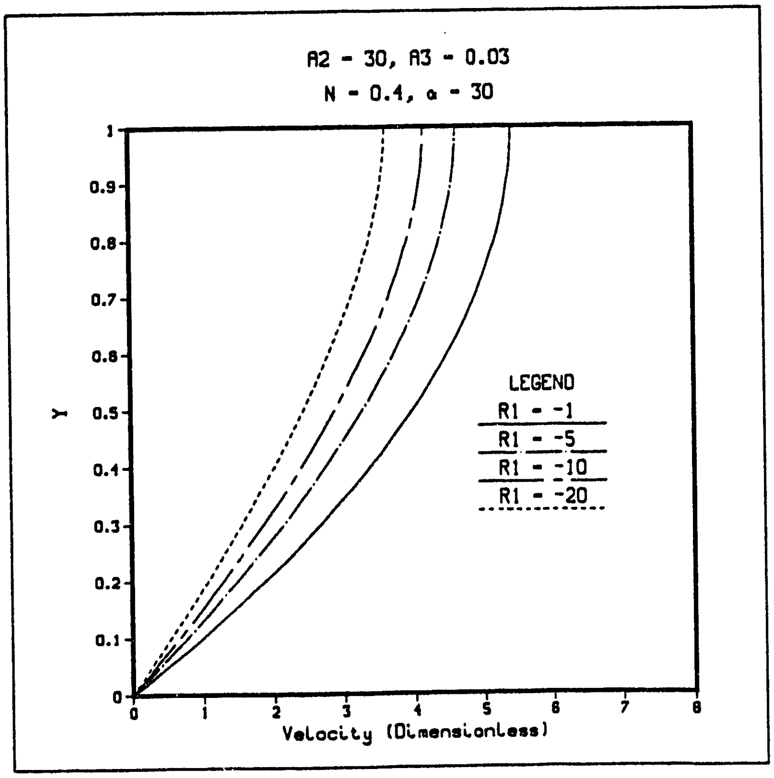

Figure 3. Effect of R_1 on the Velocity Profile

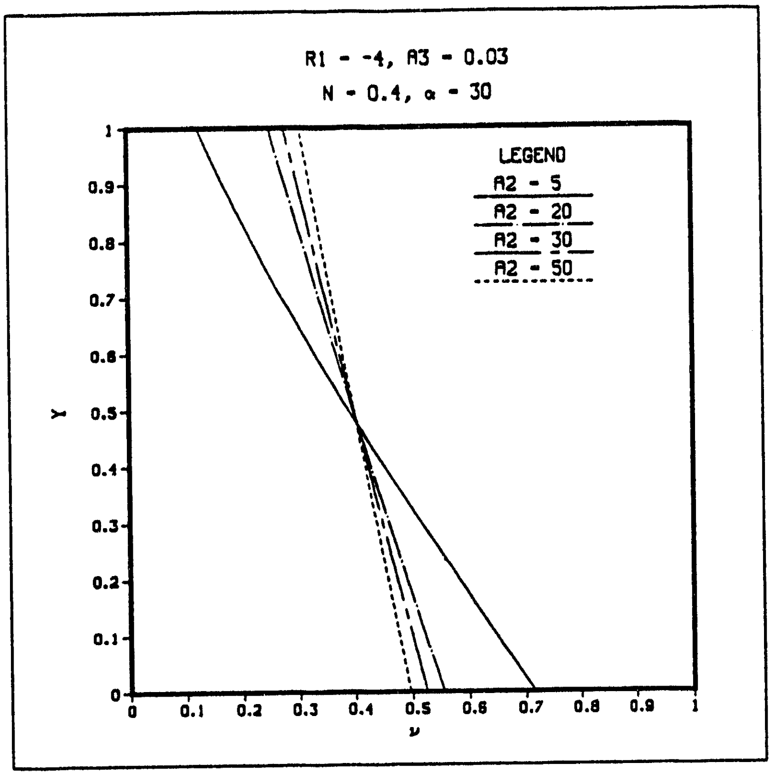

Figure 4. Effect of A_2 on the Volume Fraction

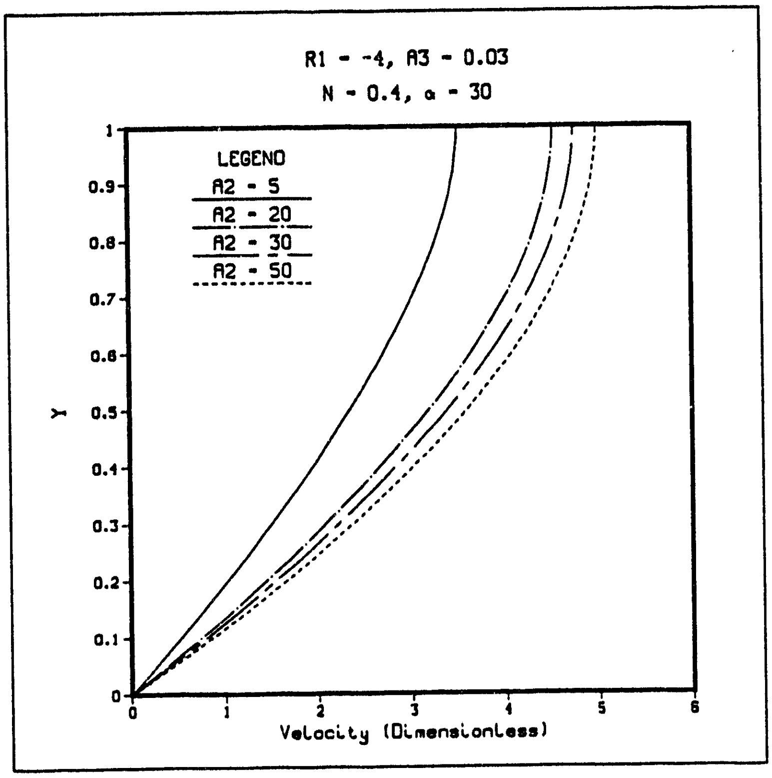

Figure 5. Effect of A_2 on the Velocity Profile

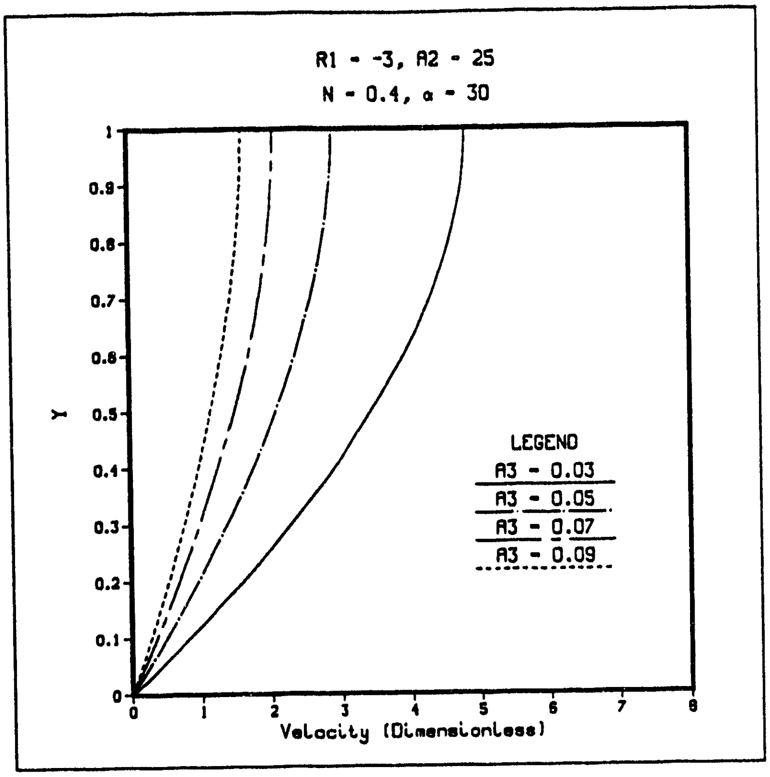

Figure 6. Effect of A_3 on the Velocity Profile

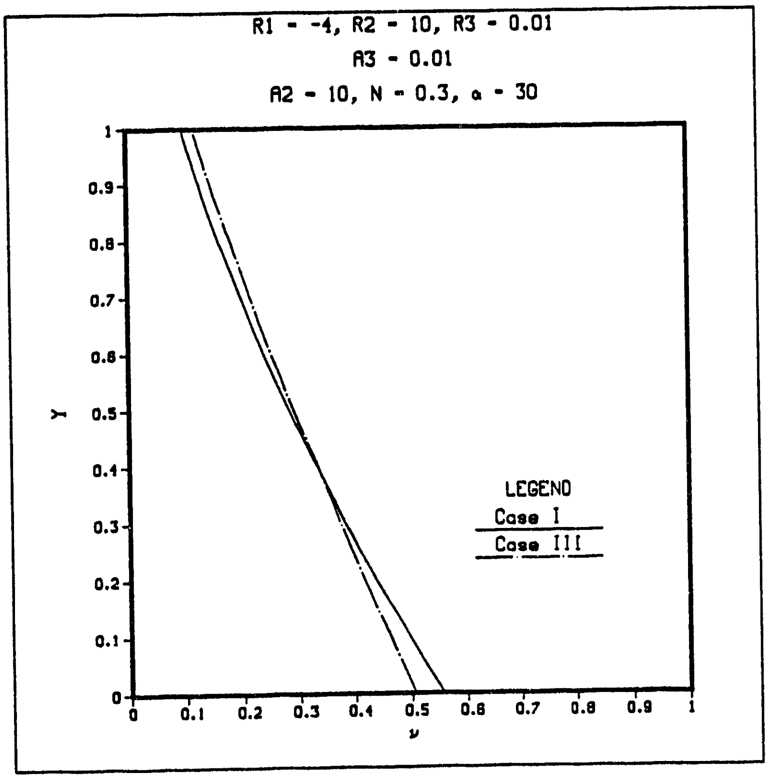

Figure 7. Comparison of Volume fraction Profiles

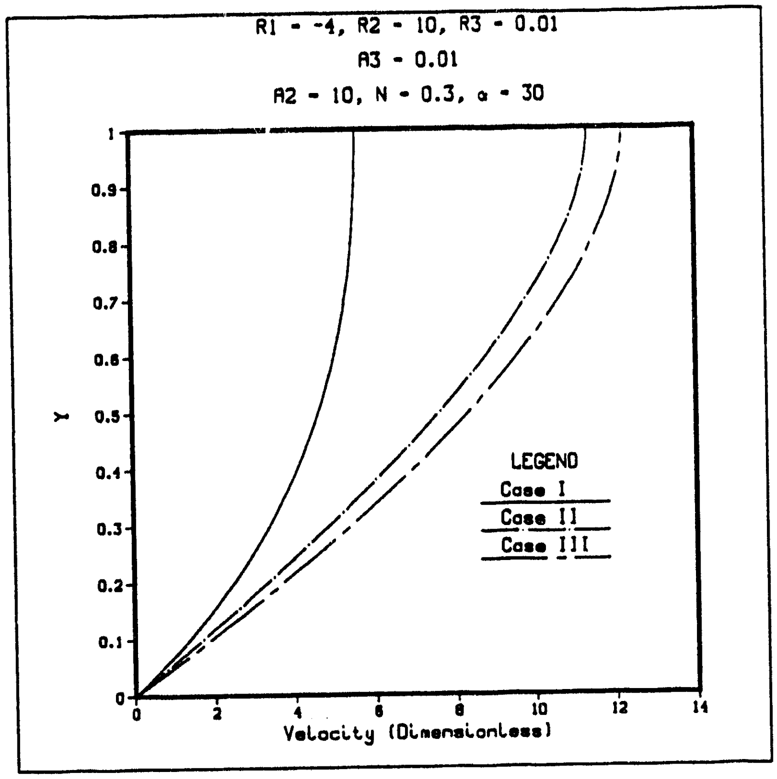

Figure 8. Comparison of Velocity Profiles

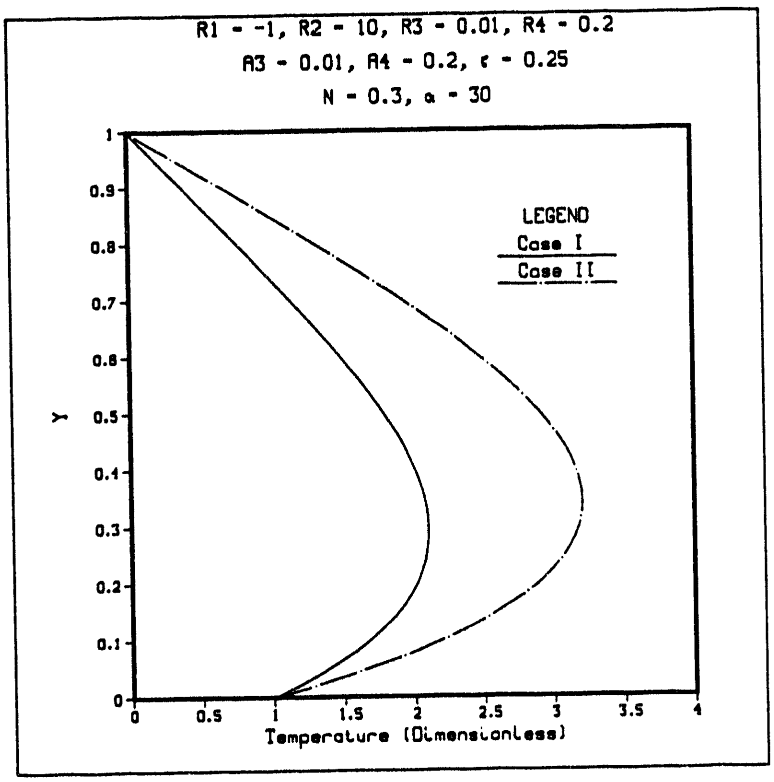

Figure 9. Comparison of Temperature Profiles

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