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A NEW TECHNIQUE FOR DYNAMIC LOAD DISTRIBUTION WHEN TWO MANIPULATORS MUTUALLY LIFT A RIGID OBJECT PART 1: THE PROPOSED TECHNIQUE*

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A NEW TECHNIQUE FOR DYNAMIC LOAD DISTRIBUTION WHEN TWO MANIPULATORS MUTUALLY LIFT A RIGID OBJECT PART 1: THE PROPOSED TECHNIQUE [1]

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Abstract

A general framework for solving the dynamic load distribution when two manipulators hold a rigid object is proposed. The underspecified problem of solving for the contact forces and torques based on the object's equations of motion is transformed into a well specified problem. This is accomplished by augmenting the object's equations of motion with additional equations which relate a new vector variable quantifying the internal contact force and torque degrees of freedom (DOF) as a linear function of the contact forces and torques. The resulting augmented system yields a well specified solution for the contact forces and torques in which they are separated into their motion inducing and internal components. A particular solution is suggested which enables the designer to conveniently specify what portion of the payload's mass each manipulator is to bear. It is also shown that the results of the previous work [1] are just a special case of the general load distribution framework described here.

INTRODUCTION

When two serial link manipulators having N_1 and N_2 joints, respectively, hold and transport a rigid body object in a three dimensional workspace, there arises a problem in dynamically distributing the payload's mass between the manipulators. This is due to the fact that the dynamics of the rigid payload can be described by six second order differential equations of motion (e.g., Newton's and Euler's equations) which are explicit functions of the twelve components of contact force and torque imparted to the object by the manipulators. Assuming that the motion of the center of mass of the object has been specified, it is easy to see that the problem of computing the contact forces[2] based on the dynamics of the object is underspecified. Indeed, there exists infinitely many solutions for the contact forces.

Approaches for distributing the load to determine the contact forces have typically posed it as an optimization problem and have suggested various performance criteria. Among these include the pseudo-inverse method [2] which yields a minimum Euclidean norm particular solution for the contact forces which induce object motion. The addition of a homogeneous solution for the contact forces (i.e., the internal contact forces which cause stress and torsion in the object but do not contribute to its motion) as done in [2] so as to minimize

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[2]contact forces implies both contact forces and torques hereinafter

a secondary criteria via gradient projection, sacrifices the minimum norm solution. Using this approach the internal contact forces become implicit variables and their calculation involves no servoing. Other optimization algorithms and criteria for minimizing the magnitudes of the motion inducing contact forces are discussed in [3]. The internal contact forces, however, are not addressed in [3]. The load distribution approach in this paper avoids optimization techniques such as the pseudo-inverse method and extends the results of a previous paper [1]. The internal forces are explicit variables to be controlled to track reference trajectories.

In [1], a rigid body dynamical model was derived in the joint space for the aforementioned configuration. It was then transformed to separate it into two sets of equations. One set of equations characterized the motion of the closed chain and contained no forces of contact. The other set was used to calculate an independent subset of the contact forces, specifically those between manipulator two and the object. A composite control architecture was suggested which led to an explicit decoupling of the force- and position-controlled degrees of freedom (DOF).

The underspecified dynamic load distribution problem and the internal contact forces which arise in this closed chain configuration were not addressed in [1]. Moreover, the force-controlled DOF were quantified by the contact forces imparted by manipulator two to the object. The contact forces between manipulator one and the object were implicit variables in the model and thus inaccessible for control purposes. No motivation for selecting the force controlled variables in this way was provided in [1].

This paper seeks to resolve these problems by introducing a general procedure for solving the dynamic load distribution when two manipulators lift and transport a rigid body object. The load distribution procedure is incorporated into the modeling framework [1] and a solution for the contact forces is suggested which permits the designer to conveniently specify what amount of the payload's mass each manipulator is to assume. The problem of controlling the internal contact forces to track reference trajectories is addressed in Part 2.

MANIPULATOR AND OBJECT DYNAMICS

This section presents the equations of motion of the individual manipulators and the payload. The composite dynamics of the manipulators are given by:

$$\begin{bmatrix} {}^1\tau \\ {}^2\tau \end{bmatrix} = \begin{bmatrix} D_1 & 0_{N_1\times N_2} \\ 0_{N_2\times N_1} & D_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} J_{1w}^T f_{c1} \\ J_{2w}^T f_{c2} \end{bmatrix} \tag{1}$$

where $0_{k\times m}$ denotes a $(k \times m)$ matrix of zeros and superscript T denotes a matrix transpose. The joint positions of manipulator $i(=1,2)$ are represented by vector $q_i = [q_{i1}, q_{i2}, \ldots, q_{iN_i}]^T$ and the joint torques applied to the joint actuators by the vector ${}^i\tau = [{}^i\tau_1, {}^i\tau_2, \ldots, {}^i\tau_{N_i}]^T$. The $(N_i \times N_i)$ symmetric, positive definite inertia matrix is $D_i(q_i)$, and the Coriolis, centripetal, and gravity forces for manipulator i are described by the $(N_i \times 1)$ vector $C_i(q_i, \dot{q}_i)$. The $(6 \times N_i)$ matrix $J_{iw}(q_i)$ is the Jacobian matrix for manipulator i, which is assumed to have full rank six. The (6×1) contact force vector f_{ci} expressed in the stationary world coordinate system (X_W, Y_W, Z_W) is shown in Figure 1. It is comprised of a (3×1) force vector ${}^{iw}f_{N_i,N_i+1}$ and a (3×1) torque vector ${}^{iw}n_{N_i,N_i+1}$ shown in Figure 1:

$$f_{ci} = \left[{}^{iw}f^T_{N_i,N_i+1},\ {}^{iw}n^T_{N_i,N_i+1}\right]^T . \tag{2}$$

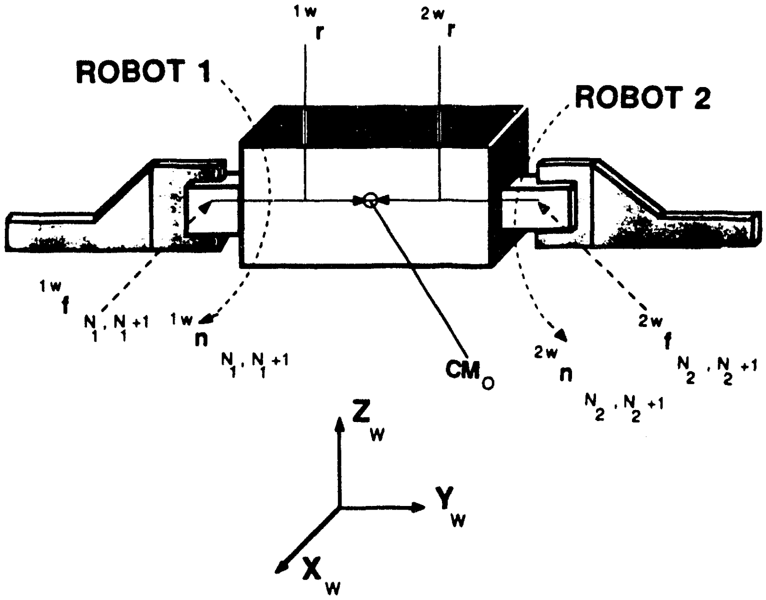

FIG 1. Freebody diagram for the rigid object.

It is convenient to express Newton's and Euler's equations for the rigid object in a compact form:

$$Y = L \begin{bmatrix} f_{c1} \\ f_{c2} \end{bmatrix} . \tag{3}$$

In eq. (3) , Y is a (6×1) vector defined by:

$$Y = \begin{bmatrix} m_c I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & K_c \end{bmatrix} \begin{bmatrix} \dot{v}_c \\ \dot{\omega}_c \end{bmatrix} + \begin{bmatrix} -m_c g \\ \Omega_c K_c \omega_c \end{bmatrix} \tag{4}$$

where all Cartesian vectors are with respect to the world coordinate system. In eq. (4) , m_c is the mass of the rigid object, and K_c is the (3×3) symmetric inertia matrix of the object about its center of mass. The (3×1) vector g represents the gravitational acceleration of the object. The (6×1) vectors $[v_c^T , \omega_c^T]^T$ and $[\dot{v}_c^T , \dot{\omega}_c^T]^T$ denote the Cartesian velocity and acceleration of the center of mass of the load, respectively, with $(v_c , \dot{v}_c)$ being the translational and $(\omega_c , \dot{\omega}_c)$ the rotational components. In eq. (4) , $(\Omega_c K_c \omega_c)$ is a (3×1) vector arising from the cross product expression $(\vec{\omega}_c \times (K_c \omega_c))$ where $\Omega_c(\omega_c)$ is a (3×3) skew symmetric matrix [1].

Using forward kinematic relationships, eq. (4) can be expressed in the joint space [1]:

$$\begin{aligned} Y &= \Lambda \left\{ \left(L_i^T\right)^{-1} \left(J_{iw}\, \ddot{q}_i + \dot{J}_{iw}\, \dot{q}_i\right) + \left(\dot{L}_i^T\right)^{-1} J_{iw}\, \dot{q}_i \right\} \\ &+ \begin{bmatrix} -m_c g \\ \Omega_c K_c \begin{bmatrix} 0_{3\times3}, & I_{3\times3} \end{bmatrix} J_{iw}\, \dot{q}_i \end{bmatrix} \end{aligned} \tag{5}$$

where Λ is the coefficient matrix of $[\dot{v}_c^T, \dot{\omega}_c^T]^T$ in eq. (4) .

The (6×12) matrix $L(q_1, q_2)$ in eq. (3) is an explicit function of the (6×6) contact force transmission matrices $L_1(q_1)$ and $L_2(q_2)$:

$$L = \begin{bmatrix} L_1, & L_2 \end{bmatrix} \tag{6}$$

where matrix $L_i (i = 1, 2)$ is defined by [1]:

$$L_i = \begin{bmatrix} I_{3\times3} & 0_{3\times3} \\ \begin{bmatrix} 0 & {}^{iw}r_z & -{}^{iw}r_y \\ -{}^{iw}r_z & 0 & {}^{iw}r_x \\ {}^{iw}r_y & -{}^{iw}r_x & 0 \end{bmatrix} & I_{3\times3} \end{bmatrix} \tag{7}$$

where $I_{k\times k}$ denotes a $(k \times k)$ identity matrix and ${}^{iw}r = [{}^{iw}r_x, {}^{iw}r_y, {}^{iw}r_z]^T$ is a vector emanating from the point where force ${}^{iw}f_{N_i, N_i+1}$ acts on the object whose head coincides with point CM_o, the center of mass of the rigid object (see Figure 1). L_i is positive definite and whose determinant is equal to one.

In [1], six rigid body kinematic constraints imposed on the joint velocities were derived as functions of the Jacobian and force transmission matrices:

$$\left[(L_1^T)^{-1} J_{1w}, -(L_2^T)^{-1} J_{2w}\right] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = 0_{6\times1} . \tag{8}$$

The six constraints result in the number of degrees of freedom in the closed chain system reducing from $(N_1 + N_2)$ to $(N_1 + N_2 - 6)$.

A GENERAL FRAMEWORK FOR LOAD DISTRIBUTION

There are infinitely many solutions for the contact forces based on eq. (3) . The approach proposed here for solving for $[f_{c1}^T, f_{c2}^T]^T$ is based on a methodology developed in [4, 5] for expressing a set of n generalized velocities whose values are restricted by $k (< n)$ bilateral constraints as a linear function of a new set of $(n - k)$ independent pseudovelocities. It is also inspired in part by the authors' recent work which used the method [4, 5] to develop position and force control laws for a nonholonomic, omnidirectional platform [6] and to resolve the kinematic redundancy of a serial link manipulator [7].

To solve the dynamic load distribution problem, a new vector variable $\epsilon = [\epsilon_1, \epsilon_2, \ldots, \epsilon_6]^T$ is introduced. The number of components of ϵ is equal to the dimension of the null space of matrix L and reflects the fact that the number of position controlled DOF lost in the closed chain is equal to the number of DOF gained for controlling the internal contact forces [8]. ϵ is defined by:

$$\epsilon = M \begin{bmatrix} f_{c1} \\ f_{c2} \end{bmatrix} . \tag{9}$$

The (6×12) matrix $M(q)$ in eq. (9) is selected such that the (12×12) composite matrix $S(q)$, defined by:

$$S = \begin{bmatrix} L \\ M \end{bmatrix} \tag{10}$$

is nonsingular. The symbolic determinant of S is given by:

$$|S| = \left|M_2 - M_1 L_1^{-1} L_2\right| \tag{11}$$

where eq. (6) has been used and where $M_1(q)$ and $M_2(q)$ are (6×6) submatrices of M:

$$M = \begin{bmatrix} M_1, & M_2 \end{bmatrix}. \tag{12}$$

It is convenient to partition the inverse of S into two matrices:

$$S^{-1} = \begin{bmatrix} \Phi, & \Psi \end{bmatrix} \tag{13}$$

where $\Phi(q)$ and $\Psi(q)$ are (12×6) matrices. Eqs. (10) and (13) imply that $L\,\Phi = I_{6\times6}$, $L\,\Psi = 0_{6\times6}$, $M\,\Phi = 0_{6\times6}$, $M\,\Psi = I_{6\times6}$, and $\Phi\,L + \Psi\,M = I_{12\times12}$.

Eqs. (3) and (9) can be solved for the contact forces:

$$\begin{bmatrix} f_{c1} \\ f_{c2} \end{bmatrix} = \Phi\,Y + \Psi\,\epsilon \tag{14}$$

in which eq. (13) has been invoked. The first term $\{\Phi\,Y\}$ on the right of eq. (14) is a particular solution to eq. (4) and are the components of $[f_{c1}^T, f_{c2}^T]^T$ which cause the object to physically move. The second term $\{\Psi\,\epsilon\}$ is the homogeneous solution and are the internal components which cause stress and torsion in the object but do not contribute to its motion.

The symbolic solution for the contact forces given by eq. (14) is significant because it indicates that the designer can specify the distribution of the payload's mass between the two manipulators by the choice of M. To demonstrate this concept using examples, it is convenient to determine a symbolic solution for matrices Φ and Ψ in terms of matrices $\{L_1, L_2, M_1, M_2\}$ using the method of inverse by partitioning [9]:

$$\Phi = \begin{bmatrix} I_{6\times6} + L_1^{-1}\,L_2\,\Delta\,M_1 \\ -\Delta\,M_1 \end{bmatrix} L_1^{-1}, \tag{15}$$

$$\Psi = \begin{bmatrix} -L_1^{-1}\,L_2 \\ I_{6\times6} \end{bmatrix} \Delta \tag{16}$$

where the (6×6) matrix Δ is defined by:

$$\Delta = \left(M_2 - M_1\,L_1^{-1}\,L_2\right)^{-1}. \tag{17}$$

Example 1. In this example it is shown that the result of [1] can be obtained by an application of the general framework provided here. In the previous work, the closed chain dynamics in the joint space were obtained by the following procedure: (i) solve eq. (3) for f_{c1}; (ii) substitute for f_{c1} in eq. (1) using its solution obtained in step i; and (iii) eliminate vector Y in the resulting equations by applying eq. (5) with $i = 1$.

Motivated further by a desire to provide physical insight into the modeling procedure in [1], suppose matrix M is selected as:

$$M = \begin{bmatrix} 0_{6\times6}, & I_{6\times6} \end{bmatrix}. \tag{18}$$

Substituting eq. (18) into eqs. (15) and (16) gives:

$$\begin{bmatrix} \Phi, & \Psi \end{bmatrix} = \begin{bmatrix} L_1^{-1} & -L_1^{-1}\,L_2 \\ 0_{6\times6} & I_{6\times6} \end{bmatrix}. \tag{19}$$

Substituting eq. (19) into eq. (14) and inserting the result into eq. (1) reveals that we have obtained the model of [1] with $\epsilon = f_{c2}$. The procedure in [1] unknowingly distributed the load such that the contact forces imparted by manipulator 2 are purely internal thus manipulator 1 alone supports the entire mass of the object. That is to say, only manipulator one induces the payload to physically move.

THE PROPOSED CHOICE FOR M

Suppose M is chosen to be:

$$M = \begin{bmatrix} -c_2 L_1, & c_1 L_2 \end{bmatrix} \tag{20}$$

where c_1 and c_2 are scalars whose values are restricted as follows:

$$c_1 + c_2 = 1, \quad 0 \leq c_i \leq 1, \quad i = 1,2. \tag{21}$$

Substituting eq. (20) into eqs. (15) and (16) yields the symbolic solution for $[\Phi, \ \Psi]$:

$$\begin{bmatrix} \Phi, & \Psi \end{bmatrix} = \begin{bmatrix} c_1 L_1^{-1} & -L_1^{-1} \\ c_2 L_2^{-1} & L_2^{-1} \end{bmatrix}. \tag{22}$$

By simply choosing values for $\{c_1, c_2\}$ such that eq. (21) is satisfied, the designer can specify how much of the load each manipulator is to support. The designer can base the selection of $\{c_1, c_2\}$ on the load carrying capacities of the individual manipulators.

Example 2. In this example the values of $\{c_1, c_2\}$ are determined such that the payload is equally distributed between the manipulators. Eq. (14) yields the solution for the motion and internal components of the contact forces acting at the contact points between the manipulators and object. However, it is insightful to determine the equivalent motion and internal forces acting at the center of mass of the object. (By equivalent we mean a force that has the same external effect on the object.) This is accomplished by premultiplying the upper and lower six rows of eq. (14) by matrices L_1 and L_2 respectively:

$$\begin{bmatrix} L_1 f_{c1} \\ L_2 f_{c2} \end{bmatrix} = \begin{bmatrix} c_1 I_{6\times6} \\ c_2 I_{6\times6} \end{bmatrix} Y + \begin{bmatrix} -I_{6\times6} \\ I_{6\times6} \end{bmatrix} \epsilon \tag{23}$$

where eq. (22) has been invoked. $c_i Y$ is the equivalent motion inducing force acting at the center of mass due to manipulator i, and it is easy to see that selecting $c_1 = c_2 = 1/2$ distributes the payload equally between the manipulators. With this choice, the equivalent motion and internal stress inducing forces in eq. (23) are orthogonal vectors. Interestingly, eq. (23) with $\epsilon = 0_{6\times1}$ is similar to the optimized result suggested in [3] for an equal distribution of the load. It should be noted, however, that the earlier work [3] did not properly account for the force transmission matrices $\{L_1, L_2\}$ which arise in the equations of motion for the rigid object. Furthermore, the internal contact forces were not addressed in [3].

An added benefit of selecting M by eq. (20) is demonstrated by evaluating the determinant of S using eq. (11) :

$$|S| = |c_1 L_2 + c_2 L_2| = 1 \tag{24}$$

since $|L_i| = 1$ [1]. Therefore the algorithm proposed to calculate $[f_{c1}^T, f_{c2}^T]^T$ using eq. (14) is singularity free when M is defined by eq. (20) .

CONCLUSION

A new method for resolving the dynamic load distribution problem which arises when two manipulators hold a rigid object has been proposed. The method is based on the fact that six internal force controlled DOF arise due to the loss of six position controlled DOF in the closed chain configuration. A vector variable ϵ was introduced which parameterizes the internal force controlled DOF. It was defined as a linear function of the contact forces. The equation defining ϵ together with the equations of motion for the object yielded a well specified solution for the contact forces. A choice for the matrix M which relates $[f_{c1}^T, f_{c2}^T]^T$ and ϵ was suggested which enables the designer to conveniently specify how much of the load each manipulator is to bear by selecting two scalars $\{c_1, c_2\}$ such that $c_1 + c_2 = 1$. Additionally, it was demonstrated that the modeling procedure in [1] used to obtain the dynamics of the closed chain in the joint space is just a special case of the general dynamic load distribution method given here.

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