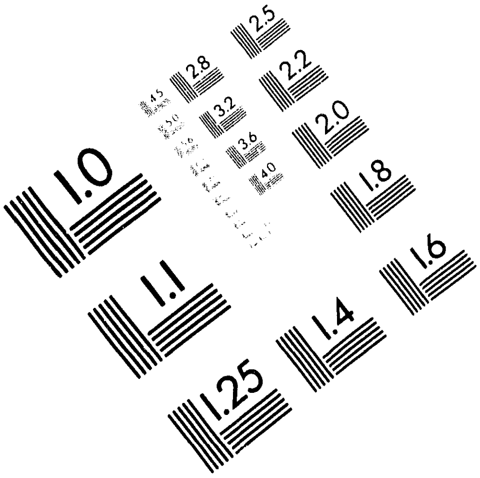

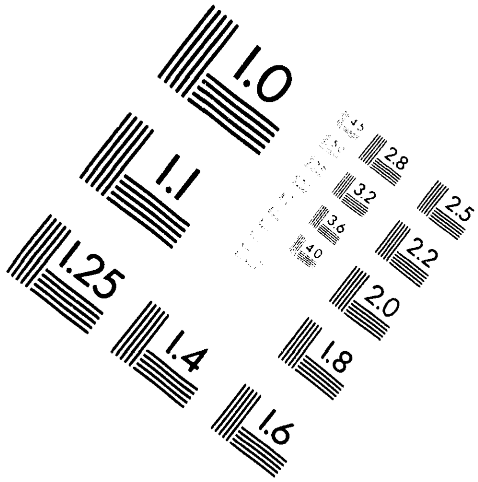

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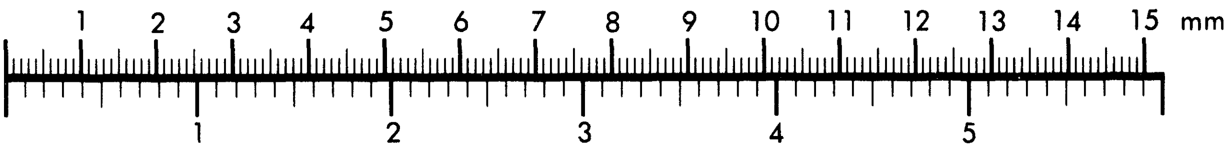

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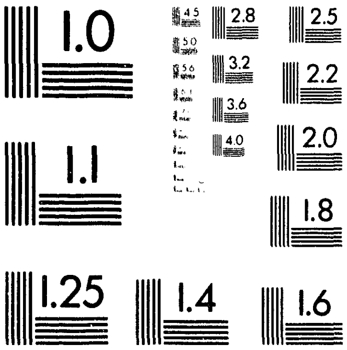

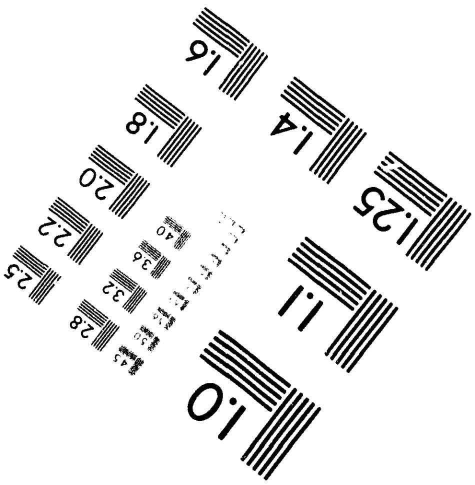

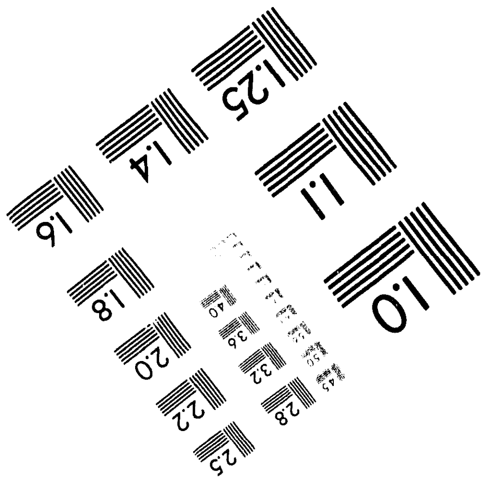

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A NEW TECHNIQUE FOR DYNAMIC LOAD DISTRIBUTION WHEN TWO MANIPULATORS MUTUALLY LIFT A RIGID OBJECT PART 2: DERIVATION OF ENTIRE SYSTEM MODEL AND CONTROL ARCHITECTURE*

Michael A. Unseren
Oak Ridge National Laboratory
Center for Engineering Systems Advanced Research
P.O. Box 2008
Oak Ridge, TN 37831-6364, U.S.A.

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A NEW TECHNIQUE FOR DYNAMIC LOAD DISTRIBUTION WHEN TWO MANIPULATORS MUTUALLY LIFT A RIGID OBJECT PART 2: DERIVATION OF ENTIRE SYSTEM MODEL AND CONTROL ARCHITECTURE [1]

M.A. Unseren
Oak Ridge National Laboratory
Center for Engineering Systems Advanced Research
P.O. Box 2008, Bldg. 6025, MS 6364
Oak Ridge, TN 37831-6364

Abstract

A rigid body model for the entire system which accounts for the load distribution scheme proposed in Part 1 as well as for the dynamics of the manipulators and the kinematic constraints is derived in the joint space. Direct application of the method in [1] to obtain the model results in an inertia matrix for the entire system which is not symmetrical. A technique is presented for expressing the object dynamics in terms of the joint variables of both manipulators which leads to a positive definite and symmetric inertia matrix. The model is then transformed to obtain reduced order equations of motion and a separate set of equations which govern the behavior of the internal contact forces. The control architecture proposed in [1] is applied to the model which results in the explicit decoupling of the position- and internal contact force-controlled degrees of freedom (DOF).

INTRODUCTION

To incorporate the dynamic load distribution scheme proposed in Part 1 [2] into a computer simulation of the multirobot closed chain system or controller design process, it is necessary to derive a rigid body model for the entire system which accounts for the aforementioned scheme as well as for the dynamics of the manipulators and the kinematic constraints. Such a model is derived here in the joint space. Direct application of the method in [1] to obtain the model results in an inertia matrix for the entire system which is not symmetric. To overcome this problem, a technique is suggested to express the object's dynamics in terms of the joint variables of both manipulators. This leads to an inertia matrix which is positive definite and symmetric. Applying the procedure in [1], the model is separated into a reduced order model and and a functional relation for the internal contact forces. The control architecture proposed in [1] is applied to the separated form of the model which results in the explicit decoupling of the position- and internal contact force-controlled DOF.

A RIGID BODY MODEL IN THE JOINT SPACE

A rigid body model is derived in the joint space for the entire closed chain dual-manipulator system in this section. In the ensuing development it is convenient to partition matrix Φ defined in eq. $(13)^{+}$ [2] into two matrices:

[1]Research sponsored by the Engineering Research Program, Office of Basic Energy Sciences, of the U.S. Department of Energy, under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

[2]Superscript + denotes that the referenced equation is in Part 1

$$\Phi = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} \quad (1)$$

where $\Phi_i(q)$ has dimension (6×6). Also, to express the dimension of matrix/column vector quantities compactly, let $N_{12} = N_1 + N_2$.

The closed chain dynamics in the joint space are obtained by first substituting for $[f_{c1}^T, f_{c2}^T]^T$ in eq. $(1)^+$ using eq. $(14)^+$:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} D_1 & 0_{N_1 \times N_2} \\ 0_{N_2 \times N_1} & D_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} c_1 J_{1w}^T L_1^{-1} \\ c_2 J_{2w}^T L_2^{-1} \end{bmatrix} Y + (A^*(q))^T \epsilon \quad (2)$$

where eq. $(22)^+$ has been invoked. The $(N_{12} \times 6)$ matrix $(A^*)^T$ in eq. (2) is termed the internal contact force coefficient matrix. It is defined by:

$$(A^*)^T = \begin{bmatrix} -J_{1w}^T L_1^{-1} \\ J_2^T L_2^{-1} \end{bmatrix} \quad (3)$$

where the superscript $*$ signifies that the matrix/vector quantity is different from the unstarred quantity in eq. (10) of [1]. It is assumed that $(A^*)^T$ has full rank six.

If eq. $(5)^+$ is applied with $i = 1$ to eliminate vector Y from eq. (2) (as was done in [1]), then the coefficient matrix of the vector of joint accelerations $[\ddot{q}_1^T, \ddot{q}_2^T]^T$ has the following structure:

$$D' = \begin{bmatrix} D_1 + c_1 J_{1w}^T L_1^{-1} \Lambda \left(J_{1w}^T L_1^{-1}\right)^T & 0_{N_1 \times N_2} \\ c_2 J_{2w}^T L_2^{-1} \Lambda \left(J_{1w}^T L_1^{-1}\right)^T & D_2 \end{bmatrix}. \quad (4)$$

D' is the inertia matrix for the entire system. It is positive definite and thus nonsingular. But in general D' is not symmetrical. Unfortunately, we need to invert the (6×6) matrix $(A^* (D')^{-1} (A^*)^T)$ which may or may not be nonsingular when D' is not symmetrical.

There is another, and better approach for expressing Y in the joint space which leads to a system inertia matrix which is positive definite and symmetric. Eq. $(5)^+$ permits Y to be expressed as a function of the joint variables of either one of the manipulators but not both. However, by a mathematical observation, the following relation holds true:

$$Y = \sum_{i=1}^{2} d_i Y_i \quad (5)$$

where $Y_i = Y(q_i, \dot{q}_i, \ddot{q}_i)$ for $i = 1, 2$ and where d_1 and d_2 are scalars whose values are constrained:

$$d_1 + d_2 = 1, \quad 0 \le d_i \le 1, \quad i = 1, 2. \quad (6)$$

Substituting for Y in eq. (2) using eq. (5) yields the closed chain dynamics in the joint space:

$$\tau = D^*(q)\,\ddot{q} + C(q, \dot{q}) + H_A^*(q, \dot{q}) + H_B^*(q, \dot{q})\,\dot{q} + (A^*(q))^T \epsilon \quad (7)$$

where $\tau = [{}^1\tau^T, {}^2\tau^T]^T$ and $\ddot{q} = [\ddot{q}_1^T, \ddot{q}_2^T]^T$. The $(N_{12} \times 1)$ vector $C(q, \dot{q}) = [C_1^T, C_2^T]^T$, where C_i is defined in conjunction with eq. $(1)^+$.

The $(N_{12} \times N_{12})$ matrix $D^{*}(q)$ in eq. (7) is defined by:

$$D^{*} = \begin{bmatrix} D_1 + c_1 d_1 J_{1w}^T L_1^{-1} \Lambda \left(J_{1w}^T L_1^{-1}\right)^T & c_1 d_2 J_{1w}^T L_1^{-1} \Lambda \left(J_{2w}^T L_2^{-1}\right)^T \\ c_2 d_1 J_{2w}^T L_2^{-1} \Lambda \left(J_{1w}^T L_1^{-1}\right)^T & D_2 + c_2 d_2 J_{2w}^T L_2^{-1} \Lambda \left(J_{2w}^T L_2^{-1}\right)^T \end{bmatrix} . \quad (8)$$

D^{*} is the new inertia matrix for the entire system. If the scalars $\{d_1, d_2\}$ introduced in eqs. (5) and (6) are selected as:

$$d_i = c_i \quad (9)$$

for $i = 1, 2$, then D^{*} is positive definite and symmetric. In this case the matrix quantity $(A^{*}(D^{*})^{-1}(A^{*})^T)$ is positive definite and therefore nonsingular [3].

The $(N_{12} \times 1)$ vector $H_A^{*}(q, \dot{q})$ and $(N_{12} \times N_{12})$ matrix $H_B^{*}(q, \dot{q})$ in eq. (7) are defined as:

$$H_A^{*} = \begin{bmatrix} c_1 J_{1w}^T L_1^{-1} \\ c_2 J_{2w}^T L_2^{-1} \end{bmatrix} \begin{bmatrix} -m_c g \\ \Omega_c K_c [0_{3\times3}, I_{3\times3}] \begin{bmatrix} c_1 J_{1w}, & c_2 J_{2w} \end{bmatrix} \dot{q} \end{bmatrix}, \quad (10)$$

$$H_B^{*} = \begin{bmatrix} c_1^2 J_{1w}^T L_1^{-1} \Lambda \frac{d}{dt} \left\{ \left(J_{1w}^T L_1^{-1}\right)^T \right\} & c_1 c_2 J_{1w}^T L_1^{-1} \Lambda \frac{d}{dt} \left\{ \left(J_{2w}^T L_2^{-1}\right)^T \right\} \\ c_1 c_2 J_{2w}^T L_2^{-1} \Lambda \frac{d}{dt} \left\{ \left(J_{1w}^T L_1^{-1}\right)^T \right\} & c_2^2 J_{2w}^T L_2^{-1} \Lambda \frac{d}{dt} \left\{ \left(J_{2w}^T L_2^{-1}\right)^T \right\} \end{bmatrix} \quad (11)$$

where eq. (9) has been invoked. It should be mentioned that the quantities $\{D, H_A, H_B\}$ in eq. (10) of [1] are just special cases of eqs. (8),(10), and (11) with $c_1 = 1$, $c_2 = 0$.

Eq. (7) accounts for the dynamics of all components of the closed chain but does not satisfy the rigid body kinematic constraints in eq. $(8)^{+}$. To form a joint space model for the entire closed chain, it is convenient to transform the constraints such that the coefficient matrix of $\dot{q}$ is the transpose of matrix $(A^{*})^T$ in eq. (3). Premultiplying eq. $(8)^{+}$ by $(-I_{6\times6})$ accomplishes this:

$$A^{*}\dot{q} = 0_{6\times1}. \quad (12)$$

It should be mentioned that the form of eqs. (7) and (12) has been obtained for a broad class of constrained mechanical systems in [4, 5] using the method of Lagrange undetermined multipliers [6]. However, the issues of dynamic load distribution and relating ϵ to internal contact forces were not addressed in [4, 5].

Differentiating eq. (12) gives the acceleration constraints:

$$A^{*}\ddot{q} + \dot{A}^{*}\dot{q} = 0_{6\times1}. \quad (13)$$

The $(N_{12} + 6)$ scalar equations comprising eqs. (7) and (13) can be used to accomplish a forward dynamics simulation of the system where the quantities $\{\ddot{q}, \epsilon\}$ are unknowns when the joint torques τ are specified. However, the N_{12} equations comprising eq. (7) exceed the DOF of the entire system ($= N_{12} - 6$). Also, this form of the model is not particularly useful for the controller design process. To alleviate these problems, a reduced order model is derived in the next section.

REDUCED ORDER MODEL AND INTERNAL FORCE DETERMINATION

Linear transformations are applied to the closed chain dynamics in eq. (7) to separate the model into two sets of equations. The sets of equations govern the motion of the closed chain and the behavior of the internal component of the contact forces, respectively. Additionally, linear transformations which express the coupled joint velocities $\{\dot{q}\}$ and accelerations in terms of new independent pseudovariables [7, 8] are applied to eliminate $\{\dot{q}, \ddot{q}\}$ from the equations.

The $(N_{12} \times 1)$ pseudovelocity and pseudoacceleration vectors are defined by:

$$\nu = B\dot{q}, \tag{14}$$

$$\dot{\nu} = B\ddot{q} + \dot{B}\dot{q}. \tag{15}$$

The $((N_{12}-6) \times N_{12})$ matrix $B(q)$ in eqs. (14) and (15) is selected so that the composite $(N_{12} \times N_{12})$ matrix T, defined by:

$$T = \begin{bmatrix} A^* \\ B \end{bmatrix} \tag{16}$$

is nonsingular. It is convenient to partition the inverse of T into two matrices:

$$T^{-1} = [\Pi,\ \Sigma] \tag{17}$$

where $\Pi(q)$ is a $(N_{12} \times 6)$ matrix and $\Sigma(q)$ a $(N_{12} \times (N_{12}-6))$ matrix. Eq. (17) implies that $A^*\Pi = I_{6\times6}$, $A^*\Sigma = 0_{(6\times(N_{12}-6))}$, $B\Pi = 0_{((N_{12}-6)\times6)}$, $B\Sigma = I_{((N_{12}-6)\times(N_{12}-6))}$ and $(\Pi A^* + \Sigma B) = I_{N_{12}\times N_{12}}$.

Eqs. (12) and (14) and eqs. (13) and (15) can be solved for $\{\dot{q}, \ddot{q}\}$:

$$\dot{q} = \Sigma\nu, \tag{18}$$

$$\ddot{q} = \Sigma\dot{\nu} - \left[\Pi\dot{A}^* + \Sigma\dot{B}\right]\Sigma\nu \tag{19}$$

in which eq. (17) has been invoked.

Premultiplying eq. (7) by the nonsingular matrix $[\Sigma, (D^*)^{-1}(A^*)^T]^T$ and utilizing the properties of eq. (17) obtain:

$$\Sigma^T D^* \Sigma\dot{\nu} = \Sigma^T\left\{\tau - C - H_A^* + \left(D^*\left[\Pi\dot{A}^* + \Sigma\dot{B}\right] - H_B^*\right)\Sigma\nu\right\}, \tag{20}$$

$$A^*(D^*)^{-1}(A^*)^T\epsilon = A^*(D^*)^{-1}\{\tau - C - H_A^* - H_B^*\Sigma\nu\} + \dot{A}^*\Sigma\nu \tag{21}$$

in which eqs. (18) and (19) have been applied. The reduced order equations of motion for the entire closed chain system are given by eq. (20) which may be solved for the pseudoaccelerations as a function of the variables (q, ν, τ). The internal contact forces $\{\epsilon\}$ have been eliminated from eq. (20) which in turn are calculated as a function of the variables (q, ν, τ) using eq. (21). The separated form of the model is useful for the controller design. This is discussed next.

CONTROL ARCHITECTURE

The problem considered is to derive a control law for the N_{12} joint torques $\tau = [{}^1\tau^T, {}^2\tau^T]^T$ so that the variables $\{\epsilon, \dot{\nu}\}$ quantifying the internal contact force- and position- controlled DOF can be controlled independently. This can be accomplished by applying the control law proposed in [1] to completely decouple eqs. (20) and (21) . The composite control $\{\tau\}$ is the sum of an $(N_{12} \times 1)$ primary controller τ^p and an $(N_{12} \times 1)$ secondary controller τ^s which are defined by:

$$\tau^p = -\left\{D^*\left[\Pi \dot{A}^* + \Sigma \dot{B}\right] - H_B^*\right\} \Sigma \nu + C + H_A^*, \tag{22}$$

$$\tau^s = (A^*)^T \tau_1^s + D^* \Sigma \tau_2^s . \tag{23}$$

In eq. (23) , τ_1^s and τ_2^s are (6×1) and $((N_{12} - 6) \times 1)$ vectors, respectively, representing control variables to be determined.

The composite control $(\tau = \tau^p + \tau^s)$ defined by eqs. (22) and (23) is substituted into eqs. (20) and (21) . The resulting equations, under the assumption of perfect knowledge of the nonlinear terms in the model, leads to the closed loop system:

$$\dot{\nu} = \tau_2^s, \tag{24}$$

$$\epsilon = \tau_1^s \tag{25}$$

in which eq. (17) has been invoked.

Suppose τ_2^s is selected to servo the pseudovariable error, and τ_1^s for servoing the internal contact force error. Since eqs. (24) and (25) are completely decoupled, the secondary controller components τ_1^s and τ_2^s are non-interacting controllers for internal contact force and position, respectively.

In [1], the control architecture decoupled the control of $\dot{\nu}$ and f_{c2}. At that time f_{c2} was viewed simply as an independent subset of the contact forces. It was not understood that f_{c2} is in fact a pure internal force, as was shown in Example 1 of Part 1 where the result of [1] was demonstrated to be a special case of the proposed load distribution scheme with $\epsilon = f_{c2}$. The control law $(\tau = \tau^p + \tau^s)$ in fact decouples the position- and internal force-controlled DOF.

CONCLUSION

Direct application of the method in [1] to derive the rigid body model in the joint space results in a system inertia matrix that is not symmetric. A method for describing the dynamics of the held object in terms of the joint variables of both manipulators was presented which led to a symmetric inertia matrix. Finally, the control architecture proposed in [1] was used to decouple the internal force controlled DOF and the position controlled DOF. Previously, it decoupled the control of the pseudovariables and an independent subset of the contact forces, namely, those between manipulator two and the object.

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