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# PROMPT NEUTRON LIFETIME IN A STRONGLY HETEROGENEOUS LATTICE (U)

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A paper proposed for presentation at the
*1992 ANS Topical Meeting on Advances in Reactor Physics*
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## ABSTRACT

The prompt neutron lifetime is an important input parameter to the solution of the point reactor kinetics equations. In the past, many methods of determining the prompt neutron lifetime included the assumption that the thermal neutron lifetime was a good approximation to the prompt neutron lifetime. It has been found that strong heterogeneities within a lattice cause the calculation of the thermal neutron lifetime to significantly overpredict the prompt neutron lifetime. Therefore, a new method for the calculation of the prompt neutron lifetime is presented. This new method, the Alpha Search Method (ASM), is based on the time eigenvalue and does not suffer from the overprediction of the neutron lifetime as approximated by the thermal neutron lifetime. The causes of this overprediction, as well as comparisons of lifetimes computed by the ASM and other methods will be presented.

## INTRODUCTION

The prompt neutron lifetime ($\ell_p$) is an important input parameter to the solution of the point reactor kinetics equations. It is defined as the average time between a prompt neutron emission and its subsequent capture. The thermal neutron lifetime ($\ell$) is a measure of the average time a neutron spends at thermal energies before being captured. For a thermal reactor, the majority of the time between a neutron's birth and it's capture is spent at thermal energies. Over the years, several methods have been developed to calculate the prompt neutron lifetime, or approximations to it. Many of these approximations, such as the thermal Inverse Velocity method, rely on "conventional wisdom" which dictates that for thermal reactors, the lifetime is dominated by thermal neutrons, and consequently little error accrues from the use of the thermal neutron lifetime as an approximation to the prompt lifetime. This is not necessarily true in strongly heterogeneous lattices such as those used at the Savannah River Site (SRS).

Explicit calculations of the prompt neutron lifetime have shown that calculation of the thermal neutron lifetime can result in the overprediction of the prompt lifetime by as much as 15% in a typical SRS lattice.

Therefore, a new method for calculating the prompt neutron lifetime has been developed. The Alpha Search Method (ASM) utilizes the results of an alpha (time eigenvalue) search performed by the GLASS lattice physics code to compute the full spectrum lifetime. This method has been found not to succumb to the same overprediction as the methods using the thermal neutron lifetime.

The Inverse Velocity method and the problems associated with its use in strongly heterogeneous lattices as well as the formulation of the new ASM will be presented. The Eigenvalue method has been chosen as a further comparison method to the two mentioned above, and results of these three methods for several different media will be shown and discussed.

## DISCUSSION

### INVERSE VELOCITY

Historically, the Inverse Velocity method has been used at SRS for the calculation of the neutron lifetimes. Based on the assumption that the thermal neutron lifetime is a good approximation of the prompt neutron lifetime, the following procedure was used to calculate the thermal lifetime.

Typically, GLASS is run and standard two-group macroscopic cross sections, such as the average thermal absorption cross section ($\overline{\Sigma_{a2}}$), are edited. A utility code, AVEVEL, evaluates the energy and space averaged inverse velocities ($\overline{1/v}$) over specified multigroup intervals. If the interval coincides with the thermal group in GLASS, then the Inverse Velocity method predicts the thermal neutron lifetime from the formula: [1]

$$\ell = \frac{\overline{1/v}}{\overline{\Sigma_{a2}}\left(1 + L^2B^2\right)}$$

The term $(1+L^2B^2)$ accounts for leakage from the lattice. It was expected that the thermal neutron lifetime calculated by this method would be close to the prompt lifetime in SRS lattices because:

1) most reactions are thermal
2) for $1/v$ absorbers, lifetime is independent of neutron energy
3) in a highly enriched tritium producing charge, U-235 and Li-6 closely approximate $1/v$ absorbers.

Calculations for several different reactor media however, show that this method of determining the thermal neutron lifetime overpredicts the prompt lifetime value by as much as 15%.

The strong heterogeneities found in SRS lattices have been shown to cause the overprediction of the thermal neutron lifetime and consequently the prompt neutron lifetime. The overprediction was determined to arise from the heterogeneity of the lattice with absorbers concentrated in a fuel assembly at the center of the cell. The reason can be seen from the schematic cross section dependence shown in Figure 1. For a homogeneous lattice, the cross section of a $1/v$ absorber is a straight line when plotted against the logarithm of the neutron energy. This behavior is also true in a heterogeneous lattice if the intracell flux profile is independent of neutron energy. In typical SRS lattices, the epithermal neutron flux is relatively flat across the cell. The thermal flux however is depressed in the fuel assembly and the fast flux is peaked. Application of advantage factors (ratios of flux in components to the cell average flux) by energy gives the effective functional cross section shape shown. Thus, the effective cell-averaged cross section for a $1/v$ absorber in a heterogeneous lattice does not have a $1/v$ energy dependence. This effect makes the cell averaged lifetime for thermal neutrons significantly longer than for epithermal and fast neutrons and hence is an overestimation of the full spectrum lifetime. Therefore, a method to calculate lifetime which does not include this assumption is needed. The Alpha Search Method is one such method.

## ALPHA SEARCH METHOD

A new formulation for the neutron lifetime, based on the time eigenvalue, has been developed which does not incorporate the assumption that the prompt neutron lifetime can be modeled by the thermal neutron lifetime. This method - the Alpha Search Method (ASM) - uses the alpha search (time eigenvalue) option in GLASS to calculate the lifetime. The alpha search option in GLASS is an iterative procedure using an effective absorption cross section of [1]

$$\Sigma_a(eff) = \Sigma_a - \frac{\alpha}{v}$$

in a perturbation analysis. A search is performed to compute the value of $\alpha$ (the stable asymptotic period) which achieves a desired k'. Given a guess of $\alpha$ and an initial k, GLASS uses the following formula to calculate the change in $\alpha$ ($\delta\alpha$) required to produce a desired k'.

$$\delta\alpha = \left(\frac{k' - k}{k}\right)\left(\frac{v\Sigma_a}{1 + L^2B^2}\right)$$

The relationship between this GLASS calculated time eigenvalue and the neutron lifetime can be seen if the time dependent diffusion equation is studied.

$$\frac{\partial n}{\partial t} = D\nabla^2\Phi - \Sigma_a\Phi + \upsilon\Sigma_f\Phi$$

Using the relationship that

$$\Phi = nv;\ \text{therefore}\ \frac{\partial n}{\partial t} = \frac{1}{v}\frac{\partial \Phi}{\partial t}$$

this equation becomes

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} = D\nabla^2\Phi - \Sigma_a\Phi + \upsilon\Sigma_f\Phi$$

A basic premise is that the flux can be separated in space and time. The flux therefore can be represented as: [2]

$$\Phi(r,t) = \Psi(r)T(t)$$

substituting this relationship into the diffusion equation and dividing by

$$\Psi(r)T(t)$$

yields

$$\frac{1}{T}\frac{dT}{dt} = v\left[\frac{D\nabla^2\Psi}{\Psi} + \left(\upsilon\Sigma_f - \Sigma_a\right)\right] = -\alpha$$

Thus the original partial differential equation becomes the two following ordinary differential equations.

$$\frac{dT}{dt} = -\alpha T$$

and

$$vD\nabla^2\Psi + v\upsilon\Sigma_f\Psi - v\Sigma_a\Psi = -\alpha\Psi$$

The second of which can be rearranged to

$$vD\nabla^2\Psi + \left(v\upsilon\Sigma_f - v\Sigma_a + \alpha\right)\Psi = 0$$

The solution to the first equation is of the form

$$T(t) = T(0)e^{-\alpha t}$$

Since the lattice is assumed to be in a near critical state, the following approximation can be used in the second equation.

$$\nabla^2\Psi = -B^2\Psi$$

After substitution of this relationship into the modified second differential equation and division by the flux is performed the equation becomes

$$\alpha = vDB^2 - v\upsilon\Sigma_f + v\Sigma_a$$

Using the definition

$$L^2 = \frac{D}{\Sigma_a}$$

the time eigenvalue can be written as

$$\alpha = v\Sigma_a\left(1 + L^2B^2\right)\left(1 - \frac{\upsilon\Sigma_f/\Sigma_a}{1 + L^2B^2}\right)$$

It is from this equation for alpha that the formulation for the lifetime can be determined. If k is defined to be [3]

$$k = \frac{\upsilon\Sigma_f/\Sigma_a}{1 + L^2B^2}$$

then by substituting this expression for k and using the definition of finite lattice neutron lifetime given earlier, the equation for alpha can be rewritten as

$$\alpha = \frac{(1-k)}{\ell_p}$$

which can be rearranged to solve for $\ell_p$ to give

$$\ell_p = \frac{(1-k)}{\alpha}$$

where k is the eigenvalue with $\alpha = 0.0$.

Thus, from a single GLASS calculation with a k search (with $\alpha$ equal to zero) and an $\alpha$ search (with k equal to one), the inputs are determined to calculate $\ell_p$. Since this method does not involve the thermal neutron lifetime, it is not influenced by the effects of the heterogeneities in the lattice. This method is much simpler than that employed in the thermal Inverse Velocity method, and will be shown to be more accurate. In order to provide a measure of the accuracy of the Alpha Search Method, results from an independent calculation of the lifetime will be compared.

EIGENVALUE METHOD

As an independent check of the Alpha Search method's accuracy, calculations were performed using the "Eigenvalue Method". [4,5] This method works by adding an artificial $1/v$ absorber (with a 1 barn cross section at 2200 m/sec) to each group and each region of the lattice. Keff values with and without this absorber are then used in the formula

$$\ell_p = \frac{\Delta k/k}{\Sigma_a * v}$$

which for this artificial $1/v$ absorber corresponds to

$$\ell_p = \frac{\Delta k/k}{N * 2.2E5}$$

where N is the number density of $1/v$ absorber in atm/b-cm.

## RESULTS

Tables 1 and 2 show the results of calculations of the neutron lifetime by several different methods: the Inverse Velocity method(AVEVEL), the Eigenvalue method, and the Alpha Search method. As can be seen, the thermal Inverse Velocity method predicts up to a 15% longer lifetime than that calculated by the other methods. Calculations were performed to determine the reason for the discrepancy. These calculations were performed on a homogeneous lattice, a two region cell of U-235/Li-6 and D2O, and an explicit representation of the tritium producing charge with control (Mark 22 geometry). The results from these calculations make several points apparent:

1) The thermal neutron lifetime calculated by the Inverse Velocity method for a homogeneous mix of U-235, Li-6 and D2O shows much closer agreement with that from the Alpha Search method than the same comparison for a heterogeneous lattice. It was also noted that as the amount of heterogeneity was increased, the relative error between the Alpha Search method and the thermal Inverse Velocity method increased.

2) By using the Inverse Velocity method for the neutron lifetime, but collapsing to a single "Few-group" over the entire multigroup structure, one can get very close agreement with the Alpha Search method.

Cases were run for the homogeneous lattice by all three methods: Inverse Velocity (AVEVEL), Alpha Search, and Eigenvalue. Table 1 shows excellent agreement between the average Inverse Velocity method (using one group) and the Alpha Search method. It can also be seen that as the amount of $1/v$ absorber is decreased, the lifetime as calculated by the Eigenvalue method approaches the value given by the Alpha Search method.

The formulation of the Eigenvalue method is analytically independent of the magnitude of the $1/v$ added. Since lifetimes by the Eigenvalue method tend toward those from the Alpha Search method as the added $1/v$ absorber tends toward zero, it appears that the Alpha Search method is equivalent to extrapolating the Eigenvalue method to zero perturbation. Unfortunately, it cannot be demonstrated that the Eigenvalue method asymptotically approaches the Alpha Search method due to a lack of significant digits in the GLASS calculations of $\Delta k$ as the concentration of $1/v$ absorber gets smaller.

Similar calculations were performed for a two region cell and for the explicit tritium producing assembly (Mark 22 geometry). Table 2 shows the results. Once again there is good agreement between the Alpha Search method and the Inverse Velocity method (using 1 group). The Eigenvalue method continues to trend towards the lifetime calculated by the ASM as the concentration of $1/v$ decreases. The reason for the discrepancy between the ASM results and the apparent asymptotic value from the Eigenvalue calculations lies in the method in which GLASS treats the multigroup spatial distributions in the two cases. In the ASM, these spatial distributions are calculated only for the initial k (eigenvalue) search with alpha equal to zero; the same shape is assumed for the alpha search. In the Eigenvalue method, the spatial distributions are computed separately for the reference condition and for the added $1/v$. It can be seen that the overprediction of the neutron lifetime is most pronounced in the Mark 22 geometry case. The Inverse Velocity method for the thermal lifetime overpredicted this case (which had the largest amount of heterogeneity) by 15%.

## CONCLUSIONS

Due to heterogeneous effects, the thermal Inverse Velocity method historically used at the Savannah River Site for the determination of prompt neutron lifetimes is inadequate. This method overpredicts the correct value of the prompt neutron lifetime by as much as 15%. In order to correct this deficiency a new method, the Alpha Search method which incorporates the alpha search option in GLASS, has been developed and tested. This method gives close agreement with values obtained by applying the Inverse Velocity method over the entire spectrum, but is much simpler to perform. The Alpha Search method compares favorably with the Eigenvalue method, is considerably easier to implement, and is not dependent on the amount of $1/v$ absorber that is added to the lattice. For these reasons the Alpha Search method is now the recommended procedure for evaluating neutron lifetimes in SRS lattices.

It should be noted that a precise value for the prompt neutron lifetime is essential only when calculations are performed near prompt critical. Since this is seldom the case, changes to the neutron lifetime as calculated by the Alpha Search method are not expected to impact the results of previous calculations. Sensitivity analysis has been performed on the point kinetics equations used in safety analysis to verify this result.

## REFERENCES


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## Table 1. Lifetimes in Homogeneous Media

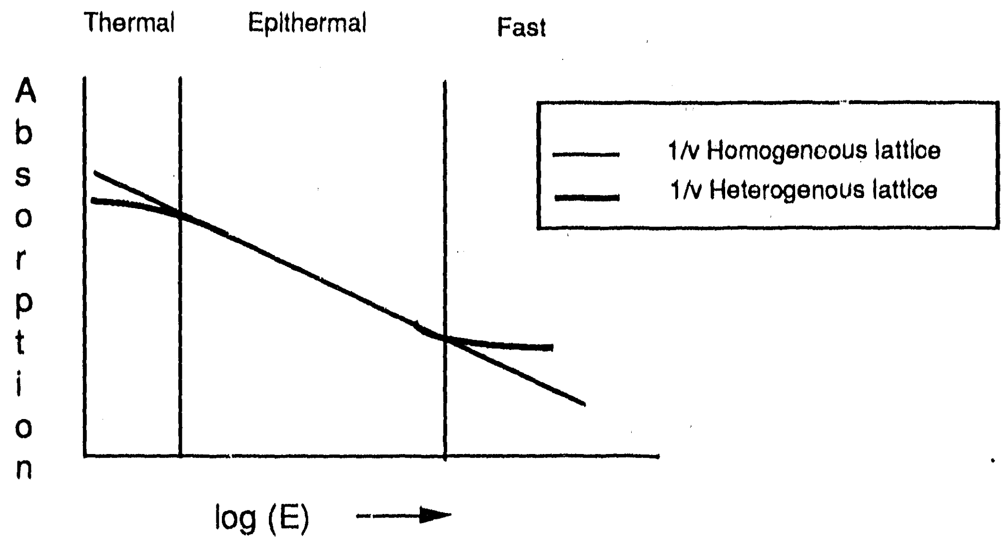
| Method | absorber (atm/b-cm) | lifetime (x10-6 sec) | percent difference |
|---|---|---|---|
| Alpha Search | 0 | 51.90 | - |
| AVEVEL (1 group) | 0 | 51.90 | 0.0 |
| Eigenvalue | 1.0E-3 | 51.34 | -1.1 |
| Eigenvalue | 5.0E-4 | 51.62 | -0.5 |

## Table 2. Lifetimes in Heterogeneous Media

| Method | absorber (atm/b-cm) | lifetime (x10-6 sec) | percent difference |
|---|---|---|---|
| Two region cell | | | |
| Alpha Search | 0 | 50.2 | 0.0 |
| AVEVEL (1 group) | 0 | 50.0 | -0.4 |
| AVEVEL (2 group) [a] | 0 | 52.3 | 4.1 |
| Eigenvalue | 1.0E-3 | 49.4 | -1.6 |
| Eigenvalue | 5.0E-4 | 49.8 | -0.8 |
| Eigenvalue | 1.0E-4 | 50.1 | -0.2 |
| Mark 22 | | | |
| Alpha Search | 0 | 127 | 0.0 |
| AVEVEL (1 group) | 0 | 125 | -1.6 |
| AVEVEL (2 group) [a] | 0 | 146 | 15.0 |
| Eigenvalue | 1.0E-3 | 119 | -6.3 |
| Eigenvalue | 5.0E-4 | 121 | -4.7 |
| Eigenvalue | 1.0E-4 | 123 | -3.1 |

[a] Using thermal group only

Figure 1. Effect of Heterogeneity on a 1/ν Absorber



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