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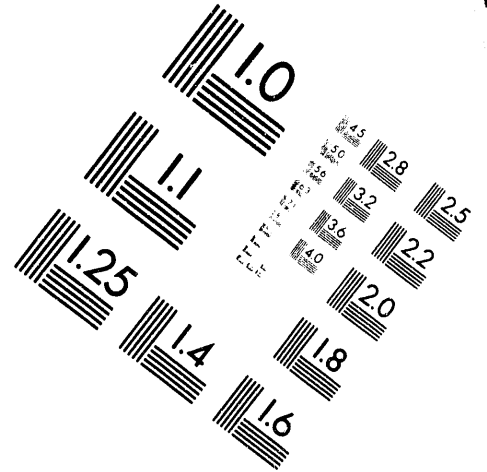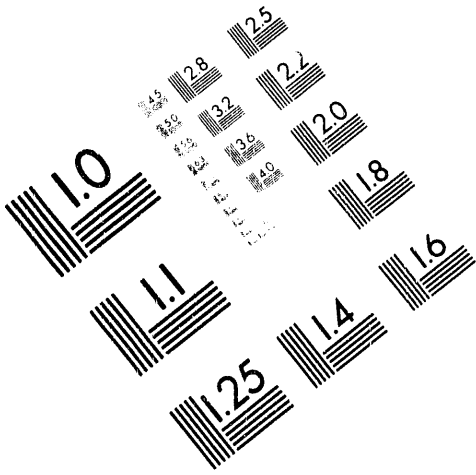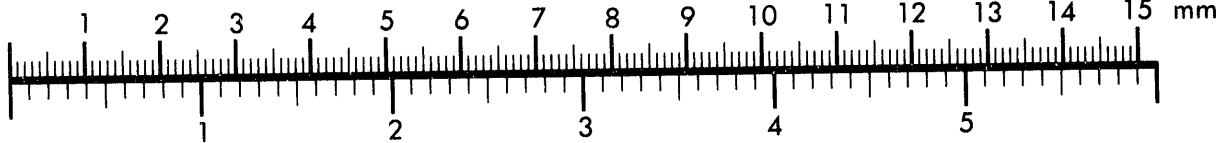**Association for Information and Image Management**
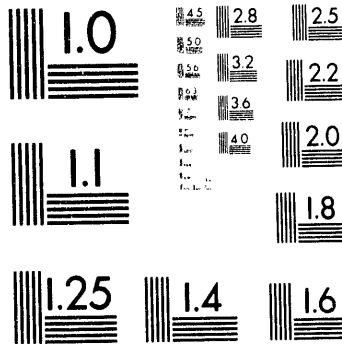
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Centimeter

1　2　3　4　5　6　7　8　9　10　11　12　13　14　15 mm

1　　2　　3　　4　　5

Inches

1.0

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1.25　1.4　1.6

2.8　2.5

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3.6

4.0　2.0

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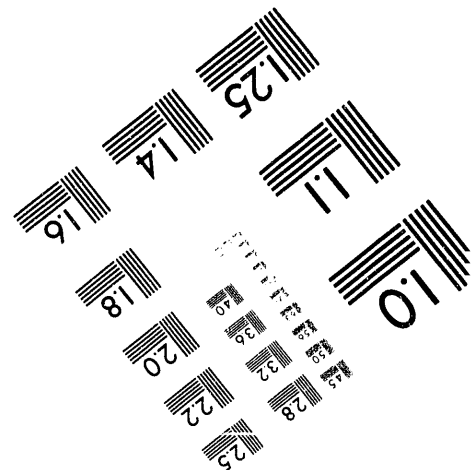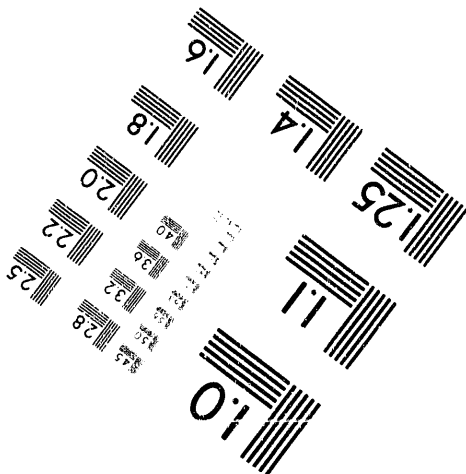
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Vibration Suppression by Modulation of Elastic Modulus Using Shape Memory Alloy[1]

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1.0 Introduction

Most common applications of "smart materials" actuators involves obliging them to undergo some generalized displacement in response to a specified stimulus. A slightly different approach is suggested in this paper. Here, we consider an application in which a modulus rather than a displacement is manipulated. Further, we present a class of problems for which such a smart material can be used to address very simply a problem of rotating equipment.

The first portion of this paper proposes a method of fabricating a material whose modulus can be changed substantially through the application of a specified stimulus. The particular implementation presented here indirectly exploits the large deformation associated with shape memory alloys to achieve the desired modulation of stiffness.

The next portion of this paper discusses a class of vibration problems for which such materials have a serious potential for vibration suppression. These are problems, such as the spinning up of rotating machinery, in which the excitation at any time lies within a narrow frequency band, and that band moves through the frequency spectrum in a predictable manner.

Finally, an example problem is examined and the utility of this approach is discussed.

2.0 A Smart Material with Modulated Elastic Modulus

Here we present an approach for modulating the elastic moduli of materials by introducing a reinforcing material whose contribution to the stiffness of the composite can be turned on and off. We first consider an elastic matrix structure through which an array of holes has been drilled. Then strands of shape memory alloy (SMA) are threaded through the holes and knobs (or knots) are placed at the ends of the strands, leaving just a little slack. (See Figure 1 and Figure 2.) Since the SMA can move freely through the matrix, it contributes nothing to the stiffness of the composite structure.
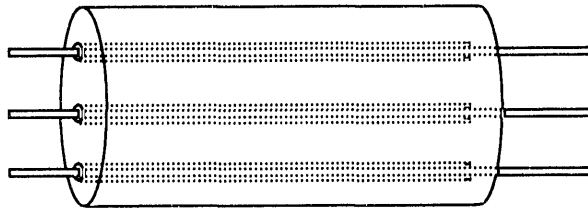
FIGURE 1. The matrix material with holes through which shape memory alloy (SMA) is threaded. The free movement of the SMA with respect to the matrix prevents it from contributing to the stiffness of the composite structure.

We now take advantage of a fundamental feature of shape memory alloy: when shape memory alloy is subjected to a change of temperature it is caused to jump to an equilibrium phase appropriate to that temperature. In particular, it can be switched from the extended, stress-free state shown in Figure 2 to the shortened state shown in Figure 3. In the shortened state, all slack is taken up, the knobs are pulled into the matrix, and the SMA is in tension.

This state, in which the SMA is contracted, the knobs fully couple the extensional deformation of the SMA to that of the matrix material. One may now use any one of the standard formulae to estimate the extensional modulus of the composite structure. A good
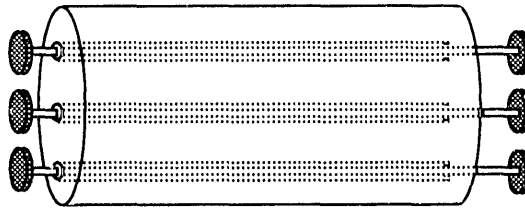
FIGURE 2. The matrix material with holes through which shape memory alloy (SMA) is threaded. Even though there are knobs on the ends of the SMA strands, there is still enough slack to permit free movement. Again, the free movement of the SMA with respect to the matrix prevents it from contributing to the stiffness of the composite structure
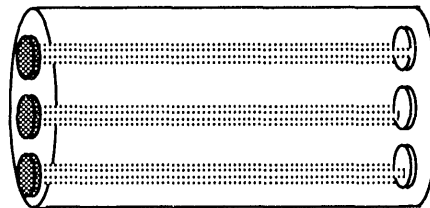

FIGURE 3. The matrix material with holes through which shape memory alloy (SMA) is threaded. The SMA has been activated, causing a jump to a phase with a shorter equilibrium length, taking up all slack. The knobs are now pressing against the matrix material, completely coupling extensional deformations of the two materials.

review of alternative methods of performing these calculations is presented in Reference 1. A first approximation of the stiffening effect of the SMA is obtained by assuming uniform strains within the composite. This analysis results in a stiffness increase proportional to the relative stiffness of the SMA and the matrix material, to the ratios of the cross sectional areas of the SMA and the matrix materials, and to the percentage of the SMA strands that have been activated to contract.

The phase transformations of the SMA is reversible, so that a return to the previous temperature will result in a return to the configuration shown in Figure 2. By raising and dropping temperature, on can cause repeated reversals in the stiffness of the composite. The issue of response time becomes primarily an issue of the rates of heating and cooling. Heating is usually achieved by running a current through the SMA itself while cooling is usually achieved through convective and diffusive processes. However, response times can be accelerated by using more aggressive cooling techniques Reference 2.

It is important to note that the effect that is being targeted here is to modulate stiffness rather than to impose a deformation, as has been done often before; see Reference 4. A. Baz et al in Reference 3 have used shape-memory alloy embedded in a composite to achieve a more gradual modulation in stiffness, though the physical mechanism is still a point of controversy.

Because applications described below require a change in modulus just from one stage of a transient spinning process to another, rather than within each rotation, particularly fast response times are not necessary.

It should be noted that the strategy presented above for modulating the stiffness of a composite structure could be applied with materials other than shape memory alloy, so long as those materials can be caused to undergo large static deformation through some outside stimulus. Classes of such materials besides shape memory alloys include thermo-elastic, piezo-electric, and magnetostrictive materials.

A further note should observe that the crude configuration shown above could be extended by placing the SMA near the surface of a beam, and staggered along the length of the beam. This configuration, shown in Figure 4, permits effective modulation of the bending modulus of the beam. If the beam is originally in a spiral configuration, one now has a spring whose rate is modulated.
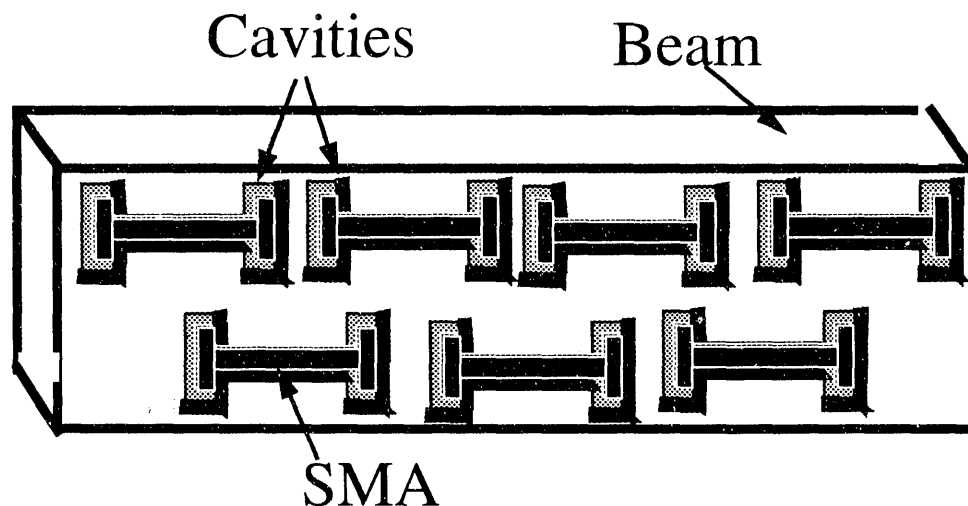

FIGURE 4. A staggered configuration of shape memory alloy fillers in an elastic matrix.

3.0 Application to a Class of Vibration Problems

General Problem Description

The vibration suppression capabilities of the smart material described above will be demonstrated by application to a class of rotating vibration problems. This class of problems is defined by the excitation frequency of the system being an integer multiple, n, of the angular velocity of the system so that during spin-up, the excitation frequency passes through the natural frequency of the system.

Paradigm Problem Description

We consider here the simplest such case, that of a rigid disk pinned to a rigid shaft. The connection includes a keyway so that the disk must rotate with the shaft, though it may wobble from side to side. A torsional spring serves to restore the disk to its normal configuration. This simple model, used in other dynamics analysis in Reference 5, is meant to account in a very approximate way for shaft or disk flexibility. The disk rotates between frictionless spring loaded plates. A number, n, of small sinusoidally shaped projections on the surface of the disk cause a sinusoidal excitation. The torsional spring is oriented such that a line connecting the peaks of the projections on the disk would be perpendicular to the axis of rotation of the torsional spring. This system is shown in Figure 5.
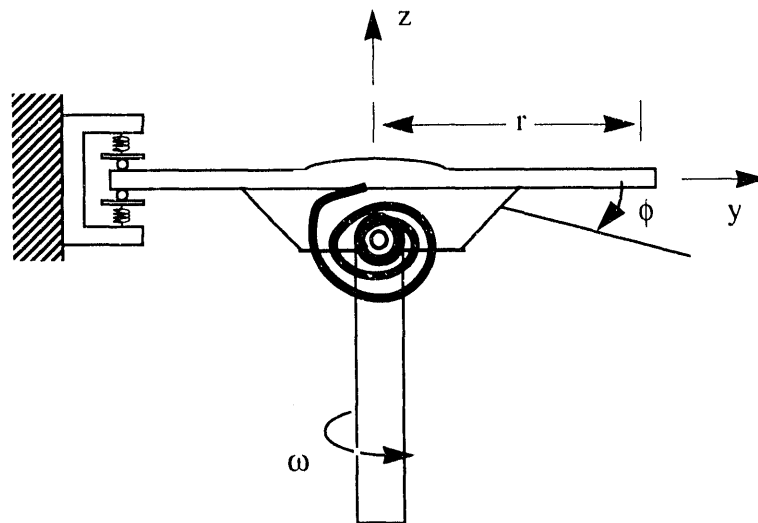

FIGURE 5. Disk/Rotor System

The governing equations can be derived in many ways (the authors have used the method presented in Reference 6). The equation for the angle ϕ, by which the disk is off normal is:

$$I'_{yy}\ddot{\phi} + c\dot{\phi} + \phi\,(\beta k + I_{yy}\omega^2) = A\sin(n\omega t) \qquad \text{(EQ 1)}$$

The multiplier β is ordinarily unity, but will be used later on as a "stiffening" multiplier. An interesting feature of this system is that the excitation frequency at resonance is:

$$\omega^2 = \frac{\dfrac{k}{I_{yy}} - \left(\dfrac{c}{2I_{yy}}\right)^2}{n^2 - 1} \qquad \text{(EQ 2)}$$

so that excitation will pass through the damped natural frequency of the system only for n greater than one.

The following quantities are used to make the equation of motion dimensionless:

$$\omega_o = \sqrt{\frac{k_o}{I_{yy}}} \qquad\qquad \text{natural frequency for the nonspinning system.}$$

$$\alpha = \frac{\omega}{\omega_o} \qquad\qquad \text{dimensionless frequency}$$

$$T = \omega_o t \qquad\qquad \text{dimensionless time}$$

$$\xi = \frac{c}{2I_{yy}\omega_o} \qquad\qquad \text{dimensionless damping ratio}$$

$$\tilde{A} = \frac{A}{\omega_o^2 I_{yy}} \qquad\qquad \text{dimensionless excitation amplitude}$$

$$\Phi(T) = \phi\left(\frac{t}{\omega_o}\right) \qquad\qquad \text{dimensionless deflection amplitude}$$

The dimensionless equation of motion is:

$$\Phi'' + 2\xi\Phi' + (\beta + \alpha^2)\Phi = \tilde{A}\sin(n\alpha T) \qquad \text{(EQ 3)}$$

Stiffness modulation will be affected by changing β, representing a gain factor for stiffness.

The spin-up profile for the class of problems considered is:

$$\omega = \begin{cases} rt & 0 < t < t_o \\ rt_o & t > t_o \end{cases} \qquad \text{(EQ 4)}$$

This spin-up profile causes the differential equation of motion to contain coefficients are explicit functions of time, and the forcing sinusoid to be a function of t^2. The ramping

nature of the forcing function contains a narrow band of frequencies about the instantaneous frequency of nrt. This results in the system's damped natural frequency being excited prior to the intersection of the instantaneous excitation frequency and the system's damped natural frequency.

Smart Material Application -- Stiffness Scheduling

Stiffness scheduling is defined as adapting the stiffness of the structure to reduce disturbance sensitivity. (This approach has been investigated in Reference 7 and Reference 8, in which the stiffnesses at the supports of rotors are modulated.) Since the smart material described above can take on two distinct elastic moduli, the system will have two distinct damped natural frequencies. If the frequency responses for these two systems can be sufficiently separated, then the frequency response of the smart system can be greatly reduced.

Here we consider a system with the following dimensionless parameters:

$$\xi = 0.02$$
$$n = 2$$
$$\tilde{A} = 0.1$$
$$\alpha = \{ \begin{array}{ll} rT & 0 < T < 200.0 \\ 200r & T > 200.0 \end{array}$$
$$r = 0.005$$

A fourth order Runge-Kutta method was used to integrate the governing equation for this system for three cases:

- the dimensionless stiffness is set to a higher value;

- the dimensionless stiffness β is set to a relatively low value;

- and dimensionless stiffness is appropriately switched during the ramp up of spin.

4.0 Results

Figure 6 shows the dimensionless tilt as a function of dimensionless time for the case where the torsional spring is held in a stiffer mode throughout: $\beta = 4$. For that case, a large resonance behavior is seen near a dimensionless time of 120. A beating occurs for a while after the shaft reaches its terminal speed. More significantly, a fairly large oscillation remains at long times, after the disk has reached terminal speed.

Figure 7 shows the corresponding curve for the case of $\beta = 1$. As expected, the resonance-like behavior occurs earlier in the process, when the excitation frequencies are lower. It is important that the behavior at long times manifests much lower amplitudes than is the case
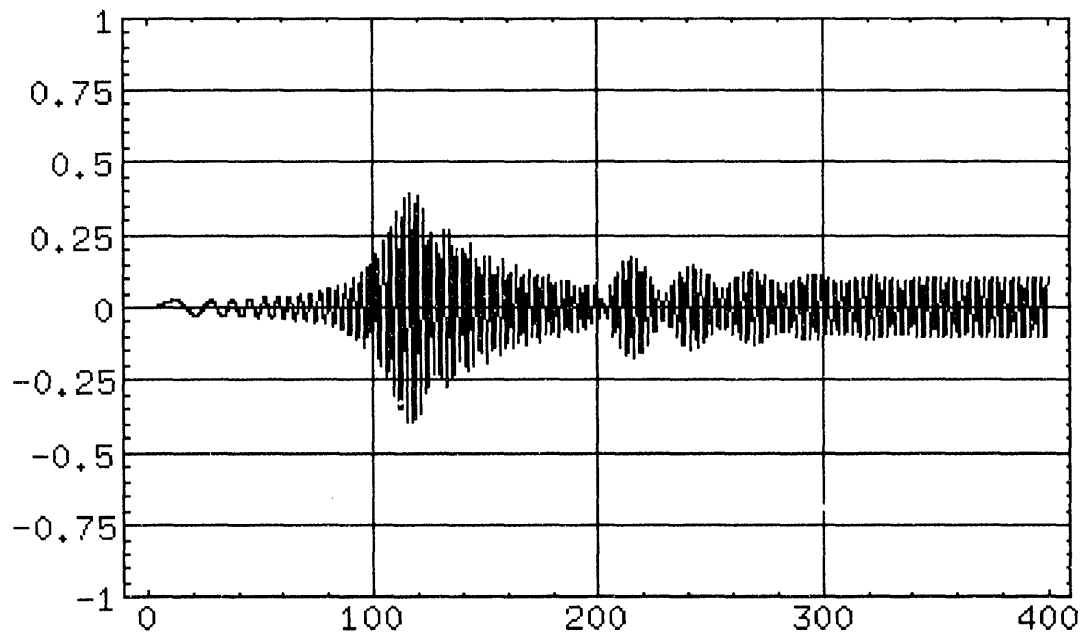
FIGURE 6. Stiffening Actuator Turned On Throughout Spin-Up.

with the stiffer spring. This difference occures because the natural frequency in this case is further from the forcing frequency.

Figure 8 shows the dimensionless tilt as a function of dimensionless time where the stiffness parameter, β , is initially held at a value of 1.0 and instantaneously changed to a value of 4.0 at dimensionless time 98.0.

Note that the maximum amplitude in the resonance regime is slightly more than halved and that the long term oscillations are substantially lower in amplitude than those of the stiff system. Of course, this high frequency response is due to the stiffness of the structure at long times being that of the material shown in Figure 7, and the forced response at those frequencies are similar.

5.0 Conclusions

The smart material described here can be used in conjunction with stiffness scheduling to give a reduction in vibration amplitude for a class of rotating systems where the excitation is proportional to the spin rate. The time at which to change stiffness is based on the spin-up profile being known in advance. This permits the best switching time to be chosen based on the frequency components of the excitation and the stiffness switching transients.
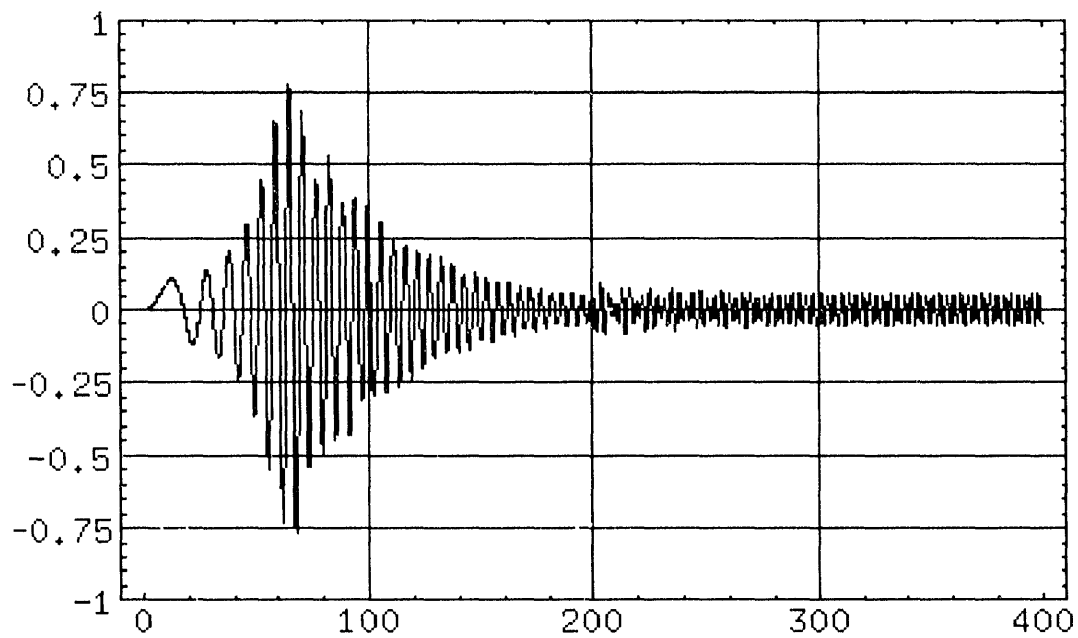
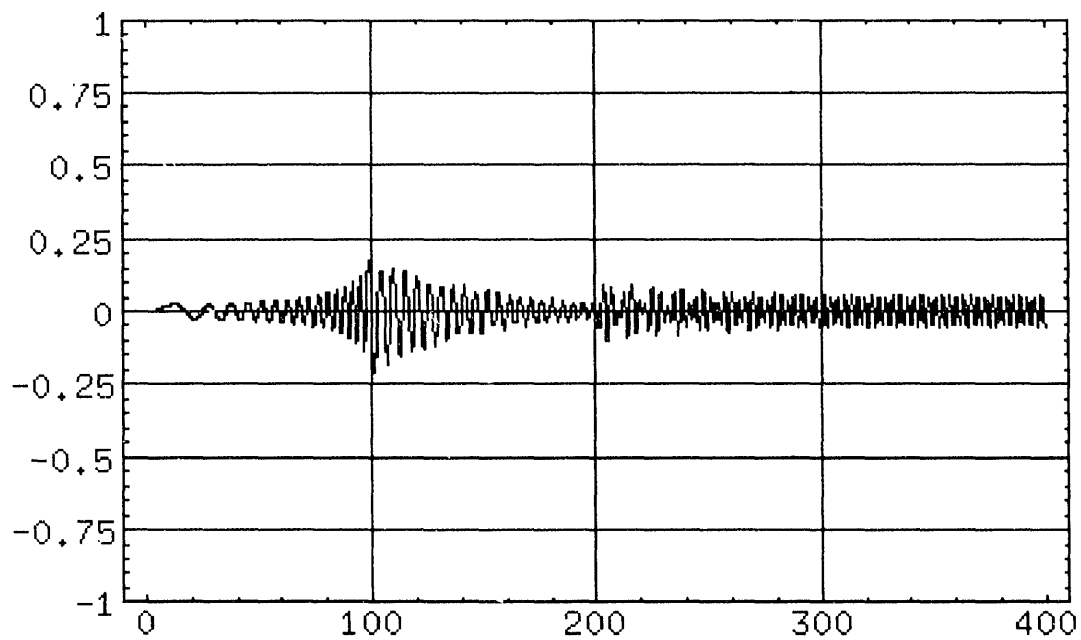**FIGURE 7. Stiffening Actuator Turned Off Throughout Spin**

FIGURE 8. Amplitude Response with Stiffness Scheduling.

6.0 Future Work

A control law for choosing the stiffness switching time is being developed which does not require exact knowledge of the spin-up profile. Further, basic work in the materials issues of such actuators must be done.

DISCLAIMER

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