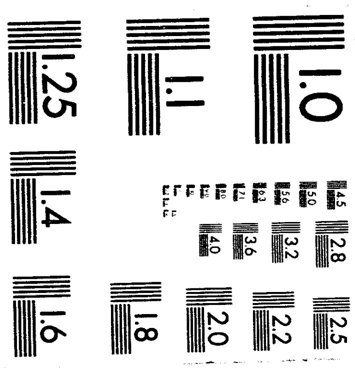

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SLAC–PUB–6397
January, 1994
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# DECOHERENCE, DETERMINISM and CHAOS*

H. Pierre Noyes

*Stanford Linear Accelerator Center*
*Stanford University, Stanford, California 94309*

Presented at the *Fifteenth Annual International Meeting*
of the **ALTERNATIVE NATURAL PHILOSOPHY ASSOCIATION**
Department of History and Philosophy of Science
Free School Lane, Cambridge, England
September 9-12, 1993

* Work supported by the Department of Energy, contract DE–AC03–76SF00515.

Abstract

We assume that "fields" are to be measured by the acceleration of a "test particle" which belongs to a class of particles whose ratios of charge to mass and gravitational to inertial mass are Lorentz invariant. We relate the measurement accuracy in space, $\Delta\ell$, and in time, $\Delta t$, by the scale invariant definition of two constants $c$, and $\kappa$: $\frac{\Delta\ell}{c\Delta t} \equiv 1$; $\frac{\Delta\ell^2}{\kappa\Delta t} \equiv 2\pi$. Taking the experimental velocity resolution $\Delta v_x = \Delta\ell/T\Delta t = N\Delta\ell/NT\Delta t$ we derive the bracket expression $[x, v_x] \propto \kappa$ where $x = N\Delta\ell$. Then it is a *deductive* consequence that the only fields which can act on such particles are structurally indistinguishable from electromagnetic and gravitational fields in the sense that they satisfy the finite difference version of the free space Maxwell equations and Einstein geodesic equations. Such a scale invariant theory becomes the proper correspondence limit for any relativistic particle theory which breaks scale invariance by taking $m_e\kappa = \hbar$. Here we use $m_e$ because it defines the threshold distance for position measurement, $h/2m_ec$, below which the non-classical process of electron-positron pair creation is observed, and above which that phenomenon cannot be *directly* observed. The coherence length $L = NT\Delta\ell$ specifies the maximum distance within which quantum mechanical interference effects can be observed. For non-overlapping "wave packets" of this length, the *deterministic* classical equations with particulate sources and sinks apply. But the characterization of a deterministic system as chaotic requires a specification of boundary conditions to a precision which violates the constraint due to measurement accuracy or electron-positron pair creation. Hence the number of degrees of freedom used in a model fixes whether the system is quantum coherent or classically decoherent but (approximately) deterministic and limits the applicability of chaos theory, removing certain paradoxes.

# 1. INTRODUCTION

The Gordian knot that I tried to disentangle in this paper — rather than just slashing at it with Occam's Razor in my usual cavalier fashion — is the intricate connection between classical and quantum coherence, classical determinism, and deterministic chaos. In the event, Clive Kilmister characterized my paper as reminiscent of the South Sea Bubble. I respond to some of his remarks early on in Chapter 3 (at the end of Section 3.2 and beginning of section 3.3).

In my opinion, the approach I present here has broad implications with respect to both the correspondence limit of relativistic quantum mechanics[1] and "wave function collapse"[2]. Here we will emphasize only the connections between coherence, determinism and chaos. I will pursue the impact of this analysis on the foundations of quantum mechanics on another occasion.

The phenomenon of "coherence" occurs in any wave theory. Experimentally it shows up when two or more distinct beams combine to produce interference fringes. "Decoherence" then corresponds to the disappearance of these interference fringes when the experimental parameters are changed. In theoretical language the difference between coherent and decoherent situations can often be accounted for by assuming that the beams correspond to "wave packets" which can interfere only when they overlap. In chapter 2 we present a specific geometrical paradigm which makes this qualitative idea into a class of measurements that cover both classical (electromagnetic) and quantum (deBroglie) wave interference.

The critical step in this analysis is the assumption that measurement accuracy must be taken seriously as a logical (epistemological, ontological, metaphysical,...?) constraint which can be *fixed* in a quantitative sense by *context*. Starting from the usual "meaning" of length and time measurement in the physics community, we argue that this means that *practice* sets a fixed bound on the shortest length and the shortest time that can be meaningfully specified experimentally by any current technology. We call these *intervals* $\Delta\ell$ and $\Delta t$ respectively. Then, any measured space or time interval can be specified by an *integer* (or a range of integers) times



these dimensional (in the physicist's sense) units. We also assume that the limiting velocity for information transfer is a fixed rational fraction unique to any system of space and time measurements we invoke, which constrains our length-time units by the scale-invariant equation $\Delta\ell = c\Delta t$.

Unfortunately our derivation of bracket expressions from these assumptions has so far failed to pass muster with Clive Kilmister. So Chapter 3 indicates what needs to be done rather than what has been actually demonstrated to his satisfaction. Rather than go into that controversy here, we simply note that *if* our contention is correct these finite and discrete bracket expressions derived from measurement accuracy allow us to take over, practically unaltered, Feynman's proof of the Maxwell equations[3] as reconstructed by Dyson[4] and the generalization of the proof to the Einstein gravitational geodesic equations given by Tanimura.[5] Since we have discussed in more detail elsewhere my attempt to take over the Tanimura proof[6,7] , details are omitted.

Chapter 4 reminds us that if the usual *local* deterministic conclusions are drawn from the field equations and the Lorentz force law, the predictions of the theory become ambiguous. We interpret this fact as due to the naive assumption that difference equations imply a unique continuum limit. We conclude that local determinism is meaningful only when we can accept *incoherence* between radiation sources, radiation field, and radiation sinks as a valid *approximation*.

Chapter 5 makes use of the fact that practically all solutions of classical, "deterministic" equations are chaotic in the sense that one must supply as much information in the boundary conditions as the "prediction" is supposed to yield. This clearly vitiates the concept of "determinism" as usually employed when physics is invoked to support the philosophical concept that goes by that name. If our analysis is correct, boundary conditions which violate the restriction to finite measurement accuracy because of their precision are *inconsistent* with the operationally *meaningless* or *pseudo* precision required to obtain chaotic predictions. This dissolves the paradox of "chaotic determinism".

# 2. COHERENCE

## 2.1 The Geometrical Paradigm

To give form to our discussion of coherence and decoherence, we use the devices schematically illustrated in figure 1. We assume, initially, that the "source" labeled by a question mark emits charged particles with a unique charge-to-mass ratio and a unique velocity $v$. Devices which we will use to insure that, to some finite accuracy, these assumptions are true are included in the figure, and will be discussed in more detail subsequently. For the moment we omit the "path extender". We start from the case when the detection screen beyond the double slit[8] exhibits a double slit interference pattern whose envelope is the single slit diffraction pattern for a slit of width $\Delta w$ and a distance $D$ from the detector array. We set the parameters such that the spacing from the center of the pattern to the first interference fringe is $s$. Then the "wavelength" $\lambda$ exhibited by this coherent interference between the beams from the two slits is measured and can be calculated from the equation

$$\lambda = \frac{ws}{D} \tag{2.1}$$

We note that $w, s$ and $D$ are length intervals that can be measured by conventional macroscopic methods such as rods calibrated against international standards. We take this as the paradigmatic case for specifying what we mean by "coherence". We emphasize that, so far, only *length* measurements are implied and hence that our diagram is *scale invariant.*

In order to measure the "coherence length" we insert into the hypothetical "path" of the particle coming from one of the slits a "path extender", schematically represented by a wedge whose sides are mirrors. One face of the wedge reflects the beam to a second mirror which returns it to the second face of the wedge, which in turn returns it to the direction it followed in the paradigmatic case. The distance $C$ from the wedge to the mirror is adjustable. $C = 0$ corresponds to the simplest double slit paradigm. We find experimentally that for a source of a particular type



the fringe system disappears when we reach a value $C_{max}$ or larger. We can then define the *coherence length* $C_{coh}$ by

$$C_{coh} \equiv 2C_{max} \tag{2.2}$$

Note that so far the definition still depends directly on geometrical measurements. Indirectly the specification depends on the ***sensitivity*** of the detector array, since the ***intensity*** of the pattern along the detector array and (if the array records individual particulate events) the ***probability*** of a particular region of the array being activated decreases as $C$ increases. The disappearance of the interference pattern is our paradigm for *decoherence.*

To go further in our analysis, we must measure the velocity $v$, or if this velocity is close to the limiting velocity for information transfer — for which we use the conventional symbol $c$ — the momentum. Then we can define a second critical parameter called the *coherence time* and symbolized by $T_{coh}$ by the relationship

$$C_{coh} = vT_{coh} \tag{2.3}$$

Use a detector array which measures the *time* of arrival of individual particles by means of a clock synchronized to the firing of the first counter in the counter telescope using the Einstein convention. In the situation where the interference fringes have disappeared, we can distinguish two paths emerging from the double slit by noting that all particles which follow the longer path arrive at the detector with a time delay greater by at least $T_{coh} = C_{coh}/v$ compared to the particles which traverse the shorter path. Various checks on this statement can depend on the measurement accuracy to which we can establish all the relevant parameters. Several such checks will occur to any experimental particle physicist. Since these checks are irrelevant to our main theme, we stop our articulation of the basic paradigm at this point, and focus on the accuracy to which we can measure velocity or momentum. The main point we wish to establish is simply that in a carefully specified context, *outside* of some coherence length or coherence time, particles can be said to follow two (or more) distinct trajectories for at least part of their history between production and detection. Inside that length, two coherent beams of the same type of particle can be made to interfere with a characteristic wavelength that can be measured geometrically.

### 2.2 Space-time velocity measurement

The "counter telescope" we have included in figure 1 consists of two devices which *record* the time of firing *or of* **not** *firing* during some time interval. The distance between the two counters is $L$ and the time delay between the two recordings is $T$. These two recordings are NO-YES *events* in that whether the individual counters do not fire ("NO") or do fire ("YES") is recorded by two distinguishable symbols in two correlated records. These records can be repeatedly examined without destroying this distinction or the sequential ordering. In this context the velocity of a particle $v$ is measured by a $YES_1, YES_2$ pair of events and is calculated by the ratio

$$v = \frac{L}{T} \tag{2.4}$$

The *accuracy* to which this constitutes — or can constitute — a *measurement* of this velocity cannot be adequately discussed in an article of this length. We simply note that what are called "particles" in high energy elementary particle physics have never been demonstrated to have velocities greater than the scale parameter

$$c \equiv 299\ 792\ 458\ m\ sec^{-1} \tag{2.5}$$

Further, there is no accepted situation in which *information* in the physical or computer science sense has been transferred at a velocity greater than this value. Demonstrable exceptions to these statements would be of extreme interest to the physics and computer science communities.



### 2.3 Energy-Momentum velocity measurement

The "magnetic selector" we have included in figure 1 can also be considered to be a device capable of measuring velocity when it is properly calibrated. However the calibration procedures are more complicated than the direct calibration of rods and clocks which suffice for space-time velocity measurement. It is here that our restriction to a particular type of particle begins to become important.

If the particle is electromagnetically neutral, or if the space-time velocity is not distinguishable from $c$ (up to the maximum value of $\mathcal{H}$ available to us), no deflection is observed and the inverse radius of curvature $\rho^{-1}$ is indistinguishable from zero. We exclude these cases for the moment because the measuring device invoked gives no information not already provided by the counter telescope. However, when a deflection (finite $\rho$) is observed, we find that for fixed $\mathcal{H}$ the radius of curvature $\rho$ changes with velocity. To cut a long story short, we find that if we measure velocity in units of $c$ by defining

$$v \equiv \beta(v)c \tag{2.6}$$

and keep the magnetic field fixed,

$$\rho^2(v) \propto \frac{\beta^2}{1-\beta^2}; \quad \rho^{-2}(v) \propto \frac{1-\beta^2}{\beta^2} \tag{2.7}$$

This clearly allows us to calibrate our magnetic field to space-time measurements and, for a particular class of particles, to specify higher and lower magnetic fields over some range by the velocity-independent (over that range) definition

$$\mathcal{H} = \frac{\rho(v)}{\rho_0(v)}\mathcal{H}_0 \tag{2.8}$$

leaving open the units in which we ultimately decide to measure magnetic fields.

If, as is often the case in high energy physics, it is more convenient to measure radius of curvature rather than space-time velocity, we can relate this approach to the space-component of the "four velocity" $(u_0, \vec{u}) = (\gamma, \gamma\vec{\beta})$ with $\gamma^2\beta^2 = \gamma^2 - 1$ and

$$\beta^2(u) = \frac{u^2}{1+u^2}; \quad \gamma^2(u) = 1 + u^2; \quad u = \pm|\vec{u}| \tag{2.9}$$

For a particular type of particle, this tells us that $u^2$ is proportional to $\rho^2$, and in a more articulated theory allows us to measure momentum by radius of curvature in a calibrated magnetic field. In this context we can ignore the (fixed) rest-mass of our "test particles" and keep our "momentum" measurements restricted to the "space-component of four velocity" or "momentum per unit mass".

Similarly, if we measure energy by the temperature rise in a calorimeter calibrated to the ideal gas law for particles of the same mass, i.e. measure pressure per unit mass rather than pressure, we can verify that this is consistent with the usual relativistic single particle kinematics

$$\frac{E^2}{m^2} = 1 + u^2; \quad \frac{E^2}{m^2} - \frac{p^2}{m^2} = 1 \tag{2.10}$$

and so on.

### 2.4 Scale invariance

We have been at some pains to remove the mass scale from our basic paradigm for "coherence" and "decoherence" because our basic argument below rests on the fact that we can derive "bracket expressions" similar to the commutators of quantum mechanics, using only measurement of space and time with accuracy bounded from below. Then it takes a *physical* phenomenon involving Planck's constant to recover quantum mechanics. This can be done in a number of ways, eg historically by the analysis of black body radiation, photo-effect, line spectra of atoms, finite size and stability of atoms measured using deviations from the ideal gas law, and so on. The cleanest breakpoint for the *relativistic* quantum



mechanics which concerns us is the creation of electron-positron pairs or the less direct but predicted and confirmed effects (eg Lamb shift, vacuum polarization in p-p scattering,...) of these degrees of freedom. This is possible because the whole idea of a "test-particle" is basic to the classical definition of "fields". Once the degrees of freedom due to the possibility of particle-antiparticle pair creation have to be included in the theory, even the concept of a "test particle" generates nonsense.

## 3. CLASSICAL FIELDS from MEASUREMENT ACCURACY

### 3.1 THE FEYNMAN-DYSON-TANIMURA PROOF

Tanimura (Ref.5) makes the following remarkable claims in his abstract:

*"R.P.Feynman showed F.J.Dyson a proof of the Lorentz force law and the homogeneous Maxwell equations, which he obtained starting from Newton's law of motion and the commutation relations between position and velocity for a single nonrelativistic particle. We formulate both a special relativistic and a general relativistic versions* [sic] *of Feynman's derivation. Especially in the general relativistic version we prove that the only possible fields that can consistently act on a quantum mechanical particle are scalar, gauge and gravitational fields. We also extend Feynman's scheme to the case of non-Abelian gauge theory in the special relativistic context."*

The formulation of the Feynman theorem as reconstructed by Dyson is simple. In Tanimura's notation:

*Given*

A single particle trajectory $x(t)$ in terms of three mutually perpendicular coordinates $x_i(t)$, $i, j, k \in 1, 2, 3$ subject to the constraints

$$[x_i, x_j] = 0; \; m[x_i, \dot{x}_j] = i\hbar\delta_{ij}; \; m\ddot{x}_k = F_k(x, \dot{x}; t) \tag{3.1}$$

*then*

the force components $F_k(x,\dot{x};t)$ can be expressed in terms of two functions $E(x,t)$, $B(x,t)$ which depend only the coordinate components $x_i$ and the time $t$ and not on the velocity components $\dot{x}_j$; these functions are related to the force by the component equations

$$F_i(x,\dot{x};t) = E_i(x,t) + \epsilon_{ijk} < \dot{x}_j B_k(x,t) > \tag{3.2}$$

and $E, B$ satisfy the equations

$$div\ B = 0;\ \partial B/\partial t + rot\ E = 0 \tag{3.3}$$

Here the Weyl ordering $<>$ is defined by

$$< ab > \equiv \frac{1}{2}[ab+ba];\ < abc > \equiv \frac{1}{6}[abc+bca+cab+acb+cba+bac],\ etc. \tag{3.4}$$

## 3.2 SCALE INVARIANT POSTULATES

The postulates can be made even simpler once one invokes scale invariance. The Feynman postulates are independent of or linear in $m$. Therefore they can be replaced by the *scale invariant* postulates

$$f_k(x,\dot{x};t) = \ddot{x}_k;\ [x_i,x_j] = 0;\ [x_i,\dot{x}_j] = \kappa\delta_{ij} \tag{3.5}$$

where $\kappa$ is any fixed constant with dimensions of area over time $[L^2/T]$ and $f_k$ has the dimensions of acceleration $[L/T^2]$. Keeping these postulates consistent with the scale parameter $c$ as the limiting velocity for information transfer can clearly be done without breaking scale invariance. This removes the apparent paradox noted by Dyson (Ref. 4) of being able to derive Lorentz invariant equations from the Galilean invariant non-relativistic commutation relations.



The remaining physical point that needs to be made clear is that the "fields" referred to in classical relativistic field theory are *defined* in terms of their action on a *single* test particle, as we did in relating $\mathcal{H}$ to 4-velocity and radius of curvature in our geometrical paradigm. Thus, if we measure the *acceleration* of that particle in a Lorentz invariant way (force per unit rest mass) *and* the force per unit charge is also defined by acceleration *and* the charge per unit rest mass of the test particle is *also* a Lorentz invariant *then* our electromagnetic field theory itself becomes an LT scale invariant theory. That is, once we replace the Feynman postulates by (3.5) and define $\mathcal{E}(x,t) = E/Q = F_E/m$ and $\mathcal{B}(x,t) = B/Q = F_B/m$, we need only derive the scale invariant version of equations (3.2), (3.3) obtained by the obvious notational change $F_i \rightarrow f_i$, $E_i \rightarrow \mathcal{E}_i$, $B_i \rightarrow \mathcal{B}_i$. Extension to gravitation makes more use of the concept of path and requires that the ratio of gravitational to inertial mass of the test particle is also a Lorentz invariant.

As noted in the introduction, Clive Kilmister objected strongly at ANPA 15 to the thesis presented in this paper. I hope to renew the discussion at ANPA 16. But first I will need him to separate his criticism of Tanimura's proof from his criticism of my generalization. So far as I know, no one has faulted the formal steps in Tanimura's proof. It follows that my scale-invariant proof is just as valid as Tanimura's in a formal sense. I assume that the acceptance of Tanimura's paper by *Annals of Physics* requires Clive to take up other issues about that proof with the author or the journal rather than with me. Otherwise, it appears that Clive must either object to postulating the commutation relations of non-relativistic quantum mechanics *or* the simple algebraic steps I have taken above to make them scale invariant and consistent with the limiting velocity of special relativity *or* to my standard use of the classical *definition* of field as the acceleration of a test particle caused by that field.

## 3.3 MEASUREMENT ACCURACY AND SCALE INVARIANT BRACKET EXPRESSIONS

Where Clive has valid objections is with regard to my various attempts to establish scale invariant bracket expressions directly from operational arguments. If he is willing to accept McGoveran's ***ordering operator calculus.*** — which I was under the impression was the case after sitting in on two day-long sessions between Clive and David McGoveran in two separate years — a rigorous derivation of scale invariant commutation relations from finiteness and discreteness already exists. The proof starts by deriving the transport operator and establishes the Lorentz transformations before going on to the bracket expressions.[9] Quantum mechanics often is claimed to view measurement accuracy restrictions as the cause or consequence of the commutation relations. This was extensively discussed by Heisenberg, Bohr and Einstein, so I thought I was just spelling out something that is usually taken for granted. I review briefly here what is needed for the Tanimura proof and try to give it heuristic support.

Replacing $\dot{x}_j$ by $v_j$, to remove the implication that we are talking about derivatives rather than finite differences, the essential equation we need to establish is

$$[x_i, v_j] = \kappa\delta_{ij} \tag{3.6}$$

where $\kappa$ is some constant proportional to $\Delta\ell^2/\Delta t$. Here our finite measurement accuracy assumption takes the form that any distance measurement $L$ can always be represented by a finite integer (which is less in magnitude than some maximum integer $N$ picked in advance) times the shortest distance $\Delta\ell$ which can be measured, directly or indirectly, using currently available technology. Since we impose the scale invariant restriction $\Delta\ell = c\Delta t$, all times are also integers in units of $\Delta t$.

We take as our paradigm for a quasi-local measurement of position $x$ and velocity $v_x$ a counter telescope whose entrance counter is a distance $x_1 = ct_1 = cn_1\Delta t$ from a reference clock and mirror and whose exit counter is a distance



$x_2 = ct_2 = cn_2\Delta t$ from the same referents. Let the time interval between the two sequential firings of the counter be $T = (n_1 + n_2)\Delta t$. Clearly, the length of the telescope is $L = x_2 - x_1 = (n_2 - n_1)\Delta\ell$. Hence the velocity of the particle causing the two firings is

$$v_x = \frac{L}{T} = \frac{n_2 - n_1}{n_1 + n_2}c \tag{3.7}$$

Note that the two clocks have been synchronized by the Einstein convention so that a light signal sent toward the mirror when the entrance counter fires and reflected back will arrive at the second counter in coincidence with the second firing.

Our paradigm gives our velocity measurement a Lorentz invariant significance. But this still does *not* allow us to assign an absolute meaning to both $x$ and $v_x$. How ever we use additional measurements to further localize $x$ within the interval $x_2 - x_1$, we cannot know its value to better than $\Delta\ell$. The best we can do is to assign it to some position which is ambiguous between $x_- \in [x_1, x_1 + \Delta\ell, x_1 + 2\Delta\ell, ...x_2 - \Delta\ell]$ and $x_+ = x_- + \Delta\ell$. Between these two locations the velocity is, as measured locally, $+c$. Thus, using only quasi-local information, the product "$xv_x$" is ambiguous depending on whether we use $x_-$ or $x_+$. Defining the difference as the bracket expression, and using $c$ for $v_x$, we have that, for local measurements, the minimum uncertainty is given by

$$[x, v_x] \equiv x_+ v_x(t) - x_- v_x(t) = c\Delta\ell \equiv -[v_x, x] \tag{3.8}$$

If we wish to include finite rotations as well as finite velocity measurements, it is convenient to define $\kappa$ as $c\Delta\ell/2\pi$ rather than $c\Delta\ell$, but we will not discuss this refinement here, as it is not needed directly in the Tanimura proof. What is needed is the assumption that we can measure position coordinates independently in three directions (within our integer restrictions), and hence that the ambiguity between position and velocity measurements along any one of these directions does not couple directly to these other directions. We can then claim that, at least heuristically, our counter telescope paradigm allows us to give relativistically invariant significance to the two basic equations

$$[x_i, x_j] = 0; \quad [x_i, v_j] = \kappa\delta_{i,j} \tag{3.9}$$

## 3.4 ADDITIONAL ALGEBRAIC RELATIONS

Subject to our requirement of not going beyond the finite limits to which our measurements can refer, the fact that the $x_i$ can be represented by integers, and the $v_j$ by rational fractions, allows us to assume that, for $\lambda, \mu$ *constants* subject to the same restrictions and $A, B, C \in x_i, v_j$, where $i, j \in 1, 2, 3$, the bracket expression has the properties

$$[\lambda A + \mu B, C] = \lambda[A, C] + \mu[B, C]$$

$$[A, \lambda B + \mu C] = \lambda[A, B] + \mu[A, C] \tag{3.10}$$

$$[A, \mu] = 0$$

The Tanimura proof refers to functions $g(x,t)$ which are not functions of $v_j$ and accelerations $a_k(x, v; t)$ which are functions of velocity as well as position, but of no "higher derivatives". Since these are also subject to our finite integer and rational fraction restrictions, we can assume that they are polynomials whose powers have context sensitive restrictions. If $n$ is the highest power of $x$ which is allowed to occur, then for any component

$$[v_i, x_j^n] = \delta_{ij}[v_i, x_i^n] = \delta_{ij}(-\kappa x_i^{n-1} + x_i[v_i, x_i^{n-1}]) = -n\kappa x_i^{n-1}\delta_{ij} \tag{3.11}$$

This allows us to specify the usual symbol $\partial g(x,t)/\partial x_k$ for all such functions we



consider by the equality

$$[v_k, g(x,t)] = -\kappa\partial g/\partial x_k \tag{3.12}$$

Note also that

$$[x_i, g(x,t)] = 0 \Rightarrow g(x,t) \; independent \; of \; v_i \tag{3.13}$$

It remains to define the symbols $[v_i, v_j]$ and $a_k(x, v; t)$ in our context. Since (within the restriction to polynomials mentioned above) we are now talking about *functions* of $x, v$, and $t$, we can introduce the concept of a *path*

$$x(t) = (x_i(t), x_j(t), x_k(t); t) \tag{3.14}$$

for the single particle we are considering. Then the bracket expression we derived above is equivalent to the definitions

$$x_i(t + \Delta t) \equiv x_i(t) + v_i(t)\Delta t$$

$$[x_i, v_j] \equiv [x_i(t + \Delta t)v_j(t) - v_i(t + \Delta t)x_j(t)] \tag{3.15}$$

$$= [x_i v_j(t) - x_j v_i(t)] \equiv \kappa\delta_{ij}$$

Taking the obvious step of saying that if time changes by $\Delta t$, then

$$v_j(t + \Delta t) \equiv v_j(t) + a_j\Delta t \tag{3.16}$$

and defining

$$[v_i, v_j] \equiv v_i(t + \Delta t)v_j(t) - v_j(t + \Delta t)v_i(t)$$

we have that

$$[v_i, v_j] + [x_i, a_j] = 0 \tag{3.17}$$

Now that we know what we mean by $[v_i, v_j]$ it is straightforward to establish the Jacobi identity

$$[A,[B,C]]+[B,[C,A]]+[C,[A,B]]=0 \tag{3.18}$$

for the symbols $A, B, C \in x_i, v_j$. The same type of argument makes it easy to establish the fact that, in our context

$$g_k(x,t) = \kappa^{-1}\epsilon_{ijk}[v_i, v_j] \Rightarrow$$

$$\partial g_k/\partial t + [v_j, \partial g_k/\partial x_j] = \kappa^{-1}\epsilon_{klm}[v_l, a_m] \tag{3.19}$$

It is then straightforward to follow through the steps of the Tanimura proof, and his generalizations, as formal algebraic derivations.

## 4. DETERMINISM FROM DECOHERENCE

We summarize the results of Chapter 3 by the conclusion that we can always attribute the acceleration of a single test particle to the classical (electromagnetic and gravitational) fields provided only that:

*a) Newton's second law holds in the sense that the acceleration of a single particle is a function only of position, velocity and time.*

*b)* $c \equiv 299\ 792\ 458\ m\ sec^{-1}$ *is the limiting velocity for information transfer.*

*c) Kepler's second law holds in the finite and discrete sense that the line from a center to a particle moving with constant velocity past that center sweeps out an area per unit time which is an integer times some appropriate constant* $\kappa$. [This point is discussed more fully in Ref. 6 and elsewhere[10,11] .]

*d) The shortest length interval we can measure is* $\Delta\ell$ *and the smallest time interval we can measure is* $\Delta t$.



*e) Our length and time units are subject to the scale invariant constraints*

$$\frac{\Delta\ell}{c\Delta t} = 1; \quad \frac{\Delta\ell^2}{\kappa\Delta t} = 2\pi$$

*f) The charge per unit mass for the test particle whose acceleration defines the electromagnetic field and the ratio of gravitational to inertial mass for the test particle whose acceleration defines the gravitational field are both invariant under the finite and discrete Lorentz boosts and rotations our finite measurement accuracy allows us to specify.*

Although our derivation of the classical field equations and the Lorentz force law uses finite differences, the temptation is almost irresistible to go to the continuum limit and interpret the result as a local, deterministic theory embedded in the continuum space-time of special relativity, or for gravitation to go on to the curved space interpretation. Even in nineteenth century physics this step has its problems. Given the field, the trajectory of the test-particle is determined, or given the trajectory, the field emitted by the test particle is determined. And given a free space field distribution, the propagation of this field forward in time can be deterministically computed (if one accepts continuum mathematics as valid). But this works only when (a) the stability of the test particle is assumed from *outside* the classical theory; (b) the reaction of the field produced by the test particle back on the particle is ignored (This "self-energy" is infinite and classically cannot be renormalized.); (c) for two or more particles the reaction of the field generated by the second particle back on the first is ignored; ... and so on. In other words, the theory is locally deterministic only if sources and sinks of the radiation are treated as incoherent. In this sense, we argue that the classical determinism is, even in its own terms, a decoherent *approximation.*

Wheeler and Feynman attempted to meet this problem back in the 1940's by making the sources and the sinks of all fields coherent and replacing the field by relativistic action-at-a-distance. I intend, on another occasion, to see if their theory could be reconstructed from the finite and discrete starting point used in this paper. But even without this detailed development, I trust I have made it clear that classical *local* determinism always implicitly assumes *decoherence.*

## 5. CHAOS FROM DETERMINISM

A great deal of attention is now being paid to the numerical solution of classical, deterministic equations of various types. It turns out that the solutions of these equations in almost all cases are chaotically unstable in the sense that even when the trajectory in phase space is bounded for a while, successive iterations of the equations produce trajectories whose departure from the early motion grows exponentially and can be characterized by a Liapunov exponent greater than unity. We can take this behavior as one definition of what we mean by *chaos.* The equations for classical fields we have derived from measurement accuracy are no exception to this generalization.

A mathematically well studied example is the restricted three body problem in which the test (or "third" ) particle has negligible mass. We consider the case in which the third particle is launched along the axis through the center of mass perpendicular to the plane containing two massive particles rotating under Newtonian gravity about that center. The position on that axis and the velocity along that axis which initiates the motion are specified as the initial "boundary condition". Then (I quote[12])

"...In particular, the following remarkable theorem can be proved. Let $t_1, t_2$... be the times at which the particle intersects the plane of motion of the other two particles. Let $s_k$ be the largest integer equal to or less than the difference between $t_{k+1}$ and $t_k$ times a constant. [The constant is the reciprocal of the period of the motion of the two particles in the plane.] Variation in the $s_k$'s obviously measures the irregularity in the periodic motion. The theorem, due to the Russian mathematicians Sitkinov[13] and Alekseev[14] as formulated in Moser,[15] is this.

*THEOREM 5. Given that the eccentricity of the elliptic orbits is positive but not too large, there exists an integer. say $\alpha$, such that any infinite sequence of terms*



*$s_k$ with $s_k \geq \alpha$, corresponds to a solution of the deterministic differential equation governing the motion of the third particle."*

with the corrolary

"*CORROLARY. Any random sequence of heads and tails corresponds to a solution of the deterministic differential equation governing the motion of the third particle.*"

In other words, if such a system could be constructed, it would provide a perfect random number generator!

The fact is that practically all solutions of classical, "deterministic" equations are chaotic in the sense that one must supply as much information in the boundary conditions as the "prediction" is supposed to yield. This clearly vitiates the concept of "determinism" as usually employed when physics is invoked to support the philosophical concept that goes by that name. The situation is sometimes summarized by the paradoxical phrase "chaotic determinism". However, if our analysis is correct, the physical situation is somewhat subtler than this. The deterministic equations for the motion of a test particle were derived initially by postulating a finite measurement accuracy. Hence, boundary conditions which violate this assumption because of their precision are *inconsistent* with the unrealizable precision required to obtain chaotic predictions. This dissolves the phenomenon of "chaotic determinism" into an artifact produced by assuming precise classical equations *and* precise boundary conditions which ignore the physical necessity of measuring particle positions and velocities.

We emphasize that our argument up to this point invokes only "classical" physics and measurement accuracy. Although we make use of a small number of algebraic relations which would not have occurred to us had we not been guided by experience with quantum mechanics [and bit-string physics!], *no* use is made of operators, Hilbert space, Planck's constant, *or* of any of the physical phenomena that led to the development of quantum mechanics. *or* of any of the conceptual baggage that the historical process which led to the construction of quantum mechanics brought in its train. Fixed measurement accuracy is enough to make "classical physics" non-commutative — *viz* finite rotations of a rigid body — independent of any quantum considerations.

We are not alone in seeing the relevance to the foundations of quantum mechanics of recent developments in *classical* physics. For example Macauley[16] remarks that

"...Born and Heisenberg argued strongly that physics should not be based upon nonobservable concepts— because of this , Max Born argued for the elimination of the continuum concept from physics. By restricting to computable numbers in classical dynamics, we take a small step in that direction. It means that formal Hilbert space theory cannot be the final foundation for quantum mechanics, because Hilbert space is built on the generalization to function space of the idea of the continuum, the completeness of the real number system (a space is complete when all the limits of all the convergent sequences in the space also belong to the space). But this introduces noncomputability into the foundations of quantum mechanics, because almost all functions that can be defined are noncomputable (see Turing, 1937)."

## 6. CONCLUSIONS

We claim by now to have made our case that modern work on fractals and chaos theory has already removed the presumption that classical physics is "deterministic". Further, we claim that in so far as classical relativistic field theory (i.e. electromagnetism and gravitation) are scale invariant, they are self-consistent only if the idea of "test-particle" is introduced from *outside* the theory. Einstein spent the last years of his life trying to use singularities in the metric as "particles" or to get them out ofthe non-lineaqrities in a grand unified theory — in vain. So classical physics in this sense *cannot* be the fundamental theory. However, we claim to have shown that if we introduce a "scale invariance bounded from below" by measurement accuracy, then Tanimura's generalization of the Feynman proof



as reconstructed by Dyson allows us to make a *consistent* classical theory for *decoherent* sources and sinks. Restoring coherence to classical physics via relativistic action-at-a distance is left as a task for the future.

*Relativistic* quantum mechanics, properly reconstructed from a finite and discrete basis, emerges in much better shape. The concept of "particles" has to be replaced by NO-YES particulate events, and particle-antiparticle pair creation and annihilation properly formulated. Much of the necessary work has, as I have claimed for some time, already been accomplished by McGoveran and myself. The breaking of scale invariance at half the electron Compton wavelength tells us where classical coherence and decoherence have to give way to quantum coherence and decoherence. The transition is smooth from the point of view of measurement accuracy. Of course much still needs to be done, but that is what ANPA is all about.

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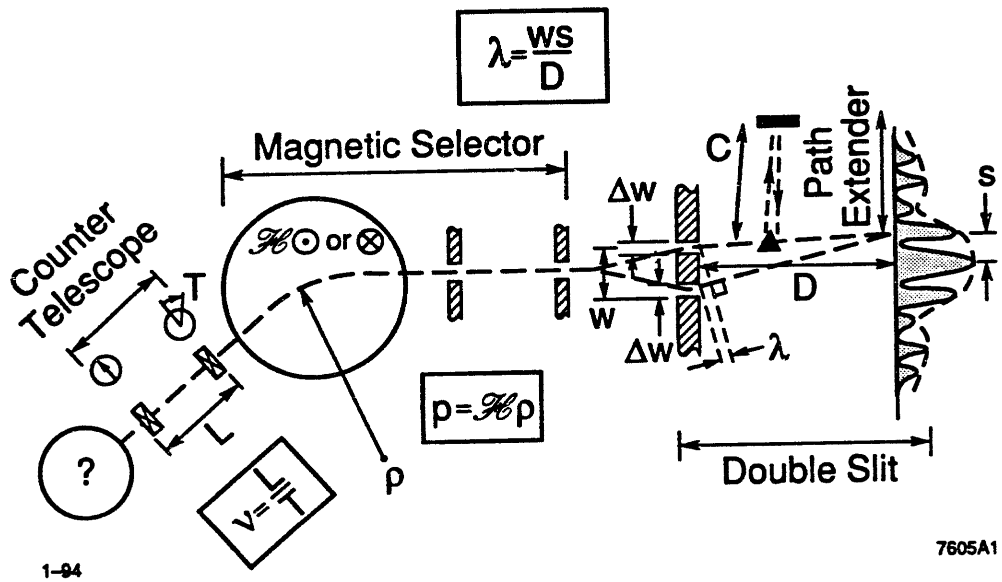
## FIGURE CAPTIONS

1) Measurement of coherence and decoherence of de Broglie waves using a counter telescope, magnetic selector, and a double slit with a path extender in one arm.



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