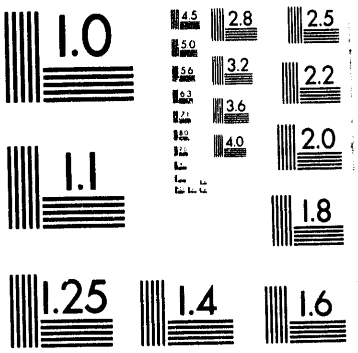
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RHIC Chromatic Correction System *

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Abstract: The chromaticity correction system, including the nonlinear correction, for the Relativistic Heavy Ion Collider (RHIC) is presented. Expected multipoles in the superconducting magnets have shown that the octupole and decapole might be large enough to reduce the momentum aperture and introduce undesirable nonlinear chromatic behavior of the machine. Simulations of these conditions have been performed with the accelerator physics tracking code TEAPOT[1]. The chromatic dependence curves were obtained by the least square fitting. A correction to the first and the second order terms were applied by using two sextupole and two octupole circuits. The decapole correction system has been applied to correct for the third order dependence on momentum. The long term tracking studies at injection did not include the decapole correction. The studies showed that the octupole correction system significantly improves the dynamical aperture at the injection. The decapole system would not be necessary at commissioning of the machine but the correction magnets will be available. At the top energy, as to be expected, the low beta quadrupoles are the dominant source of the nonlinear momentum dependence.

1. Introduction

Chromaticity could be defined as "a change of the linear optics parameters with the beam energy-momentum"(2). The linear correction is performed with a standard two sextupole circuits while the higher order correction consists of the octupole and decapole families. The horizontal and vertical chromaticities are defined as the linear tune dependence on momentum ($\delta=\Delta p/p=(p-p_o)/p$) where p_o is the momentum of particles with the designed energy E_o:

$$\xi_x = \partial\nu_x/\partial\delta \,|_{|\delta=0}, \text{ and } \xi_y = \partial\nu_y/\partial\delta \,|_{|\delta=0}. \quad ...(1)$$

From the equation of motion for the off momentum particles the betatron tune dependence on momentum can be presented as:

$$\nu_{x,y}(\delta) = \mathbf{a} + \mathbf{b}\,\delta + \mathbf{c}\,\delta^2 + \mathbf{d}\,\delta^3, \quad ...(2)$$

where **a**, **b**, **c**, and **d** could be treated as constants.

2. The Linear Chromaticity Correction

The linear chromaticity correction is a standard two circuit correction. Twenty four chromaticity sextupoles are placed at the focussing and defocusing quadrupoles within each arc, where the dispersion values are: $D_{xf}\approx 1.9$ m and $D_{xd}\approx 0.9$ m, respectively. The strengths of the sextupoles to correct for the required chromaticity at the injection and at the storage lattice with the full energy are:

at injection: $\kappa_{2f} = B''L/B\rho = 0.11\ m^{-2}$ at storage: $\kappa_{2f} = 0.20\ m^{-2}$
$\kappa_{2d} = B''L/B\rho = -0.21\ m^{-2}$ $\kappa_{2d} = -0.37\ m^{-2}$

The transfer functions of the production sextupoles were measured to be **S** =8.959 T/m/A and **S** =8.305 T/m/A, for the excitation currents of I=29.64A and I=49.35A, respectively.

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The maximum required current through the sextupoles at the top storage energy was measured **I=43 A** (with different random seeds and with all possible errors included).

The second order tune shifts induced by the sextupoles at the operating horizontal and vertical tunes, ν_x= 28.19 and ν_y= 29.18, respectively, are:

$$\partial\nu_x/\partial\varepsilon_x = -380.0, \quad \partial\nu_y/\partial\varepsilon_y = 130.0, \quad \partial\nu_y/\partial\varepsilon_x = -1200.0$$

which makes the maximum tune shift equal to $\Delta\nu_y$=0.000265 for normalized emittance of $\varepsilon_x=\varepsilon_y=6\pi\sigma^2\beta\gamma/\beta_{Twiss}=20\pi$ mm mrad, where $\beta\gamma$ is the relativistic factor, while the β_{twiss} is the betatron function (the real beam size at the β_x=50 m and at the top proton energy $\beta\gamma$=268.2 is σ_x=0.788 mm while the real emittances are $\varepsilon_x=\varepsilon_y$=0.0746 π mm mrad).

3. Second and Third Order Chromatic Correction

The tune shift on momentum and amplitude due to systematic multipoles of the higher orders could be presented[3] as:

$$\delta\nu_x=\Theta_d/2\pi*\mathbf{b_3}[3/2\beta_x\eta^2\,\delta^2+3/8\beta_x a_x^2-3/4\beta_x a_y^2]+\Theta_d/2\pi*\mathbf{b_4}[2\beta_x\eta^3\,\delta^3+3/2\beta_x\eta a_x^2\,\delta-3\beta_x\eta a_y^2\,\delta] \quad ..(3)$$

$$\delta\nu_y=\Theta_d/2\pi*\mathbf{b_3}[-3/2\beta_y\eta^2\,\delta^2-3/4\beta_y a_x^2+3/8\beta_y a_y^2]+\Theta_d/2\pi*\mathbf{b_4}[-2\beta_y\eta^3\,\delta^2-3\beta_y\eta a_x^2\,\delta-3/2\beta_y\eta a_y^2\,\delta] \quad ...(4)$$

where $\Theta_d=B\,\mathbf{l_d}/B\rho$ is in the case of the dipoles the bending angle of the dipole, while in the case of the correction elements octupole and decapole the products of $B_3=\Theta_d*\mathbf{b_3}$ and $B_4=\Theta_d*\mathbf{b_4}$ represent the strength of the octupole and decapole elements, respectively where $\Theta_d*\mathbf{b_3} = B^{III}$ **L**/Bρ and $\Theta_d*\mathbf{b_4} = B^{IV}$**L**/B$\rho$, and **L** is the length of the correction element. The horizontal dispersion is presented by η, the horizontal and vertical amplitudes of the betatron motion are $a_x=\sqrt{\beta_y\varepsilon_x}$ and $a_y=\sqrt{\beta_y\varepsilon_y}$,respectively, while β_x and β_y are the betatron functions.

The magnetic field is presented by:

$$\mathbf{B} = B_y + i\,B_x = B_o\,\Sigma\,(\mathbf{b_n} + i\mathbf{a_n})(x + iy)^n \quad ...(5)$$

The systematic multipoles at BNL, as b_3 and b_4, are defined in "units", which represents 10^{-4} part of the main dipole field at the 3/4 of the coil radius $\mathbf{R_o}$. The magnetic field measured along the horizontal axis in the middle plane is presented as:

$$B_y(x)= B_o[1 + b_1(x/\mathbf{R_o})+ b_2(x/\mathbf{R_o})^2+ b_3(x/\mathbf{R_o})^3+ b_4(x/\mathbf{R_o})^4+\ldots] \quad ...(6)$$

The 3/4 of the coil radius for the dipoles and correctors is equal to $\mathbf{R_o}$=2.5 cm. A change of the vertical magnetic field, with one unit of the octupole multipole ($\mathbf{b_3}$=1) within the dipole is equal to: $\Delta B_y = B_o*\,\mathbf{b_3}\,(x/\mathbf{R_o})^3$, or $\Delta B_y=1*10^{-4}*B_o$, for $x=\mathbf{R_o}$, while the horizontal kick at the horizontal offset "x" produced by this field is equal to: $\Delta\theta=\Delta x'=(B_o*\mathbf{b_3}*\mathbf{l_d}/B\rho)*(x/\mathbf{R_o})^3$, where $\mathbf{l_d}$ is the length of the dipole, while Bρ is the magnetic rigidity.

3.1 Octupole Correction Circuits

The betatron functions at the corrector's positions are: $\beta_{max}\approx$46.0 m, $\beta_{min}\approx$11.0 m, $\beta_{averg.}\approx$22.0 m, and $\eta_{max}\approx$1.74 m, $\eta_{min}\approx$0.76 m, $\eta_{averag.}\approx$1.10 m. The equations (3) and (4) provide a solution for the strength of the octupole and decapole correction elements. The first order chromaticity was corrected with the chromaticity sextupoles to the zero value. The vertical and horizontal tune shifts $\Delta\nu_{x,y}(\delta)$ were measured with the TEAPOT for different values of the momentum offsets δ (five values were used: δ=0, $\pm\sigma_p$, and $\pm 2\sigma_p$ where for the injection the σ_p=0.043%). The tune dependence on momentum was measured under three different conditions:

-both octupole circuits - octupoles at the focussing quadrupoles $\mathbf{B_{3F}}$ as well as the octupoles at the defocusing quadrupoles $\mathbf{B_{3D}}$ - were turned off,

-only the $\mathbf{B_{3F}}$ octupoles were turned on and

-only the $\mathbf{B_{3D}}$ octupoles were turned on.

The least square fitting routine produced three different tune versus momentum dependences. The results of the summation and of the difference of the tune shifts:

$$\Delta\nu^s_{x,y}(\delta) = \partial\nu_{x,y}(+\delta) + \partial\nu_{x,y}(-\delta) \quad \text{and} \quad \Delta\nu^d_{x,y}(\delta) = \partial\nu_{x,y}(+\delta) - \partial\nu_{x,y}(-\delta),$$

provide the octupole $\mathbf{O_{XF}}$, $\mathbf{O_{XD}}$, $\mathbf{O_{YF}}$, and $\mathbf{O_{YD}}$ and decapole $\mathbf{O_{XF}}$, $\mathbf{O_{XD}}$, $\mathbf{O_{YF}}$, and $\mathbf{O_{YD}}$ matrix elements, respectively. The horizontal and vertical tune shifts induced by the octupole correction elements can be presented with a matrix elements as:

$$\begin{pmatrix} \Delta\nu_x/2\delta^2 \\ \Delta\nu_y/2\delta^2 \end{pmatrix} = \begin{pmatrix} O_{XF} & O_{XD} \\ O_{YF} & O_{YD} \end{pmatrix} * \begin{pmatrix} N_F B_{3F} \\ N_D B_{3D} \end{pmatrix}$$

where the coefficients of the matrix $\mathbf{O}$ are calculated from the average values of the lattice parameters as:

$$\mathbf{O_{XF}} = 3/4\pi\, \beta_{max}\eta^2_{max} = 33.25\ \mathrm{m}^3, \quad \mathbf{O_{XD}} = 3/4\pi\, \beta_{min}\eta^2_{min} = 1.52\ \mathrm{m}^3$$
$$\mathbf{O_{YF}} = -3/4\pi\, \beta_{min}\eta^2_{max} = -7.95\ \mathrm{m}^3, \quad \mathbf{O_{YD}} = -3/4\pi\beta_{max}\, \eta^2_{min} = -6.34\ \mathrm{m}^3$$

with a value of the determinant of $\mathbf{D_s}$ = -198.72. The strength of the octupole correction element was calculated at the nominal value of the current through the coils of $\mathbf{I}$=50A is ($\mathbf{B_{3F}=BL/B\rho\ *1/R_0^3}$) and at the top energy with the magnetic rigidity of $\mathbf{B\rho}$=839.2 Tm. The octupole correction element strength and the systematic octupole strength of one unit is equal, to $\mathbf{B_{3F}}$=0.700 m $^{-3}$ and $\mathbf{b_3}$= 1* 10^{-4} * 1/(0.025)^3 = 6.4 m^{-3}, respectively. The results for the octupole strength were obtained from the reported(4) octupole measurements. The magnetic field of $\mathbf{B}$=0.0164 T was measured at radius @$\mathbf{R_0}$=0.025 m, with a current through the coils of $\mathbf{I}$=51 A. The effective length was $\mathbf{L}$=0.571 m.

With forty-two (N_F=N_D=42) correctors it is possible to cancel the average systematic octupole in the dipoles of: $\mathbf{b_3}$ =0.8 units. The expected systematic octupole in the dipoles was reported(3) to be at the top energy equal to $\mathbf{b_3}$ = 0.24 units.

The octupole and decapole correction elements are placed in the interaction regions according to figure 1.

3.2 Decapole Correction Circuits

The tune third order dependence on momentum is corrected with the decapole correction elements. There are two circuits of the decapole elements, $\mathbf{B_{4D}}$ and $\mathbf{B_{4F}}$, as presented in figure 1 with the total number of 42 elements. The tune horizontal and vertical tune shifts on momentum were predicted from the lattice parameters and calculated with the TEAPOT. The matrix elements calculated from the lattice parameters for the decapole tune shift are:

$$\mathbf{D_{XF}} = 1/\pi\, \beta_{max}\eta^3_{max} = 77.14\ \mathrm{m}^4, \quad \mathbf{D_{XD}} = 1/\pi\, \beta_{min}\eta^3_{min} = 1.54\ \mathrm{m}^4$$
$$\mathbf{D_{YF}} = -1/\pi\, \beta_{min}\eta^3_{max} = -18.45\ \mathrm{m}^4, \quad \mathbf{D_{YD}} = -1/\pi\, \beta_{max}\eta^3_{min} = -6.43\ \mathrm{m}^4$$

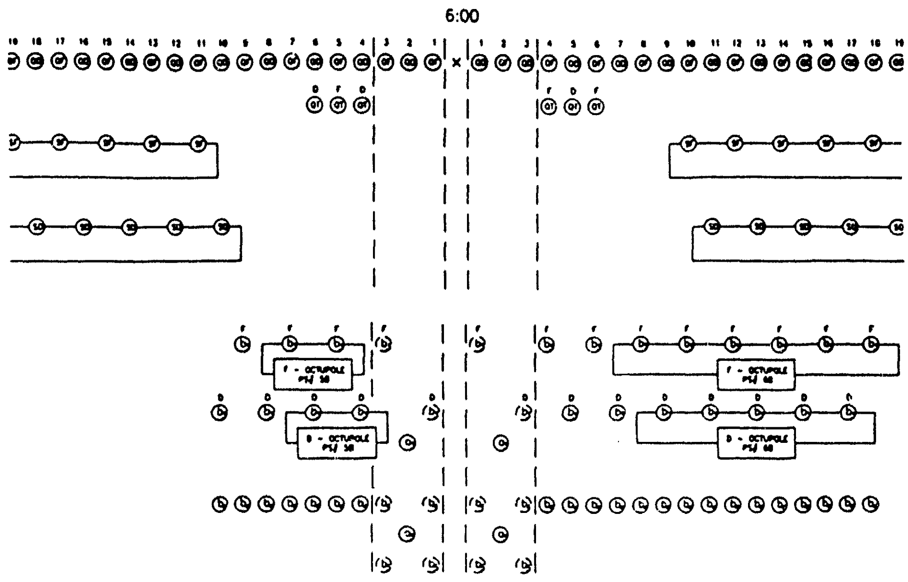

Figure 1. Octupole and decapole correction elements inside the interaction region.

The decapole tune shifts correction was simulated with the TEAPOT. The tune dependence on momentum was obtained by measuring a difference of the tune shifts:

$$\Delta\nu^{\mathbf{d}}_{x,y}(\delta) = \partial\nu_{x,y}(+\delta) - \partial\nu_{x,y}(-\delta) = 2\,\mathbf{d}\,\delta^3, \quad \text{as } \mathbf{b} = 0.$$

Again three tune dependences on momentum were obtained by the least square fitting of the particles with different momenta: $\delta = 0, \pm\sigma_p, \pm 2\sigma_p$, where $\sigma_p = 0.044\ \%$ at injection and $\sigma_p = 0.089\%$ at the top storage energy. The first dependence was obtained when both decapole circuits were turned off, the second dependence when only the decapoles close to the focussing quadrupoles when turned on, and the third dependence was obtained when only the decapoles at the defocusing quadrupoles when turned on. The matrix elements for the third order correction of the tune dependence on momentum can be presented as:

$$\begin{pmatrix} \Delta\nu_x / 2\delta^3 \\ \Delta\nu_y / 2\delta^3 \end{pmatrix} = \begin{pmatrix} D_{XF} & D_{XD} \\ D_{YF} & D_{YD} \end{pmatrix} * \begin{pmatrix} N_F \mathbf{B}_{4F} \\ N_D \mathbf{B}_{4D} \end{pmatrix}$$

$$\begin{pmatrix} N_F \mathbf{B}_{4F} \\ N_D \mathbf{B}_{4D} \end{pmatrix} = 1/\mathbf{D_d} \begin{pmatrix} D_{YD} & -\mathbf{D}_{XD} \\ -\mathbf{D}_{YF} & D_{XF} \end{pmatrix} * \begin{pmatrix} \Delta\nu_x / 2\delta^3 \\ \Delta\nu_y / 2\delta^3 \end{pmatrix}$$

$$\begin{pmatrix} N_F \mathbf{B}_{4F} \\ N_D \mathbf{B}_{4D} \end{pmatrix} = \begin{pmatrix} 0.4558 \\ 7.2965 \end{pmatrix}$$

with a value of the determinant of $\mathbf{D_d}$ = -477.245. The strength of the decapoles is calculated at the nominal value of the current through the coils of **I**=50A as $\mathbf{B_{4F}}=\mathbf{BL/B\rho} * \mathbf{1/R_o^4}$, where at the top energy the magnetic rigidity is **B**ρ=839.2 Tm. The decapole correction element strength and the systematic decapole multipole strength of one "unit" in the dipoles are equal to $\mathbf{B_{4F}}=22.89\ m^{-4}$ and $\mathbf{b_4}= 1*10^4*1/(0.025)^4 = 256\ m^{-4}$, respectively The measurements(5) showed the magnetic field of **B**=0.0154 T, at a radius of $\mathbf{R_\sigma}$=0.025 m, with a current through the coils of **I**=59A, where the effective length was equal to **L**=0.575 m. From the above result, with forty-two ($N_F= N_D$=42) elements, it is possible to cancel the systematic decapole in the dipoles of: $|\mathbf{b_4}|$ =0.5 units, while the expected systematic multipole(3) is $\mathbf{b_4}$=-0.29.

4. Conclusion

Dominant sources of the nonlinear behavior in RHIC at injection are the 9.45 m dipoles. At the top energy, with two interaction regions at β^*=1 m, the dominant sources of nonlinearities are the triplet quadrupoles. The chromaticity in RHIC is corrected at injection up to the second order. The linear chromaticity uses two standard sextupole circuits. Two octupole circuits correct the second order tune dependence on momentum. The dynamical aperture studies at injection, with all possible errors in the RHIC magnets included, has showed an average aperture of 11σ. The commissioning of RHIC will not use the decapole correctors although they will be available. At the top energy the octupoles induce to high amplitude tune shift . The average dynamical aperture of the low beta lattice showed an aperture of 5.25 $\sigma_{x,y}$. Additional families of sextupoles are being considered for the nonlinear momentum dependence correction at the top energy.

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