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Correlation, Functional Analysis and Optical Pattern Recognition

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ABSTRACT

Correlation integrals have played a central role in optical pattern recognition. The success of correlation., however, has been limited. What is needed is a mathematical operation more complex than correlation. Suitably complex operations are the functionals defined on the Hilbert space of Lebesgue square integrable functions. Correlation is a linear functional of a parameter. In this paper, we develop a representation of functionals in terms of inner products or equivalently correlation functions. We also discuss the role of functionals in neural networks. Having established a broad relation of correlation to pattern recognition, we discuss the computation of correlation functions using acousto-optics.

1. INTRODUCTION

Pattern recognition is an extremely complex field. At present, a general solution to the problem of recognizing an arbitrary object in an arbitrary background does not exist. In fact, there are very few good solutions to restricted pattern recognition problems. Many approaches to the problem have been suggested and have been or are continuing to be investigated.[1]

The evaluation of a correlation integral is a popular approach to optical pattern recognition.[2] The initial interest in correlation probably stems from the applications of the matched filter in radar signal processing and communications theory. Further, there is a natural affinity between correlation and the Fourier transformation relation in optical systems.[3,4]

Considerable research effort has been directed toward the analysis of the performance of various correlation filters. There has also been significant effort directed at the development of optical correlation systems.

In normalized form, correlation does provide a rudimentary pattern recognition operation in that it measures mathematical "correlation" between objects.[2,4,5,6,7,8] However, one would readily recognize that correlation falls short of the pattern recognition capabilities of the human mind. This clearly implies that we need a mathematical operation more complex than correlation for general pattern recognition. Perhaps, a neural network can provide the required complexity.

One such mathematical operation is called a functional. We define a functional as a numerical (real or complex) function defined on a set of functions. A correlation or inner product is then just a linear functional. In general, a functional is a nonlinear mapping of a function into the real (or complex) numbers. Fortunately, a very broad class of functionals can be evaluated as a sum of products of inner products or correlations. Thus, correlation can be viewed as a fundamental computation in a broad sense.

In the following section, we review normalized correlation and introduce the concept of normalized quadratic filters. The concept of generalized correlation is introduced in terms of functionals in Section 3. Further, since perceptron neural networks are functionals on a vector space, we discuss the relation of correlation to neural networks.

Having argued the broad relation of correlation to pattern recognition, we discuss the implementation of the computation of correlation functions using an acousto-optic correlator in Section 4. Ultimately, the success of optical pattern recognition depends upon the efficiency and efficacy of the optical components and systems.

2. NORMALIZED CORRELATION AND QUADRATIC FILTERS

The pattern recognition problem can be said to be one of discriminating between different classes of objects or signals. Frequently, the problem is one of recognition (detection) of a specific object (signal) in the presence of all other possible objects (signals). An example is the detection of a specific vehicle type in the presence of other vehicle types, terrain features, and various man-made objects. This is a very difficult problem and a general solution may not be achievable. The major problem is one of suitably defining "all other objects." The problem is considerably simplified when the number of objects to be discriminated is small. In the case of a small number of well-defined objects, there may be several relatively simple approaches to the problem.

The interest in correlation filters is, in the main, associated with the problem of recognizing an object in an arbitrary background. The problem is, then, to determine a recognition system that will cause output values for all inputs to be recognized as the object of interest to fall within a specified range. For all other objects, the output values fall outside this range. This may sound simple, but the problem is one of defining all inputs that are equivalent to the object of interest. For example, how big must the tail on the letter "Q" be before it is discriminated from the letter "O"; is a white circle (or other shapes) contained in a white square?

In general, the literature on the technique of using correlation filters for optical pattern recognition assumes or implies that pattern recognition can be effected by computing the magnitude of the correlation between the object in question and a reference (filter) function. However, it is clear that a nonlinear operation is required to accomplish pattern recognition. A naive approach to the nonlinearity is peak detection. A more sophisticated approach is to normalize the correlation function or inner product. Normalization achieves intensity invariance.

The treatment of normalization and intensity invariance in the literature is relatively sparse. Goodman[4] suggested the normalized matched filter as a means of achieving character recognition. Duda and Hart[9] suggested normalization to effect intensity invariance. Dickey and Romero[2,5] have discussed normalized correlation at length. In their papers, they use the normalized form to evaluate partial information filters and composite filters.

The use of correlation filters in pattern recognition is essentially an inner product between two functions: the object and reference functions. These functions may be considered vectors in a Hilbert space.[10] The normalization of the inner product (correlation integral) defines a unique angle between the reference and the object function. It is this angle that provides a measure of similarity between the object and reference functions.

$$c(y-x_o)=\left|\int h^*(x)s_h(x)f(x+y-x_o)dx\right|^2 \tag{1}$$

where

$$f(x)=\text{input (object) function}$$

$$h^*(-x)=\text{filter impulse response}$$

$$s_h(x)=\begin{cases}1, x\in \text{ support of } h(x)\\ 0, \text{ otherwise,}\end{cases}$$

$$x_o=\text{coordinate that maximizes the integral}$$

The indicator function has the properties

$$s_h(x)h(x)=h(x), \tag{2}$$

$$s_h^2(x)=s_h(x). \tag{3}$$

Applying the Cauchy-Schwarz inequality to Eq. (1) gives

$$c(y-x_o)\le\int\left|f(x+y-x_o)\right|^2 s_h(x)dx\int\left|h(x)\right|^2 dx. \tag{4}$$

The preceding suggests a normalized correlation function given by

$$c(y-x_o)=\frac{\left|\int h^*(x)f(x+y-x_o)dx\right|^2}{\int\left|f(x+y-x_o)\right|^2 s_h(x)dx\int\left|h(x)\right|^2 dx}\leq 1. \qquad (5)$$

It is a further property of the Cauchy-Schwarz inequality that the equality in Eq. (5) is obtained at $y = x_0 = 0$ and only if

$$h(x)=\lambda f(x)s_h(x). \qquad (6)$$

For the case $y = x_0$, Eq. (5) is equivalent to

$$\hat{c}(0)=\cos^2(\theta) \qquad (7)$$

where θ is the Hilbert space angle between h (x) and the restriction of f(x) to the support of h(x). It is interesting to note that the normalized inner product has been proposed[11] as a measure of similarity between vectors in a vector approach to automatic text retrieval.

It is the form of Eq. (5) and the λ in Eq. (6) that effects intensity invariance. Thus, it is the classical matched filter associated with white noise that maximizes the normalized correlation given by Eq. (5). If the filter used to identify an object function f(x) is not the matched filter, a normalized correlation value less than one is obtained. In this case, there are many functions different from f(x) that give the same correlation value. The difference (e.g., mean square difference) between these functions and the object function must approach zero as the normalized correlation function approaches one. Examples of normalized correlation are presented in Section 4.

For any filter other than the matched filter, the normalized correlation is less than one when the object function for which the filter was made is the input function. Further, there is always a function that gives a normalized correlation value of one. This function is just the filter impulse response. For an arbitrary filter, there are many functions that produce a correlation value equal to or greater than that produced by the object function. Gheen, Dickey, and DeLaurentis[8] discuss the arbitrary approximation of Bayes classifiers by performing a series of correlations. Javidi, Refregier and Willet[12] describe a constraint under which normalized correlation is optimum.

An extension of the correlation (inner product) approach to pattern recognition is the quadratic form (filter) given by

$$Q = |(\psi, A\psi)|^2. \tag{8}$$

where ψ is the input function restricted to the support of the target and A is a linear operator. Gheen [13, 14] has investigated the invariance properties of quadratic filters.

The quadratic form given by Eq. (8) can be normalized using the Cauchy-Schwarz inequality giving

$$\hat{Q} = \frac{|(\psi, A\psi)|^2}{\|\psi\|^2 \|A\psi\|^2} \le 1, \tag{9}$$

with equality obtained iff

$$A\psi = \lambda\psi. \tag{10}$$

Thus, we can obtain the equality in Eq. (9) for a set of normal image functions ϕ_i, if we define the operator by

$$A\psi = \sum \lambda_i (\phi_i, \psi)\phi_i, \tag{11}$$

or

$$A = \sum \lambda_i (\phi_i, \cdot)\phi_i. \tag{12}$$

If the ϕ_i are also orthogonal and the λ_i are real, we have a Hermitian operator. However, if $\lambda_i = \lambda_j$ for some i, j, we have a multiple eigenvalue. Multiple eigenvalues are not desirable in that all linear combination of ϕ_i for which $\lambda_i = \lambda_j$ satisfy the equality in Eq. (9).

It appears that there is no requirement that the ϕ_i be orthogonal. However, we would want to retain the requirement that $\lambda_i \neq \lambda_j$ for all i, j. Thus, the quadratic filter defined by Eq. (9) and Eq. (12) defines a composite filter system with the major computations consisting of inner products (correlations). There, however, does not appear to be any increase in computational efficiency of the quadratic filter over the corresponding set of correlations. In fact, there is more information available in the set of individual correlations.

3. GENERALIZED CORRELATION AND FUNCTIONALS

Normalized correlation and normalized quadratic filters discussed in the last section are relatively simple functionals. Although these functionals provide a degree of pattern recognition, it is clear (see Reference 8) that considerably more complex functionals are required for general pattern recognition.

In this section, we assume that the ability to compute an arbitrary functional on the input function is sufficient for a large class of pattern recognition problems. A functional is the transformation of a vector space X into the real numbers R,

$$F: X \rightarrow R. \tag{13}$$

The vector space X will typically be a function space. The Hilbert (function) space of Lebesgue square integrable functions (L_2) is an adequate space for modeling input (image) functions.

In the following, we show that arbitrary nonlinear functionals can be computed in terms of sums of products of linear functionals (inner products). That is, we can compute any pattern recognition functional as a sum of products of correlations. We develop this result from two approaches. One approach uses the theory of Volterra functionals. The other approach is based on the theory of perceptron neural networks.

3.1 Volterra Functionals

Volterra functionals (Volterra filters) have been applied to the nonlinear signal processing and nonlinear systems analysis.[15,16,17] Gheen [18] has investigated the invariant properties of Volterra filters for pattern recognition. Volterra[19] introduced regular homogeneous functionals of degree n,

$$F_n[f(x)] = \int\int\cdots\int k_n(x_1, x_2 \cdots x_n) f(x_1) f(x_2) \cdots f(x_n) dx_1 dx_2 \cdots dx_n. \tag{14}$$

Analogous to polynomials, he then defines regular functionals of degree n as

$$G_n[f(x)] = k_o + F_1[f(x)] + F_2[f(x)] + \cdots F_n[f(x)]. \tag{15}$$

The finite sum in Eq. (15) is readily extended to include an infinite number of terms. It is important to determine what class of functionals can be represented by infinite sums of terms given by Eq. (14). For our purposes, the answer is contained in a theorem given by Volterra.

Theorem: Every functional continuous in the field of continuous functions can be represented by

$$G[f(x)] = \lim_{n\to\infty}\Big[k_{n,o} + \sum_{1}^{r_n} \int\int\cdots\int k_{n,p}(x_1, x_2 \cdots x_p)$$

$$f(x_1)f(x_2)\cdots f(x_p)dx_1dx_2\cdots dx_p\Big], \qquad (16)$$

where the functions $k_{n,p}$ (x_1, x_2, ...x_p) are continuous functions.[19]

We only need to consider continuous because continuous functions are dense in L_2. The limit in the theorem can be appreciated by considering the quadratic functional given by

$$H_2[f(x)] = \int f^2(x)dx. \qquad (17)$$

This functional cannot be expressed by a second degree functional of Eq. (14) without the use of distributions.[15] The Dirac delta function allows one to write Eq. (17) as

$$H_2[f(x)] = \int\int \delta(x_1 - x_2)f(x_1)f(x_2)dx_1dx_2. \qquad (18)$$

Palm and Poggio[15] give a theorem corresponding to the above theorem that eliminates the limit process by employing distributions.

In general, we can expand the $k_{n,p}$ functions in terms of one-dimensional basis functions giving

$$k_{n,p}(x_1, x_2 \cdots x_p) = \sum_{i_1}\sum_{i_2}\cdots\sum_{i_p} \alpha_{i_1, i_2 - i_p}\, \psi_{i_1}(x_1) \qquad (19)$$

$$\psi_{i_2}(x_2)\cdots\psi_{i_p}(x_p),$$

where the ψ_i are the appropriate basis functions. If we substitute Eq. (19) in Eq. (16), we obtain an expression for the functional in Eq. (16) as a sum of products of inner products (equivalently: linear functionals, correlations). Clearly, examining Eq. (18) and Eq. (16) leads one to the conclusion that the computation of a general functional would require a large number of correlations.

It should be noted that Eq. (16) may not be the most compact form for expressing an arbitrary functional. For example, normalized correlation given by Eq. (5) is a continuous functional and can be expressed in the form of Eq. (16). Analogous to ordinary functions, functionals can be expressed in a functional Taylor series.[20] The terms in the Taylor series are determined using the Frechet derivative.

3.2 Neural Networks

Perceptron neural networks compute functionals on an input vector. [21-27] Cybenko[21], Hornik, Stinchcomb and White[22], and Hornik[23] show that a perceptron neural network with a single hidden layer can approximate any continuous function with arbitrary precision. DeLaurentis and Dickey[27] show that a network with two hidden layers with nonnegative linear combinations and compositions of excitatory and inhibitory response functions uniformly approximate arbitrary nonnegative continuous functions.

Perceptron neural networks can also be represented as a sum of products of inner products (linear functionals, correlations). For simplicity, we consider single hidden layer networks. These networks compute functions (functionals) given by

$$G(x) = \sum_{i}^{N} \sigma\left[(y_i, x) + b_i\right], \tag{20}$$

where σ is an activation function, x is the input vector, y_i is a vector defining the weights to hidden node i, b_i is a bias and $(\cdot,\cdot)$ denotes an inner product. Based on the Weierstrass approximation theorem, we can uniformly approximate continuous functions with polynomials. Also, continuous functions are dense in L_2. Hence, we can approximate the activation function by a polynomial,

$$\sigma(z) = \sum^{Q} B_p z^p. \tag{21}$$

Substituting Eq. (21) in Eq. (20) gives

$$G(x) = \sum_{i}^{N} \sum_{p}^{Q} B_p \left[(y_i, x) + b_i\right]^p. \tag{22}$$

Thus, we can express arbitrary neural network functionals as sums of products of correlations. As in the previous subsection, it is clear that, in general, a large number of correlations are required. It should be noted that N in Eq. (20) and Eq. (22) and Q in Eq. (21) are determined by the complexity of the functional and the precision required. Generally, without other constraints, the size of a neural network (number of nodes) will be quite large. Attempts to bound neural networks with respect to a given problem have met with little success. Koiran [29] has derived a bound on the number of hidden layer nodes that is an exponential function of the dimension of the input vector.

3.3 Functionals and Pattern Recognition

In this section, we have shown a strong relation between correlation and pattern recognition. This is based on the assumption that the computation of an arbitrary functional should be sufficient for pattern recognition problems. However, these results indicate that the amount of computation required for an arbitrary functional is very large.

It is thus clear that the real problem in optical pattern recognition is in determining what functional to calculate. Once this is done, correlation may play a large role in the computation.

4. ACOUSTO-OPTIC CORRELATOR

In Section 3 we pointed out that large numbers of correlations can be instrumental in expressing arbitrary functionals. A natural concern then arises about the ability to compute large numbers of correlations in a short time, since correlation is computationally intensive. Historically this is where optical correlators have been employed, due to the ability of optical lenses to perform two-dimensional Fourier transformations.[4] (However, we note that digital electronics are advancing to comparable speeds.[30]) In this section we briefly review the method of acousto-optic correlation, which performs space-domain correlation instead of traditional frequency-domain correlation. We also present some examples of normalized correlation, both simulated and actual.

Current real-time frequency-domain optical correlators are unable to produce the full complex values necessary for Fourier-based correlation. Approximate values (partial information filters) are typically substituted for the full complex values. This introduces some error into the final correlation result. We do not wish to debate the implications of such errors for traditional approaches to pattern recognition using a correlation followed by a thresholding operation. Rather, it is of interest to consider the impact such errors would have for the computation of arbitrary functionals via sums of products of correlations as shown in Section 3. In this situation, it is obviously desirable to reduce the error in correlations to a minimum.

Unlike the frequency-domain approach, a space-domain correlation does not require complex values. (Correlation of real functions will only require real values. Correlation of complex functions can be performed by processing real and complex parts separately.) Thus a space-domain correlator can in principle produce correlation results with less error than a frequency-domain correlator, resulting in arbitrary functionals closer to the desired form. We now highlight the fundamentals of our working acousto-optic correlator, which has been detailed elsewhere.[31]

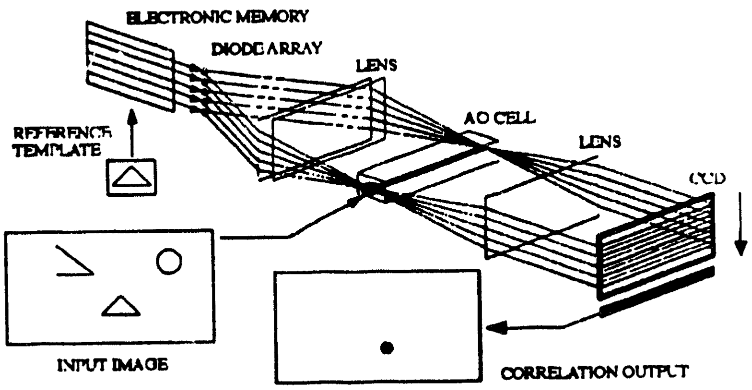

Figure 1 Acousto-optic correlator.

Our acousto-optic correlator consists of a vertical "stack" of multiple one-dimensional time-integrating correlators as shown in Figure 1. This architecture was first proposed by Psaltis.[32] The reference template (or filter) $h(t,n)$ is applied to the diode array while the input image $f(t,n)$ is applied to the acousto-optic (AO) cell. Let t be time, and n be the row number (line number) of the images. Each row of the reference template h is used to time modulate a given input diode, while each row of the input image f is input sequentially into the AO cell. The inputs are synchronized such that each combination of diode output, AO cell, and CCD row forms an individual one-dimensional (along the image rows) time-integrating correlator. To illustrate, assume for the moment we are computing C_1, the correlation of two one-dimensional functions $f(t, n_0)$ and $h(t,n_0)$. Let T_1 be the duration of one line of the input image, and T_2 be the duration of the reference template. The one-dimensional time-integrated correlation result at row n_0 of the CCD would then be

$$C_1(\tau,n_0) = \int_{T_1}^{T_1+T_2} f(t-\tau,n_0)h(t-T_1,n_0)dt, \tag{22}$$

where bias terms and constant multipliers have been ignored. Thus correlation is accomplished in the horizontal dimension, along the CCD rows.

The next step is to compute C, the two dimensional correlation of two-dimensional functions $f(t,n)$ and $h(t,n)$. Using the CCD in time delay and integrate (TDI) mode, with charge shifting in the direction of the vertical arrow in Fig. 1, accomplishes correlation in the vertical dimension. Let M be the number of rows in the input image f. The final result is then given by

$$C(\tau,n) = \sum_{i=0}^{M-1}\left[\int_{T_1}^{T_1+T_2} f(t-\tau,n-i)h(t-T_1,M-i)dt\right], \tag{23}$$

where again bias terms have been neglected.[31] This is the desired two-dimensional correlation result. Equations (22) and (23) illustrate the basic ability of this system to compute correlations.

We now present some results of normalized correlation, partly to illustrate the motivation for using nonlinear functionals. Examination of these results leads to a discussion of the hardware constraints imposed on the computations, which cause roundoff errors when computing the correlations. This will demonstrate the effects of such errors when computing a combination of correlations, such as might be done to compute a nonlinear functional.

The first example of normalized correlation consists of computer simulation results. The correlations were performed using FFTs with double precision floating point arithmetic and single precision storage.[6] Figure 2a consists of four images of the same object at four different intensities. Figure 2b is a plot illustrating the different intensities along a horizontal cut through Figure 2a.

Figure 2a. Tank image at four different intensities.

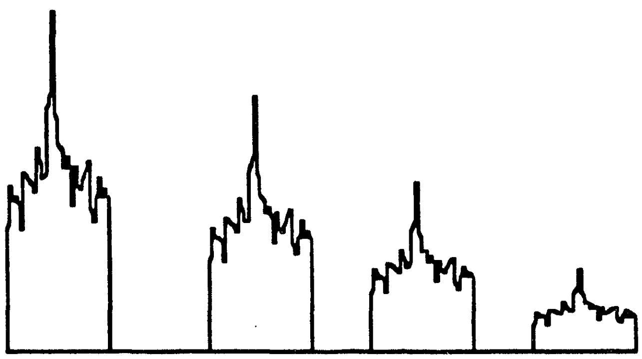

Figure 2b. Tank image intensity profiles.

Figure 2c shows the result of an unnormalized correlation on the image of Figure 2a. The matched spatial filter was used. Figure 2d shows a profile of the correlation result of Figure 2c. As expected, the correlation peaks vary with the image intensity.

Figure 2c. Result of unnormalized correlation.

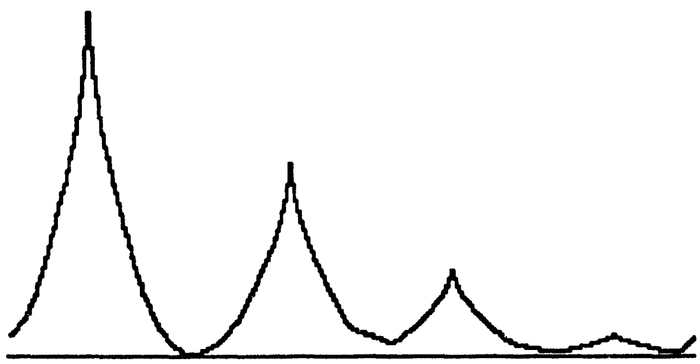

Figure 2d. Unnormalized correlation profile.

Figure 2e shows the result of a normalized correlation on the image of Figure 2a. Again the matched spatial filter was used. As expected, the normalized correlation peaks are equal, as shown in Figure 2f.

Figure 2e. Result of normalized correlation.

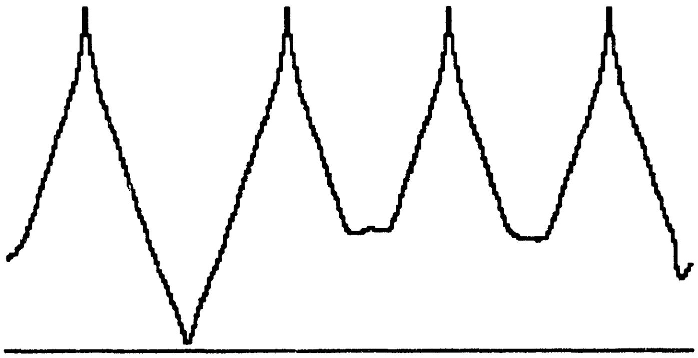

Figure 2f. Normalized correlation profile.

Figure 3a is similar to Fig. 2a, except that two false objects have been inserted into the image. Figure 3b shows the intensity profiles. Figure 3c is the unnormalized correlation result, with the corresponding profile in Figure 3d.

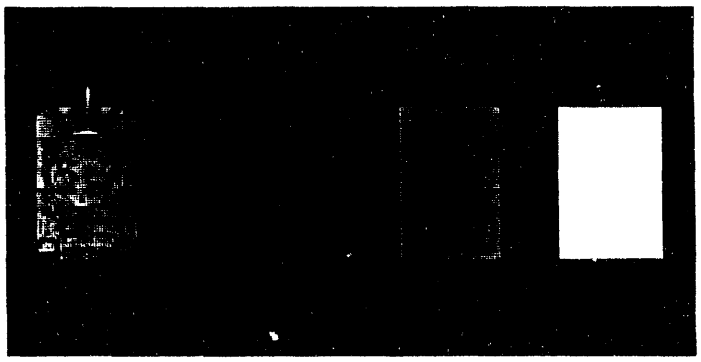

Figure 3a. Tank images with false objects.

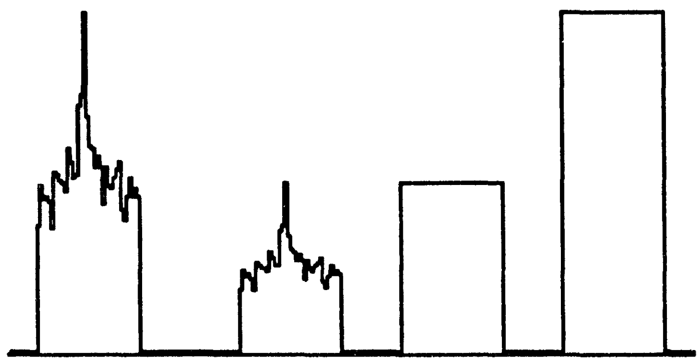

Figure 3b. Intensity profiles of tanks and false objects.

Figure 3c. Result of unnormalized correlation with Fig. 3a.

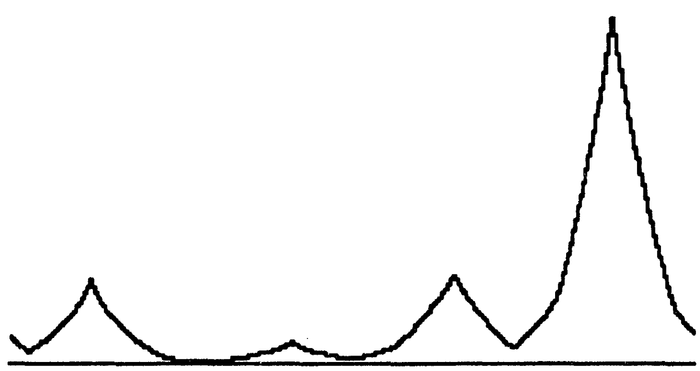

Figure 3d. Unnormalized correlation profile.

As shown in Figures 3c and 3d, the brightest false object produces the highest correlation peak, resulting in a false identification. This illustrates the undesired dependence of unnormalized correlation on scene intensity. Normalized correlation allows identification of the correct target independent of scene intensity. Figure 3e shows the result of a normalized correlation on the image of Figure 3a, with the profile shown in Figure 3f.

Figure 3e Result of normalized correlation with Fig. 3a.

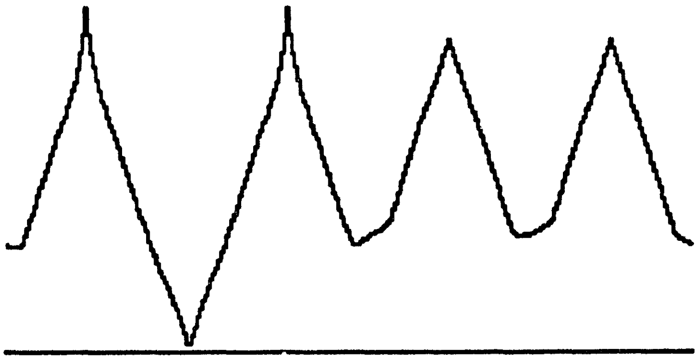

Figure 3f Normalized correlation profile.

The simulation results of Figs. 2 and 3 illustrate the results of normalized correlation when high numerical precision is used. We now show normalized correlation results generated using the AO correlator, where optical hardware limitations result in lower numerical precision. Figure 4a contains three possible targets and a bright false object in the lower right corner, similar to the situation in Fig. 3a.

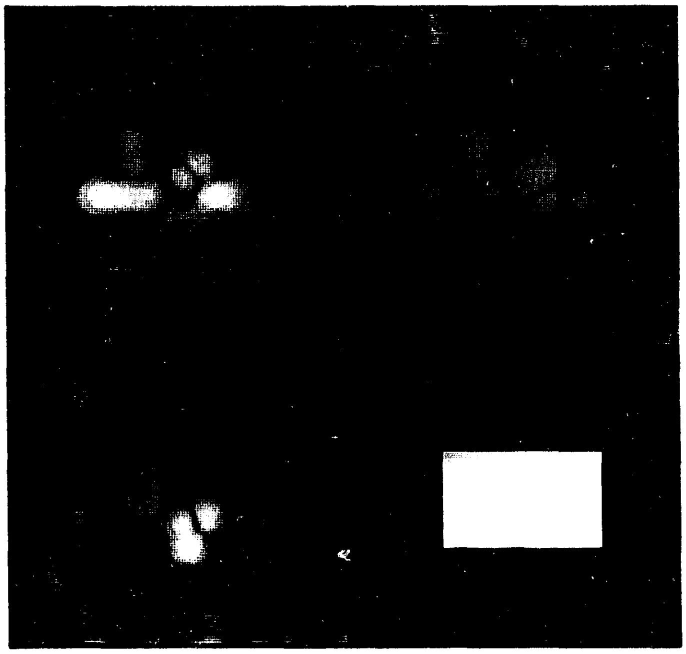

Figure 4a Targets with a false bright object.

First, this image was correlated on the AO correlator with a matched spatial filter for the target in the upper left corner. The result is shown in Figure 4b.

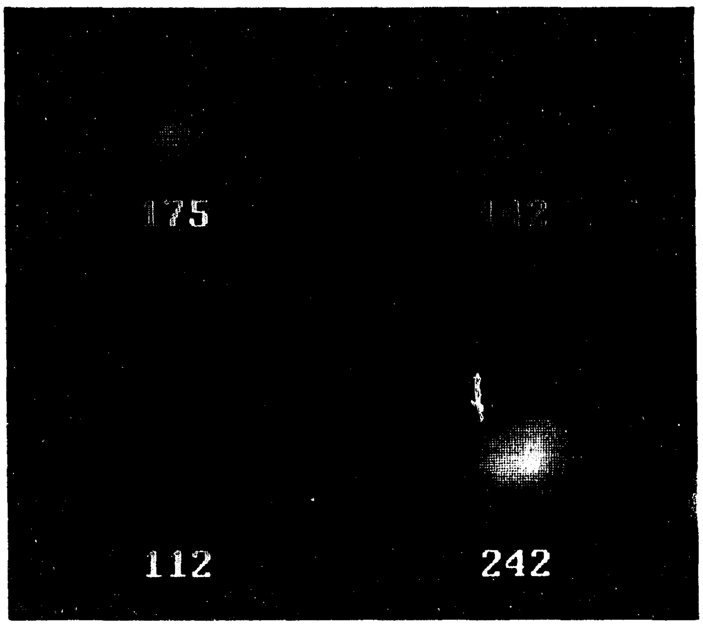

Figure 4b AO correlation (unnormalized) of Fig. 4a.

As expected, the false bright object results in the highest correlation peak, causing a false detection. The intended target has a peak well below the false peak, causing a missed detection. The image was then processed with the AO correlator using normalized correlation. The result is shown in Figure 4c.

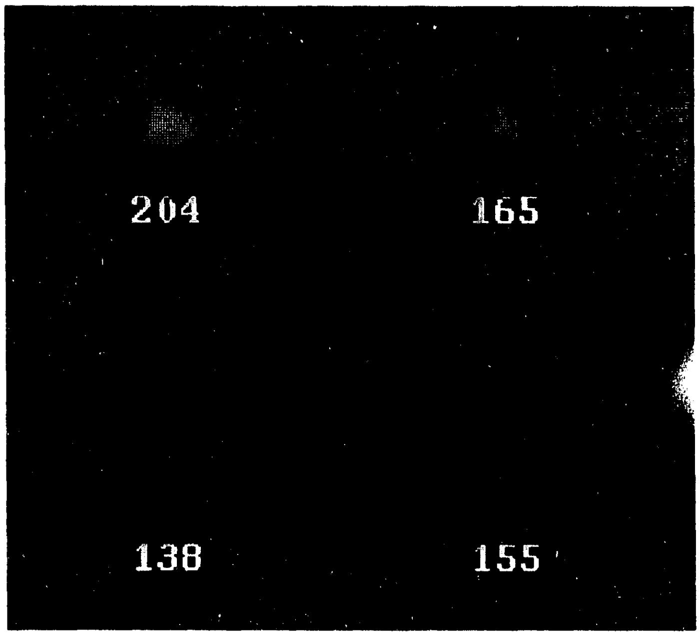

Figure 4c AO correlation (normalized) of Fig. 4a.

As expected the normalized correlation produces the highest peak for the intended target, while the false peak is much smaller. Thus the simulation results are borne out on the AO correlator.

We now address the situation of Fig. 2a, with identical targets at different intensities as shown in Figure 5a.

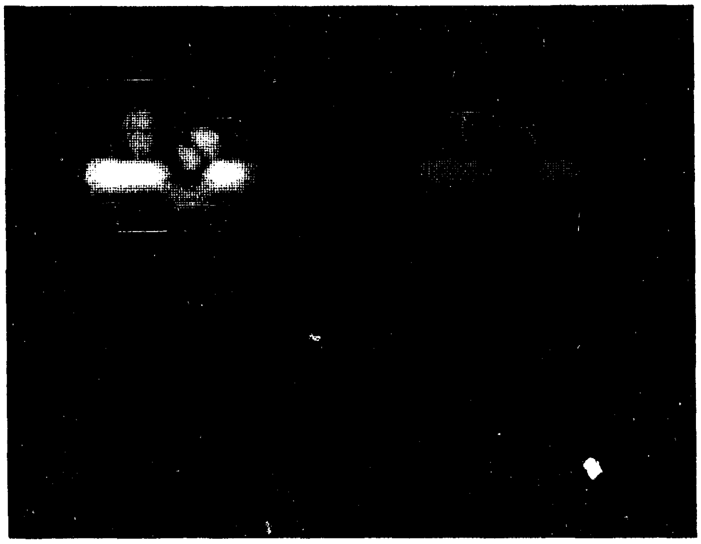

Figure 5a Identical target at four different intensities.

This scenario is of interest because it raises numerical precision issues, since the normalized correlation peaks should be equal. While the peaks are equal when floating point double precision is used for the computations, the results are quite different when precision (dynamic range) constraints of the optical system are imposed. The results of a simulated normalized correlation using low precision 8-bit integer computations are shown in Figure 5b.

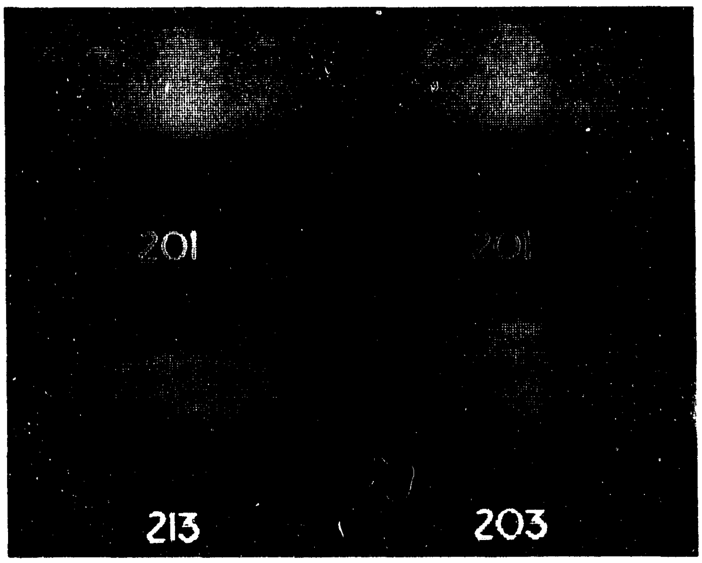

Figure 5b Simulated normalized correlation with 8-bit arithmetic.

The use of 8-bit arithmetic results in the unequal peak values as shown in Figure 5b. The normalized correlation results from the AO correlator in Figure 5c show similar effects.

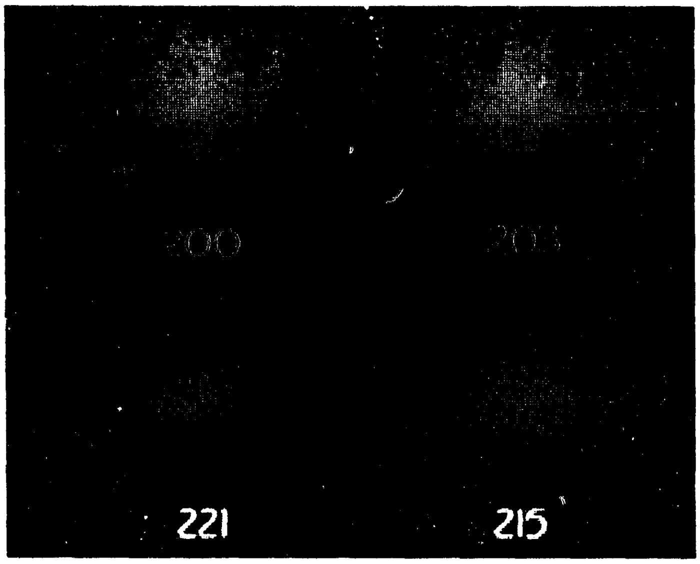

Figure 5c AO normalized correlation results

The principal source of computational error has been shown to be the limited dynamic range of the CCD accumulator in the AO correlator system.[7] Simulation results assuming a 12-bit dynamic range for the CCD show a resulting decrease in error, as shown in Figure 5d.

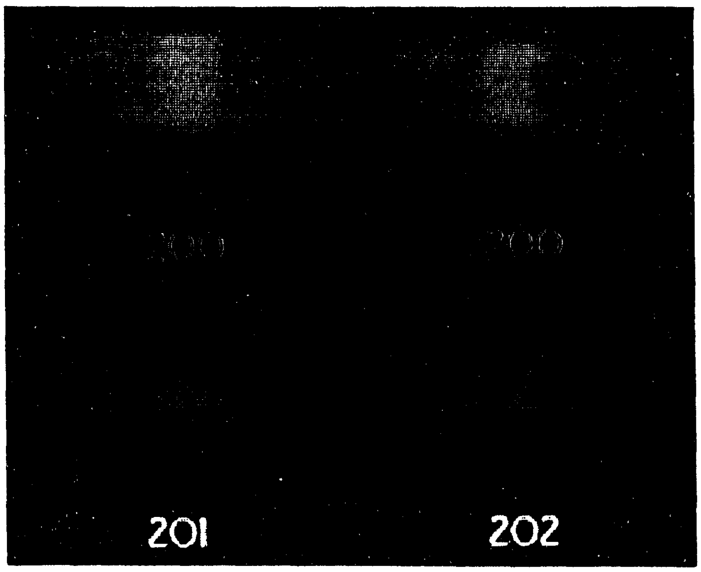

Figure 5d Simulated normalized correlation with 12-bit accumulator.

In this section we have shown examples of the utility of normalized correlation for pattern recognition. We have also reviewed the principles of the acousto-optical correlator, and pointed out the effects of system hardware limitations. Since the hardware in the existing AO correlator does not represent the state of the art (particularly the diodes and CCD) there is promise that much more accurate computations can be achieved. This would prove beneficial ıor computing either normalized correlations or another type of functional consisting of sums of products of correlations.

5. SUMMARY

In this paper, we have discussed solving general pattern recognition problems in terms of arbitrary functionals. This was motivated by the fact that the common method of correlation followed by thresholding is unable to solve many real pattern recognition problems. The problem of varying scene intensity is only one example where standard correlation fails. It has been shown that normalized correlation and normalized quadratic filters provide solutions to this problem. Intuitively, one realizes that the degree of nonlinearity introduced to the correlation method need not stop with the use of normalization. Given that normalized correlation and normalized quadratic filters are simple functionals, it is logical to consider more general functionals. Then we pointed out a fundamental relationship between nonlinear functionals and correlation, namely that arbitrary nonlinear functionals can be computed in terms of sums of products of correlations. This relationship was illustrated in terms of Volterra functionals and neural networks, two types of nonlinear functionals used for pattern recognition.

We note several implications of this relationship. First, it shows that correlations may be quite important to general pattern recognition. This is despite the fact that correlation alone is insufficient for solving many pattern recognition problems. In a sense, correlations can serve as a basis set for the space of nonlinear functionals applied to pattern recognition problems. This provides impetus to using optical correlators as high-speed correlation "engines", since a very large number of correlations may be needed to compute the desired functionals. We caution however, that it is an open question as to whether generating functionals in this manner would be advantageous or not. For example, as mentioned in Section 2, a series of normalized correlations can approximate an optimal Bayes classifier. Computationally, normalized correlation is not extraordinarily difficult, thus a series of normalized correlations might be easier to compute than a very large series of products of correlations.

Given our caution, however, we feel that this relationship offers the opportunity to approach general pattern recognition problems in an orderly manner, rather than using ad-hoc methods. The first step would be the determination of the desired functional to solve the pattern recognition problem. Whether this can be made tractable is another open question. Once the functional is determined, the next step would be to express the functional in terms of a combination of correlations. Even though the problem of determining the functional is a daunting one, framing pattern recognition problems in this way may prove useful. For instance, some problems have "known" functionals based on current approaches that "work". An example would be a neural network which appears to solve a specific pattern recognition problem. One could derive the nonlinear functional the neural network is computing. The functional can then be analyzed in terms of its constituent correlations, perhaps in a manner analgous to Fourier analysis. This may provide a good basis for comparison with alternative approaches. Rather than comparing different approaches strictly on the basis of nebulous simulation results to determine which is "best", comparisons can be made on even ground in terms of correlations. We are encouraged in this respect by the fact that two seemingly disparate approaches to pattern recognition, namely nonlinear filters and neural networks, are seen to be variations on a common theme.

Finally, we have discussed acousto-optic correlation and briefly raised some of the performance issues. If an optical correlation "engine" proves to be useful in the computation of nonlinear functionals, speed and accuracy will be of paramount importance. The principal hardware limitation in the AO correlator (for both speed and accuracy) has been identified as the CCD. Use of a state of the art CCD (with better dynamic range) can enhance the AO correlator performance. Again we note that in comparison, current hardware limitations for frequency-domain optical correlators are much more severe. The use of partial information filters, necessitated by current SLM technology, severely limits the usefulness of such correlators as correlation "engines", since errors are introduced into the correlation results. Only the development of fully complex SLMs would make frequency-domain optical correlators viable for the computation of nonlinear functionals as sums of products of correlations. Until the development of such SLM technology, we favor a space-domain correlation approach.

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