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Robot Trajectory Planning via Dynamic Programming

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ABSTRACT

The method of dynamic programming is applied to three example problems dealing with robot trajectory planning. The first two examples involve end-effector tracking of a straight line with rest-to-rest motions of planar two-link and three-link rigid robots. These examples illustrate the usefulness of the method for producing smooth trajectories either in the presence or absence of joint redundancies. The last example demonstrates the use of the method for rest-to-rest maneuvers of a single-link manipulator with a flexible payload. Simulation results for this example display interesting symmetries that are characteristic of such maneuvers. Details concerning the implementation and computational aspects of the method are discussed.

INTRODUCTION

Traditional rigid robot trajectory planning approaches have typically been based on inverse kinematics techniques [1]. Use of these techniques is appropriate in many situations, but there are important instances when it is not. Such instances arise whenever robot flexibility or dynamics are of importance. For example, inverse kinematics schemes cannot accomodate structural flexibility in high payload-to-robot weight ratios [2]. Another feature of inverse kinematics solutions is that trajectories are not unique for redundant robots. The purpose of this paper is to present an alternative approach for trajectory planning based on the method of dynamic programming which is applicable to rigid, flexible, and redundant robots.

The optimal trajectory planning problem for rigid robots is posed as a linear time-invariant system for both non-redundant and redundant configurations. The first two example problems demonstrate this approach by generating a pair of trajectories for end-effector tracking of a straight line with rest-to-rest motions of planar two-link and three-link rigid robots. The generality of the dynamic programming method is presented in the last example problem. A single-link manipulator with a flexible payload is optimally slewed through a set of rest-to-rest maneuvers. The problem is posed as a nonlinear system with state variable equality constraints and input inequality constraints. The simulation results of the single-link manipulator display interesting symmetries characteristic of such motions.

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DYNAMIC PROGRAMMING

For purposes of brevity, our discussion of the dynamic programming algorithm is restricted to a summary of its key points. A more complete description will likely appear in a forthcoming paper.

To begin, consider the initial value problem

$$\dot{x} = f(x, u, t), \qquad x(t_1) = x_1 \tag{1}$$

where $x \in R^n$ is the state, $u \in R^p$ is the input, and $(\dot{})$ denotes differentiation with respect to time, t. The input is assumed to be discretized temporally as

$$u(t) = u_k \qquad t_k \leq t < t_{k+1} \tag{2}$$

for $k = 1, \ldots, N$. Given the existence and uniqueness of the solution to the initial value problem, adjacent states in time can be related as

$$x_{k+1} = g_k(x_k, u_k) \tag{3}$$

where $x_k = x(t_k)$. The discrete optimal control problem of interest is stated as follows. Given the initial state x_1, find the inputs u_k which minimize the objective function

$$\Gamma(x_k, u_k) = \sum_{k=1}^{N} \Gamma_k(x_k, u_k) \tag{4}$$

subject to Eq. (3) and the inequality constraints

$$u_{lower} \leq u_k \leq u_{upper} \qquad k = 1, \ldots, N \tag{5}$$

Closed form solutions to the optimal control problem cannot be found in general. Consequently, one must often employ iterative procedures. The iterative approach adopted in this paper can be thought of as an adaptation of the well-known Newton's method [3]. As with Newton's method, a quadratic model is used to approximate the objective function. The distinguishing feature of the present approach is that minimization of this model is accomplished using the method of dynamic programming [4].

In order to obtain a quadratic model of the objective function in terms of the inputs, it is necessary to determine the first and second derivatives of g_k and Γ_k with respect to their arguments. The amount of effort required for this task is largely problem dependent. For example, calculation of these derivatives in the first two examples is relatively straightforward. In contrast, the derivatives of g_k in the third example must be determined with recourse to a numerical integration scheme. Such complexities arise when closed-form solutions to the initial value problem cannot be found.

The salient features of the dynamic programming algorithm are:

1. Quadratic convergence nearby the solution.
2. Inequality constraints for inputs accounted for exactly.
3. Equality constraints for states imposed using a quadratic penalty function.

Figure 1: Sketch of planar robots considered in the first two examples.

4. Optimality conditions determined from definiteness of p by p matrices.
5. Order Nnp storage required for each iteration.
6. Order Nn^3 operations required for each iteration.

The interested reader may consult a text on optimization for detailed descriptions of the various terms used above (see, e.g., Ref. [3]).

EXAMPLES

The first two examples deal with straight line tracking, rest-to-rest maneuvers of rigid two-link and three-link robots moving in a plane (see Fig. 1). Here the goal is to move the tip of the terminal link from point A to point B, tracking a straight line connecting the two points, bringing the robot to rest in time T.

Joint trajectories for such maneuvers are not unique. That is, there are an infinite number of joint time histories which meet the above stated goal. Here, the optimal trajectory is defined as the one which minimizes the function C, defined as

$$C = \sum_{k=1}^{N} \sum_{j=1}^{N_j} (u_k^j)^2 \tag{6}$$

where N is the number of time steps, N_j is the number of joints, $u_k^j = \theta_j''(\tau_k)$, $\tau_k = (k-1)t/T$, and $(\ )'$ denotes differentiation with respect to $\tau = t/T$. The function C can be thought of as a measure of the smoothness of the trajectory.

Letting $\bar{x} = x/L_1$ and $\bar{y} = y/L_1$, consider a two-link robot with $\theta_{1A} = \pi/4$, $\theta_{2A} = \pi/12$, $\bar{x}_B = 1.8$, $\bar{y}_B = -0.2$, $L_2/L_1 = 1$, and $N = 201$. The states for this problem are the joint angles and the joint velocities, and the inputs are the joint accelerations. Plots of the states and inputs as functions of τ are shown in Figure 2. Also shown in the figure is the path followed by the tip of the second link. Notice that the straight line path is tracked and the robot is at rest at the conclusion of the maneuver. We mention again that the trajectory shown in Figure 1 is one of many which can accomplish the goal of the stated maneuver. The distinguishing feature of this one is that it minimizes the function C.

For the three-link robot consider the problem in which $\theta_{1A} = \pi/4$, $\theta_{2A} = \pi/12$, $\theta_{3A} = \pi/6$, $\bar{x}_B = 2.6$, $\bar{y}_B = -0.4$, $L_2/L_1 = 1$, $L_3/L_1 = 1$, and $N = 201$. Plots of the states, inputs, and tip path are shown in Figure 3. Notice again that the straight line path is tracked and the robot comes to rest at the endpoint. It

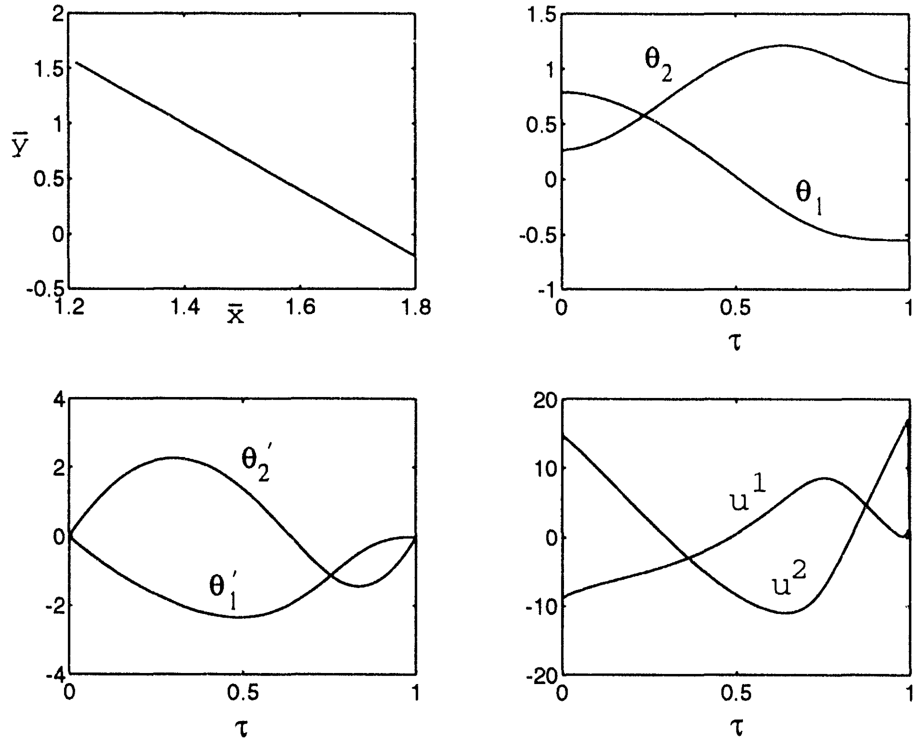

Figure 2: Straight line tracking rest-to-rest motion of two-link robot.

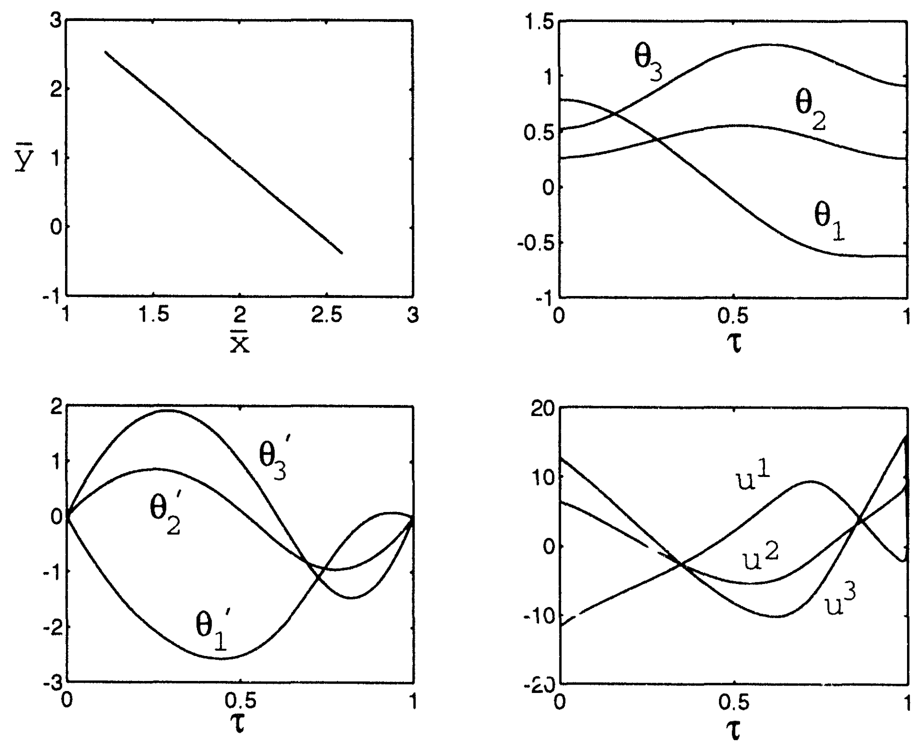

Figure 3: Straight line tracking rest-to-rest motion of three-link robot.

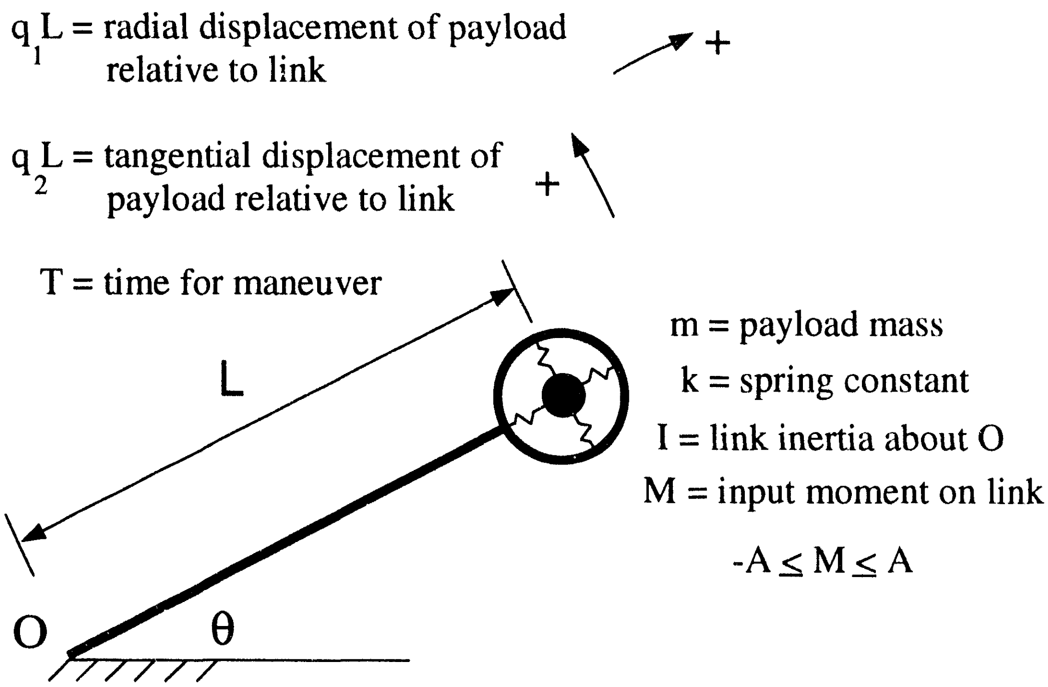

Figure 4: Sketch of single-link manipulator with a flexible payload.

is worthwhile mentioning that there is not a one-to-one correspondence between the joint angles for straight line tracking in the case of the three-link robot. This is in contrast to the situation for the two-link robot in which there is only a single value of θ_2 for each value of θ_1. The redundant degree of freedom introduced by the additional link in this example poses no computational difficulties since the minimization of C leads to a unique solution.

The final example is concerned with rest-to-rest maneuvers of a single-link robot with a flexible payload. A sketch of the model showing the relevant parameters is given in Figure 4. The equations of motion in dimensionless form are given by

$$\begin{bmatrix} c_1 + (1+q_1)^2 + q_2^2 & -q_2 & 1+q_1 \\ -q_2 & 1 & 0 \\ 1+q_1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta'' \\ q_1'' \\ q_2'' \end{bmatrix} = \begin{bmatrix} c_2 u - 2[(1+q_1)q_1' + q_2 q_2']\theta' \\ 2q_2'\theta' + (1+q_1)(\theta')^2 - c_3 q_1 \\ -2q_1'\theta' + q_2(\theta')^2 - c_3 q_2 \end{bmatrix} \tag{7}$$

where $c_1 = I/(mL^2)$, $c_2 = AT^2/(mL^2)$, $c_3 = (k/m)T^2$, and $u = M/A$ is the input subject to the constraints $-1 \leq u \leq 1$. For this example, the optimal rest-to-rest trajectory is defined as the one which minimizes the function D, defined as

$$D = \sum_{k=1}^{N} u_k^2 \tag{8}$$

where $u_k = u(\tau_k)$.

Since closed-form solutions to Eq. (7) are generally not available, the dynamic programming algorithm made use of a numerical integration scheme to

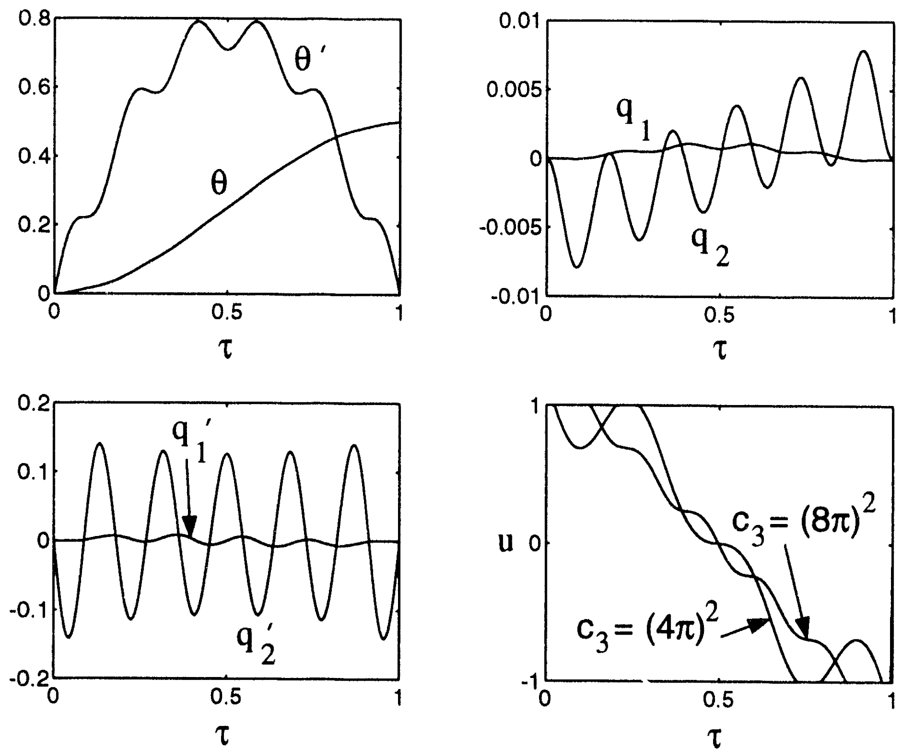

Figure 5: Rest-to-rest motion of single-link manipulator with a flexible payload.

determine the g_k's and their first two derivatives (see Eq. (3)). In this example, a fixed-step, fourth-order Runge-Kutta numerical integration scheme with $N = 201$ was used.

Results presented in Figure 5 show the states and inputs as functions of τ for a maneuver with $\theta(0) = 0$, $\theta(1) = 1/2$ radian, $c_1 = 1$, $c_2 = 5$, and $c_3 = (8\pi)^2$. The symmetries of the inputs and states about $\tau = 1/2$ are evident in the figure. Similar symmetries have been observed by Petterson and Robinett [5] for the control of a flexible beam and by Ben-Asher et al. [6] for planar, time-optimal, rest-to-rest maneuvers of undamped flexible satellites.

The effect of varying payload stiffness on the optimal torque profile is also shown in Figure 5. Results are presented for the two values of c_3 indicated in the figure. As c_3 is increased, the torque profile approaches the one for a rigid payload. As c_3 decreases, a critical value is reached below which the rest-to-rest maneuver is not possible. This lower bound is an important constraint to consider for the control of flexible payloads where the final angle and final time are specified.

CONCLUSIONS

Robot trajectory planning using a method based on dynamic programming is presented for three example problems. The first two examples demonstrate the effectiveness of the approach for end-effector tracking of a straight line for planar two-link and three-link rigid robots. The third example demonstrates the generality of the approach for nonlinear systems with input and state constraints. Based on the results of this paper, the proposed method for trajectory planning shows much promise. Additional testing and evaluation will be required, however, to assess more completely the strengths and limitations of the method.

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