ANL/APS/PP-65709

Appendix: A

Effect of the Slit Widths

The aim of the following mathematical treatment is to prove that any distortion of the gaussian intensity distribution of the beam, which may be caused by the finite width of the slits, is small, provided that the slit widths are not large with respect to the source size and divergence. The widths of the slit A (the upstream slit in the coffin, which will be called slit 1 in this Appendix) and of the slit B (the downstream slit in the hutch, called slit 2 herein) were the same, equal to 0.1 mm for the measurement of the horizontal source size and divergence, and equal to 0.025 mm in the vertical case. It turns out, as shown below, that the distortion of the phase space gaussian beam distribution is estimated to be not greater than 6% in the vertical case and is considerably smaller in the horizontal case. The procedure followed in the analysis of the experimental data (see the main procedure followed in the analysis of the experimental data (see the main text) is then justified and is a good approximation.

First we redefine some of the variables and we write again some of the formulae used in the main text:

x, y are the phase space variables of the beam emitted by the source (x is used in the main text, instead of y).

The effect of the slits on the beam is introduced by convoluting the Gaussian intensity distribution in phase space (1A) with a step function $S(x)$, such as:

$$S(x_1') \text{ and } S(x_2') = \begin{cases} 0 & \text{for } |x_1'|,|x_2'|>a \\ 1 & \text{for } |x_1'|,|x_2'|\leq a \end{cases}$$

a being the half-width of the slits. In order to evaluate the integral of (2A), a transformation of the integration variables from phase space coordinates to slit coordinates is operated by using (3A). Then the integral (2A) can be written:

$$\iint_{-\infty}^{+\infty} S(x_1')S(x_2')\exp[-(x' - x)^2/2\sigma_x^{\,2}]\cdot \exp[-(y' - y)^2/2\sigma_y^{\,2}]|J|dx_1'dx_2'$$

$$= \iint_{-a}^{+a}\exp[-(x'-x)^2/2\sigma_x^{\,2}]\cdot\exp[-(y' - y)/2\sigma_y^{\,2}]J\ dx_1'\ dx_2' \qquad (4A)$$

where J, the Jacobian, is equal to: $1/(Z_2-Z_1)$.

Let:

$$\alpha_1^{\ 2} = \frac{1}{2(z_2 - z_1)^2} \quad \frac{z_2^{\ 2}}{\sigma_x^{\ 2}} + \frac{1}{\sigma_y^{\ 2}}$$

$$\alpha_2 = \frac{1}{2\ (z_2 - z_1)^2} \quad \frac{z_1^{\ 2}}{\sigma_x^{\ 2}} + \frac{1}{\sigma_y^{\ 2}}$$

$$\alpha_{12}^{\ 2} = \frac{1}{(z_2 - z_1)^2} \quad \frac{z_1 z_2}{\sigma_x^{\ 2}} + \frac{1}{\sigma_y^{\ 2}}$$

The following is obtained by approximating the exponential in $x'_1{}^2$, $x'_2{}^2$, $x_1 x'_1$ and $x_2 x'_2$ under the integral sign in (4A) with the linear term in the series expansion (it is assumed that the slit widths are small enough to justify this approximation, as the results which are obtained below indicate). The integral in (4A) is equal to:

$$4a^2 J \quad 1 - \frac{(\alpha_1^{\ 2} + \alpha_x^{\ 2})a^2}{3}$$

$$- \frac{[(4\alpha_1^{\ 4} - \alpha_{12}^{\ 4})x_1^{\ 2} - (4\alpha_2^{\ 4} - \alpha_2^{\ 4})x_2^{\ 2} - 4\ (\alpha_1^{\ 2}\alpha_{12}^{\ 2} - \alpha_2^{\ 2}\alpha_{12}^{\ 2})x_1 x_2]a^2}{6} \quad .$$

$$\cdot \ \exp\ [-(x^2/_{2\sigma_x{}^2} + \frac{y^2}{2\sigma_y^{\ 2}})]$$

By using the values 2a=0.025 mm, x = 0.15 mm and y = 0.016 mrad, one finds that (5A) is equal to:

$$0.02 \cdot J \cdot \left[1-0.025- \int 4.36x_1^{2} + 2.35x_2^{2} - 6.4x_1x_2)\right] \cdot$$

$$\cdot \exp - \left[x^2/2\sigma_x{}^2 + y^2/2\sigma_y{}^2\right].$$

The deviation from the Gaussian behavior is small if $X_1 = X_2 = 3\sigma_x$ the above expression in X_1 and X_2 is equal to $\simeq 0.06$. Smaller corrections are expected in the case of the horizontal source size and divergence since a/σ_x is smaller than in the vertical case. It was also verified that the slit width corrections were negligible in the two following ways: a) the analytical expression (5A) was inserted in the program which calculated the coordinates of the phase space points corresponding to the various intensity levels of the experimental scan curves. No significant variations of the values of the point coordinates were obtained; b) the program which fitted the experimental scan curves with Gaussians, could also average the value of the Gaussian function over the slit area by doing a 5 x 5 mesh averaging. Even in this case no significantly different values of the source size and divergence were found.

A rough (but simple) estimate of the effect of slit width can be obtained by considering the one-dimensional case; a single Gaussian distribution folded with a rectangular slit. In this case formula (4A) is much simpler:

$$I\ (x) = \int_{-\infty}^{+\infty} S(x')\ \exp\left[-\ (x) - x')^2/2\sigma^2\right]dx'$$

$$= \exp\left[-x^2/2\sigma^2\right]\cdot\int_{-a}^{+a} \exp\left[\exp-(x'^2 - 2xx')/2\sigma^2\right)dx'$$

$$= 2\alpha\left(1 - \frac{a^2}{6\sigma^2} + \frac{a^4}{40\sigma^2} + \frac{a^2x^2}{6\sigma^4}\right)\cdot \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

where the exponential under the integral sign was approximated by:

$$\exp\left[-(x'^2 - 2xx')/2\sigma^2\right] \simeq 1 - \frac{x'^2 - 2xx'}{2\sigma^2} + \frac{(x'^2 - 2xx')^2}{8\sigma^4}$$

By taking $\sigma \simeq 0.15$ mm and $2a=0.025$ mm, (6A) is written:

$$I(x) \simeq 0.025 \cdot (0.989 + 0.051x^2)\cdot\exp\left(-\frac{x^2}{2\sigma^2}\right)$$

Any correction is small, since meaningful values of x are not greater than 3σ.

The conclusion is that: a) corrections due to slit width were small and well below the experimental errors; b) a quick estimate of the effect of slit width in emittance measurements by the two scanning slit method can be obtained by suing the formula for the one-dimensional case. This formula should give a satisfactory estimate wherever a precise correction is not needed or the slit widths are small.

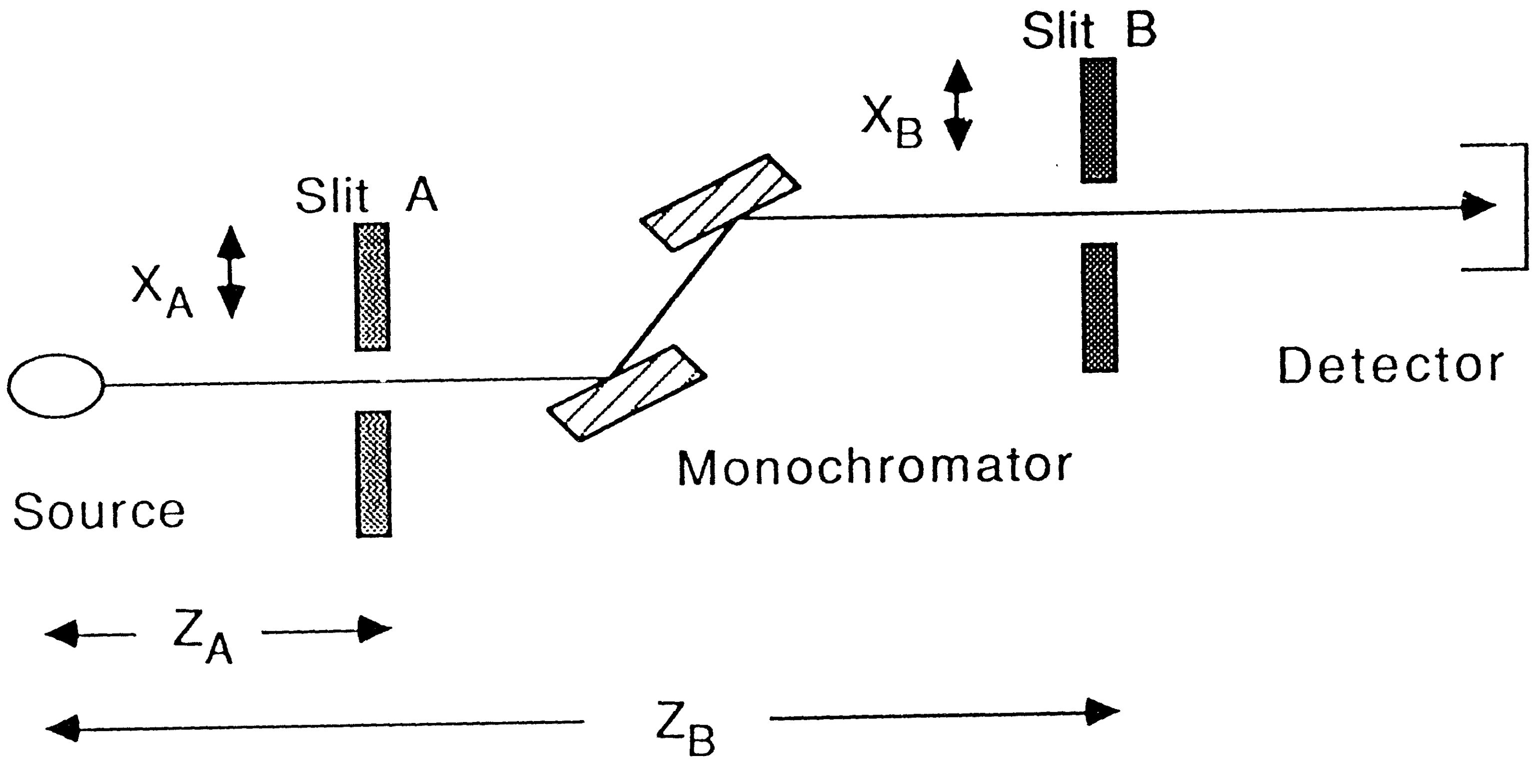
Fig. 1 Photograph of the 123 pole APS/CHESS undulator built by Spectra Technology.

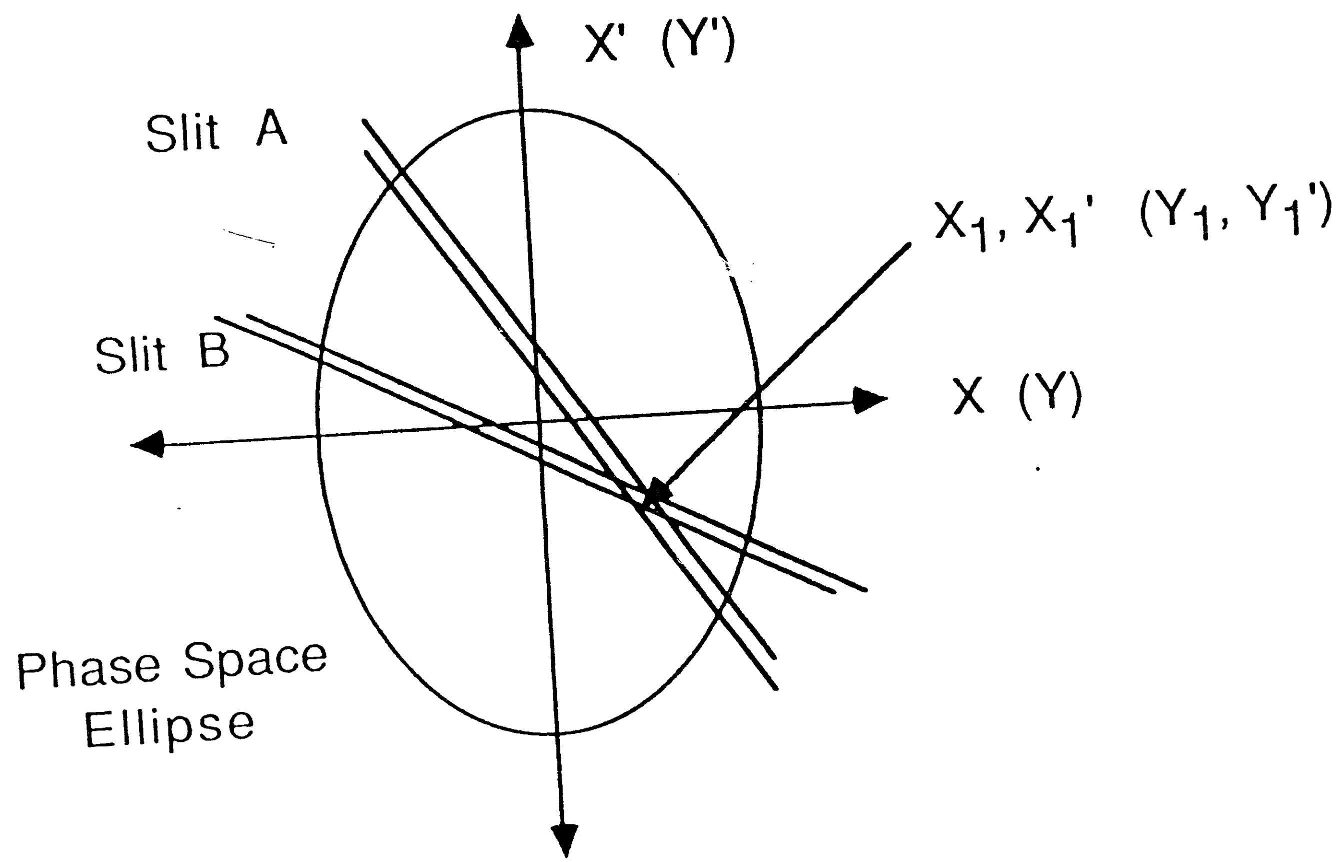
Fig. 2 Schematic of the experimental arrangement for emittance measurements. The distance from the source to Slit A and Slit B was 18.54 meters and 25.97 meters respectively. Slit sizes were 100 microns for the horizontal emittance measurements and 25 microns for the vertical emittance measurements.

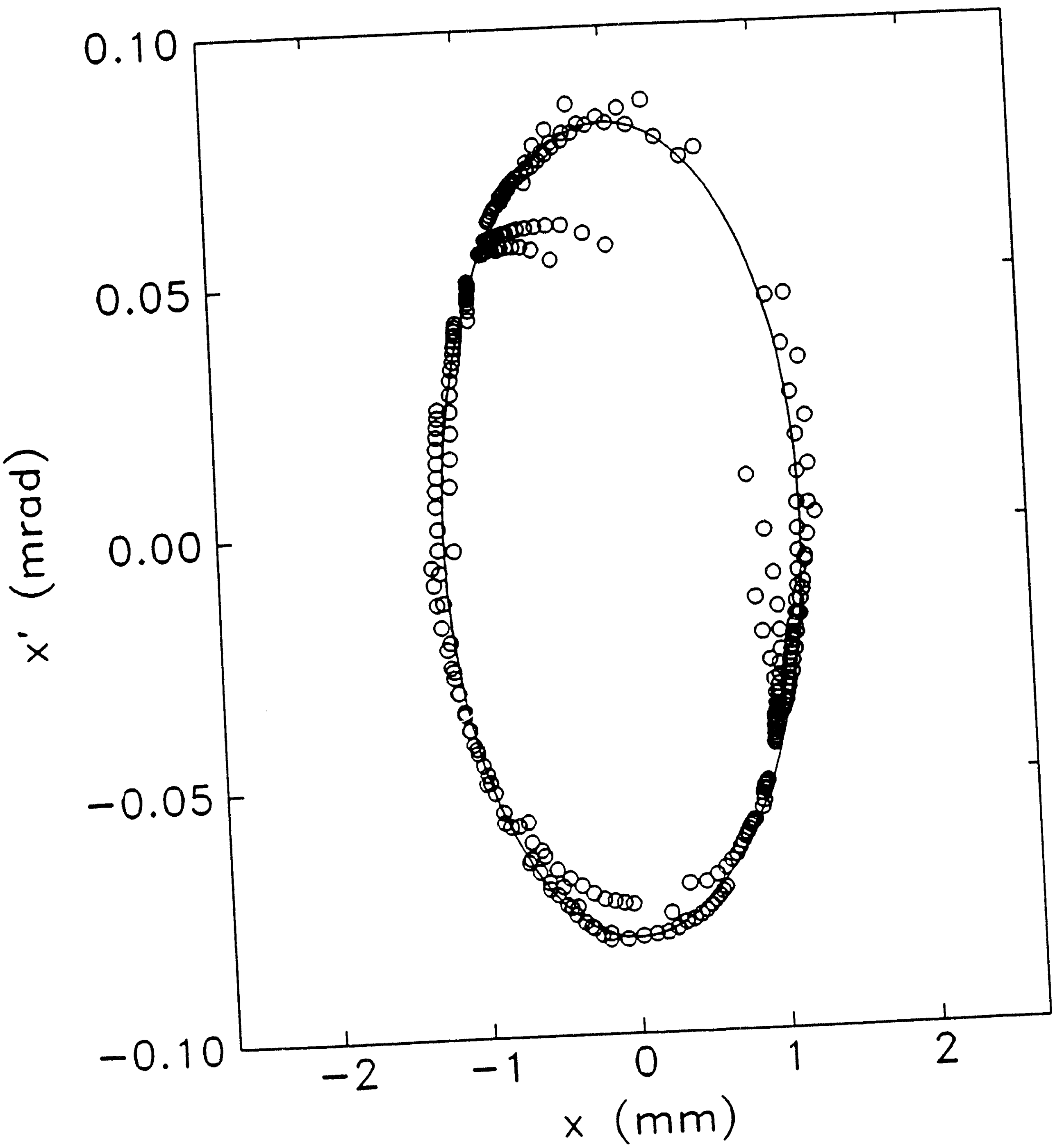
Fig. 3 The projection of the two slits onto the source phase space. Data is collected by first fixing Slit A and then recording the measured intensity as Slit B is stepped. Slit A is then moved and the procedure repeated until the entire phase space area has been rastered over.

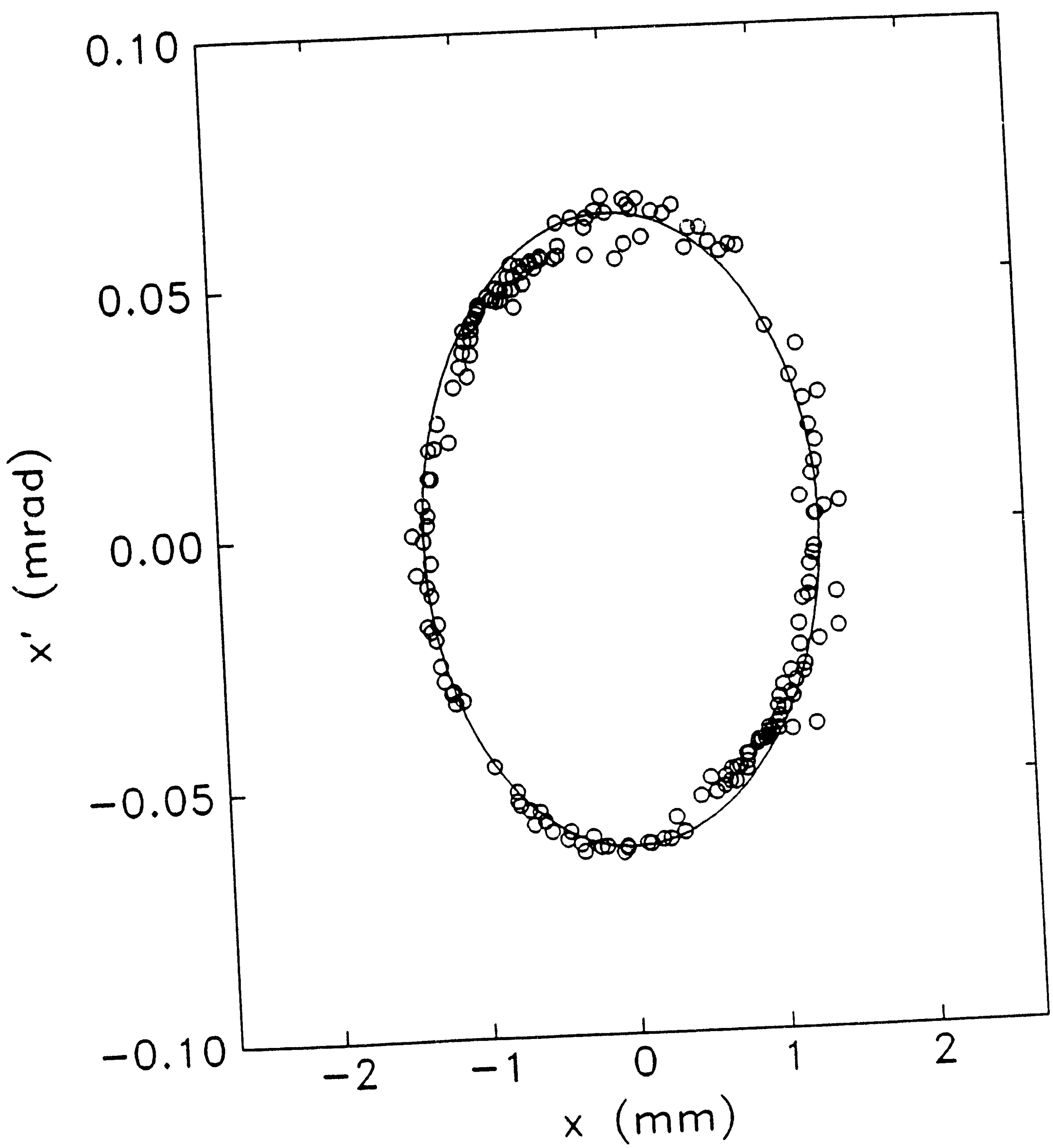
Fig. 4 The horizontal phase space data plotted using the intensity scaling (IS) analysis technique. (a) Data collected at 17 keV with 1 milliamp of beam in the storage ring. The measured (uncorrected) one sigma values for the horizontal beam size and divergence was 1.20 mm and 0.081 mrad. (b) Data collected at 7.2 keV with 2 milliamps of beam in the storage ring. The measured (uncorrected) and sigma values for the horizontal beam size and divergence was 1.33 mm and 0.062 mrad. (c) Data collected with 5 milliamps of beam in the storage ring. The measured (uncorrected) one sigma values for the beam size and divergence for this case was 1.35 mm and 0.072 mrad respectively.

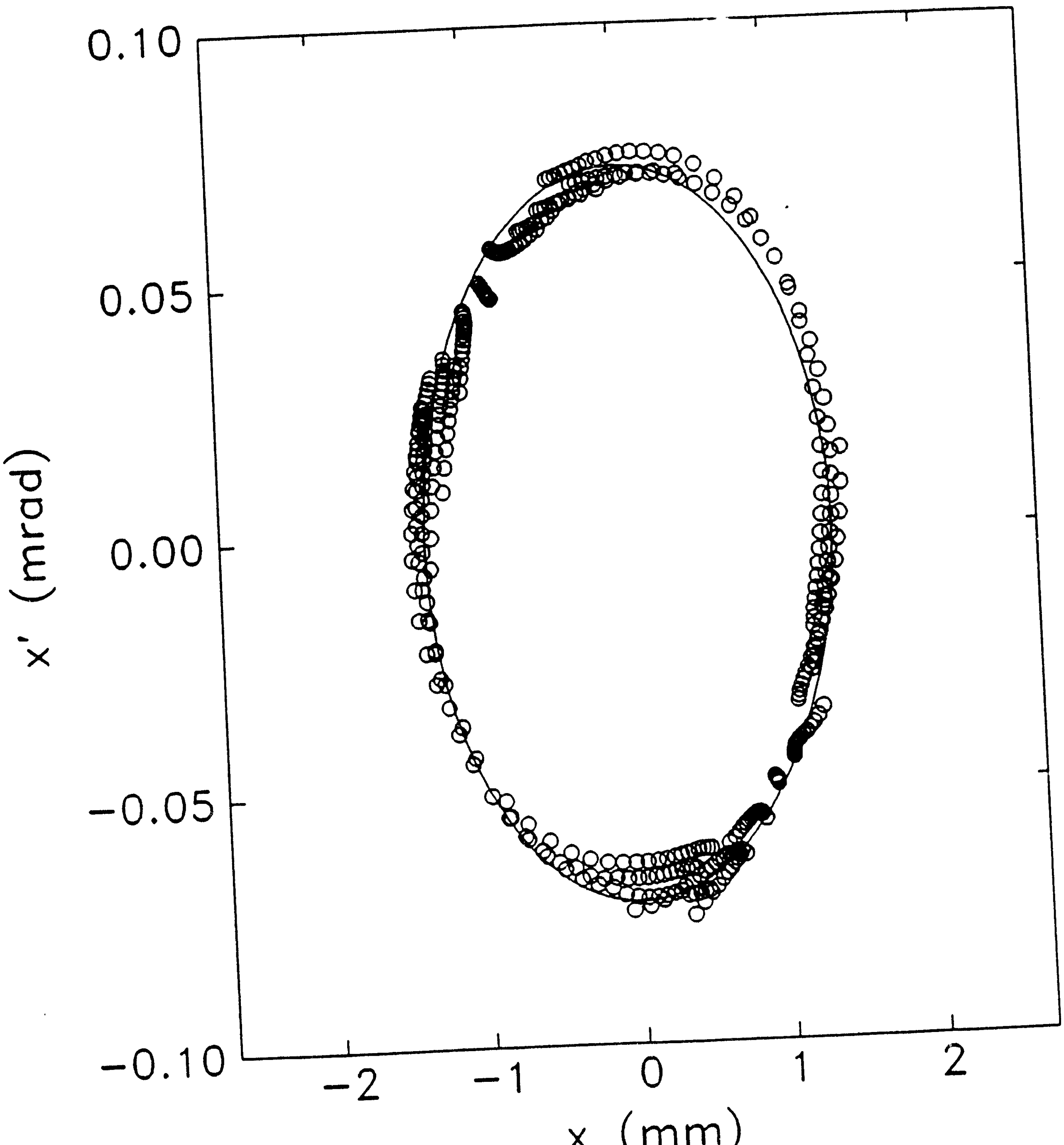
Fig. 5 The vertical phase space data plotted using the intensity scaling (IS) analysis technique taken at 1.7 milliamps. The measured (uncorrected) one sigma values for the vertical beam size and divergence was 0.14 mm and 0.017 mrad.

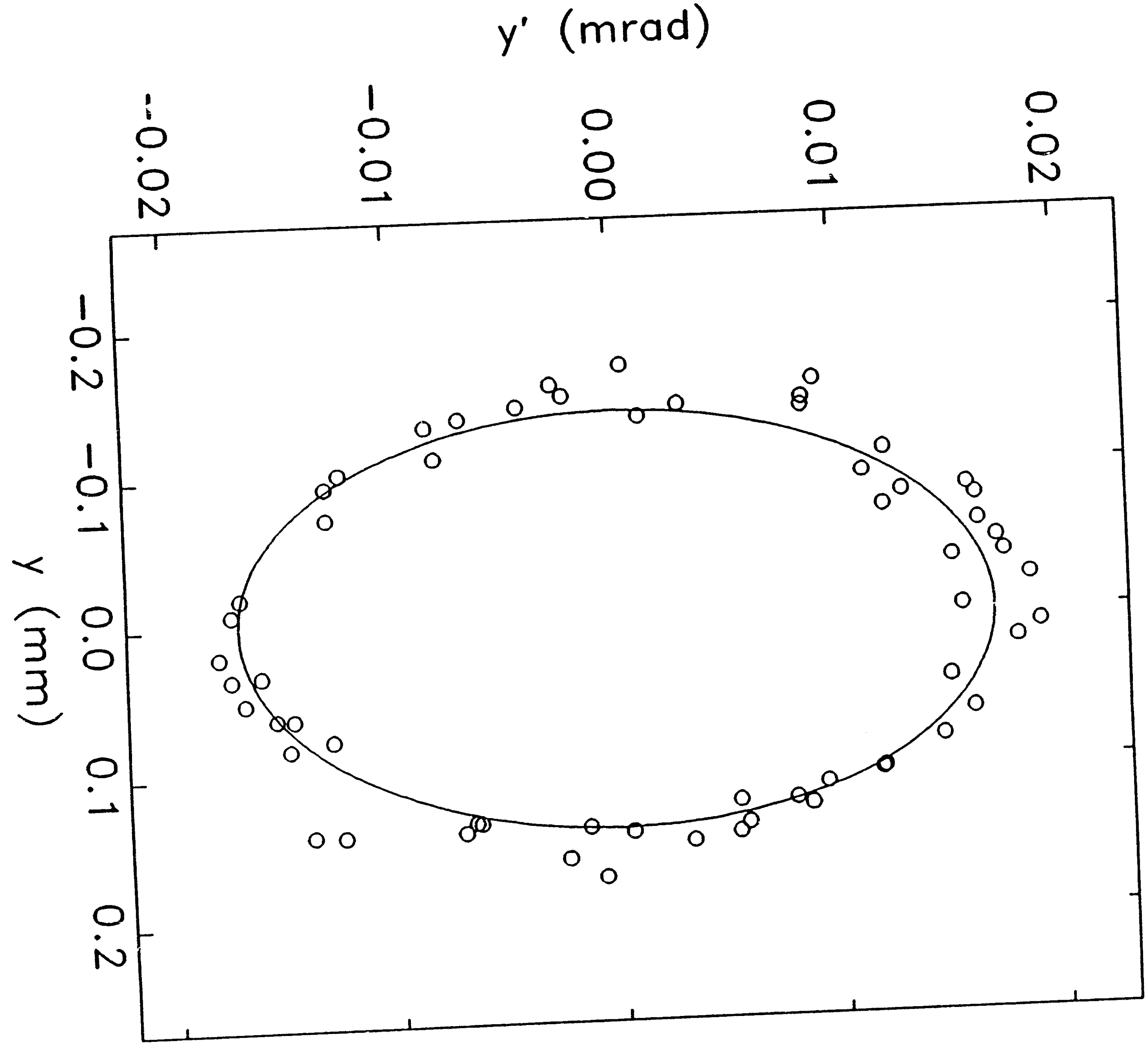

Slit B
X_B
Slit A
X_A
Detector
Monochromator
Source
Z_A
Z_B


X' (Y')
Slit A
X_1, X_1' (Y_1, Y_1')
Slit B
X (Y)
Phase Space
Ellipse


0.10
0.05
0.00
−0.05
−0.10
x' (mrad)
−2
−1
0
1
2
x (mm)


0.10
0.05
0.00
−0.05
−0.10
x′ (mrad)
−2
−1
0
1
2
x (mm)


0.10
0.05
0.00
−0.05
−0.10
x' (mrad)
−2
−1
0
1
2
x (mm)


y' (mrad)
-0.02
-0.01
0.00
0.01
0.02
y (mm)
-0.2
-0.1
0.0
0.1
0.2

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