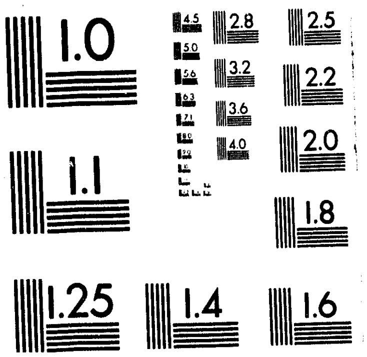
1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3
7.1

1 of 1

Orbit Correction and Analysis Workshop
Brookhaven National Laboratory
December 1-3, 1993

BNL-49654
Conf-931254--3

Skew Quadrupole Errors in the RHIC IR Triplets*

S. Tepikian and S. Peggs
Brookhaven National Laboratory, Upton, NY 11973

Abstract

From simulation studies of the RHIC lattice, we found that skew quadrupole errors and quadrupole roll misalignments in the IR triplets (where the β function can be large) produce large vertical dispersions. A simple model, using only one triplet, is applied to obtain sensitivities of vertical dispersion to skew quadrupole errors. This study revealed that measuring the vertical dispersion in the triplets is a good tool for the diagnosis of local skew quadrupole correction.

Introduction

During simulation studies of the RHIC lattice, there was one seed that had poor dynamic aperture, due partly to a large vertical dispersion of 6.58m in the triplets. Since RHIC has no vertical bending magnets in the lattice, any vertical dispersion can only come from errors. A major source of vertical dispersion is caused by coupling from the horizontal dispersion, due to skew quadrupoles.

We show that the vertical dispersion can be reduced by applying a combination of local and global decoupling. The local correctors are placed in the triplets where the β is large. Since the phase advance across the triplets is negligible, only one corrector per triplet is needed. There are also three families of skew quadrupoles in the arcs, of which two are used for global decoupling.

Only the local triplet correction is discussed in this paper. The following three sections deal with perturbation theory, a simple model, simulation and concluding remarks.

Perturbation Theory

The equation of motion for the vertical plane is

$$y'' + Ky - Mx = 0.$$

and the horizontal plane is

$$x'' - Kx - My = \delta\frac{1}{\rho}$$

where x and y $[m]$ are the particle horizontal and vertical coordinates respectively, primes are derivatives with respect to azimuth, K $[m^{-2}]$ is the quadrupole strength, M $[m^{-2}]$ is the skew quadrupole strength, ρ $[m]$ is the bending radius of curvature and $\delta = \Delta p / p$. Substituting $y = \delta\eta_y$ and $x = \delta\eta_x$ into vertical equation for the dispersions gives

$$\eta''_y + K\eta_y + M\eta_x = 0$$

MASTER

where η_x and η_y are the horizontal and vertical dispersions respectively. Assume that η_x is of

*Work performed under the auspices of the U.S. Department of Energy.

zeroth order and M is first order, then η_y is first order in M. In the triplets, the two main sources of skew quadrupoles are rotations of the quadrupoles and the skew quadrupole corrector itself leading to

$$\eta''_y + K\eta_y = (2\theta K + S)\eta_x$$

where θ [rad] is the roll angle of a quadrupole, assumed to be small, and S [m^{-2}] is the skew quadrupole strength of the corrector element. This equation is of the same form as an orbit error equation whose solution is known [1]. Hence, one source and observation point gives

$$\eta_y = \theta\left[\frac{\sqrt{\beta_y\beta_{y0}}\eta_{x0}}{f\sin(\pi\nu_y)}\cos(|\Delta\phi_y| - \pi\nu_y)\right] \equiv \theta H_\theta$$

where f is the focal length of the quadrupole, ν_y is the vertical betatron tune, β_y and β_{y0} is the vertical beta function at the reference and source point respectively, $\Delta\phi_y$ is the phase difference between the reference and source, η_{x0} is the horizontal dispersion at the source and H_θ is the sensitivity. Similarly, for the corrector we have

$$\eta_y = \frac{1}{F}\left[\frac{\sqrt{\beta_y\beta_{y0}}\eta_{x0}}{2\sin(\pi\nu_y)}\cos(|\Delta\phi_y| - \pi\nu_y)\right] \equiv \frac{1}{F}H_F$$

where H_F [m^{-1}] is the sensitivity and F [m] is the focussing strength due to the corrector.

Additionally, coupling can influence the dynamics of the particles in other ways. A convenient measure of the strength of coupling is the minimum of the difference in the transverse tunes. It can be shown [2] that the minimum tune split due to a single short quadrupole rotated by a small angle is

$$\Delta\nu_{min} \equiv |\nu_x - \nu_y|_{min} = \left|\theta\left[\frac{\sqrt{\beta_{x0}\beta_{y0}}}{f\pi}\right]\right| \equiv |\theta G_\theta|$$

and the minimum tune split due to a corrector is

$$\Delta\nu_{min} = \left|\frac{1}{F}\left[\frac{\sqrt{\beta_{x0}\beta_{y0}}}{2\pi}\right]\right| \equiv \left|\frac{1}{F}G_F\right|$$

The decoupling coefficients G_θ and G_F are also at their largest in the IR triplets.

A Simple Model

To better understand local coupling correction, we start with the ideal RHIC storage lattice tuned with two IR's at $\beta^* = 1m$ and the four others tuned to $\beta^* = 10m$. Roll errors of $1 mrad$ are applied to single quadrupoles in one of the triplets in a low β^* interaction region. Table 1 shows the beta functions, phase advance quadrupole strengths and the sensitivities of the triplet, corrector and the arc quadrupoles.

Table 1: Optical properties of the IR triplet quadrupoles and the local skew quad corrector, in a low beta storage insertion. Parameters for the arc F (horizontally focussing) and the arc D (vertically focussing) quadrupoles are included for comparison. Positive f implies vertical focussing. The vertical tune is taken to be $\nu_y = 29.185$. L is the length of the quadrupoles and μ_x and μ_y are the phase advance from the crossing to the element's center.

Name	$\beta_x\,[m]$	$\beta_y\,[m]$	$\eta_x\,[m]$	$\mu_x/2\pi$	$\mu_y/2\pi$	$L\,[m]$	$f\,[m]$	H	G
Quad 1	718	668	0.520	0.2448	0.2433	1.44	12.03	44	18.3
Quad 2	1354	550	0.731	0.2455	0.2444	3.40	-5.25	-117	-52.3
Quad 3	575	1313	0.490	0.2463	0.2453	2.10	8.46	116	32.7
Corrector	845	999	0.586	0.2459	0.2451			446	146.2
F quad	49.6	9.8	1.843			1.11	-11.02	-2.5	-0.6
D quad	10.4	48.6	0.940			1.11	10.66	6.5	0.7

From examining the sensitivities, H and G (with the source and reference point at the center of the element), listed in Table 1, we conclude that the linear coupling is a much more serious problem than vertical dispersion. For example, it would be convenient to set tolerances on the misalignment roll angle θ so that:

> **1:** The minimum tune split caused by one triplet quadrupole is much less than the nominal tune split of $\Delta\nu = 0.01$ (nominal RHIC tunes are $\nu_x = 28.190$ and $\nu_y = 29.180$).
>
> **2:** The vertical dispersion created is less than 0.1 meters.

Taking Quad 2 as the worst case example, the minimum tune split condition leads to a requirement that $\theta << 0.19 mrad$, while the vertical dispersion condition requires $\theta << 0.85 mrad$. The first of these two numbers is practically unobtainable. Hence, it will be necessary to correct local coupling sources in the RHIC triplets.

Simulation

The above discussion is tested by simulating $1 mrad$ rolls in the three triplet quadrupoles, denoted as θ_1, θ_2 and θ_3, firing up the corrector with a focal length, F_c and seeing the results. Simulations were performed using MAD [3]. Table 2 gives the results of applying the rolls in each of the individual quadrupoles in the triplets as well as turning on the corrector. The results for the vertical dispersion and tune split agree with the predictions of the G and H sensitivities given in Table 1, except in the sixth row, where the strength of the corrector becomes too large and the approximation.

The seventh row of Table 2 combines the rolls of the three quadrupoles in the triplets (with roll of Quad 2 reversed from the third row). From the last row in Table 2, we show that these rolls can be corrected with a single corrector. Furthermore, both the vertical dispersion and the tune split are corrected. This shows that vertical dispersion could be used as monitor to set the corrector.

Table 2: Results of a numerical experiment, adding selected errors to elements in one low beta triplet of the RHIC lattice. Horizontal and vertical fractional tunes were adjusted to be exactly $\nu = 0.185$, so that $\Delta\nu = \Delta\nu_{min}$ is due solely to coupling. η_{y1}, η_{y2}, η_{y3} and $\eta_{y\,corr}$ are the vertical dispersions at the center of the elements.

	θ_1 $[mrad]$	θ_2 $[mrad]$	θ_3 $[mrad]$	F_c^{-1} $[km^{-1}]$	η_{y1} $[m]$	η_{y2} $[m]$	η_{y3} $[m]$	$\eta_{y\,corr}$ $[m]$	$\Delta\nu$
1	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0.044	0.040	0.055	0.055	0.018
3	0	1	0	0	-0.138	-0.125	-0.193	-0.168	0.052
4	0	0	1	0	0.085	0.077	0.118	0.104	0.033
5	0	0	0	-0.5	-0.209	-0.189	-0.291	-0.254	0.074
6	0	0	0	-1	-0.718	-0.649	-1.001	-0.872	0.157
7	1	-1	1	0	0.339	0.308	0.474	0.414	0.106
8	1	-1	1	-0.707	-0.004	-0.002	-0.004	-0.003	0

Conclusion

Skew quadrupole errors in the low beta triplets contribute a significant amount of linear coupling and vertical dispersion to RHIC. Since the phase advance across the triplet is small, only a single skew quadrupole corrector is needed to correct vertical dispersion and linear coupling. We have demonstrated using a simple model that the skew quadrupole errors in the triplets can be corrected by this scheme. Furthermore, the vertical dispersion, which can be measured with the two dual plane BPM's at the ends of the triplets, can be used as a diagnostic to set the corrector. Note that only the vertical dispersion excited by the triplets is corrected. Since this is a major part, subtraction of the background vertical dispersion caused by skew quadrupole errors in the rest of RHIC may not be necessary.

References

[1] E. D. Courant and H. S. Snyder, "Theory of the Alternating-Gradient Synchrotron", Annals of Physics, 3, 1-48, (1958)
[2] S. Peggs, "Coupling and Decoupling in Storage Rings", IEEE Trans. Nucl. Sci., **NS-30**, 2460 (1983)
[3] H. Grote and F. C. Iselin, "The MAD Program User's Reference Manual", CERN/SL/90-13 (AP)

DATE FILMED

4/7/94

END