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**Momentum Transfer Dependence of Medium Effects in the (e,e') Longitudinal Response**

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Momentum transfer dependence of medium effects in  
the  $(e,e')$  longitudinal response

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Recent  $^{56}\text{Fe}(e,e')$  data at momentum transfer  $|\vec{q}| = 1.14$  GeV displays behavior which is qualitatively different from that of lower momentum transfers. An explanation of this difference is offered based on an analysis of the longitudinal response in nuclear matter. An ansatz is made for the momentum dependence of the nucleon self-energy functions in the nuclear medium which suppresses medium effects for momenta above the nucleon mass scale. This suppression is shown to improve the agreement with the high momentum transfer data, and offers a motivation for further experimental investigation in the momentum transfer region between 0.5 and 1.0 GeV.

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The essential characteristics of the longitudinal response obtained from the recent Stanford Linear Accelerator Center (SLAC) data [1] for  $^{56}\text{Fe}(e,e')$  at a momentum transfer of 1.14 GeV appear to be reproduced by the free relativistic Fermi gas model [2], while the longitudinal response extracted from data [3] for  $^{56}\text{Fe}(e,e')$  at 0.55 GeV is better described by mean field theory (MFT) in nuclear matter [4]. For comparison, in Fig.1 a representative sample of both data sets is shown along with the calculated response curves for the free Fermi gas (dotted line) and MFT in nuclear matter (solid line). These two models are limiting cases with respect to the treatment of the medium. That is, no medium effects are included in the free Fermi gas, while in nuclear matter the effect of the medium is constant throughout space. The fact that the data approaches the Fermi gas and nuclear matter descriptions at high and low momentum transfer respectively, implies that there is a net  $|q|$  (momentum transfer) dependence present in the influence of the medium on the longitudinal response. The goals of the present investigation are to demonstrate that the qualitative difference between the measured longitudinal response at 0.55 GeV and that at 1.14 GeV suggests a suppression of medium effects in the higher momentum transfer data, and to identify a scale at which this suppression should become apparent.

There is in general a characteristic length associated with a momentum dependence. Here a scale  $\Lambda$  is naively introduced for the momentum dependence of the nucleon self-energy in matter,  $\Sigma(\mathbf{k})$ . This dependence is chosen to suppress medium effects for momenta  $|\mathbf{k}|$  greater than  $\Lambda$ ; for simplicity a step-function dependence is assumed. The value of  $\Lambda$  is then adjusted to give qualitative agreement with both the low and high momentum transfer data for the longitudinal response, and is thereby constrained to lie approximately between 0.8 and 0.9 GeV. The dynamical origin of this suppression is not clear. Although existing calculations of the nucleon self-energy in nuclear matter including correlations [5] show indications of a decrease in the self-energy with increasing momentum, the slope is small and the calculations are inconclusive at the 1.0 GeV scale. It can also be speculated based on recent developments in the description of the interaction between composite hadrons that such an effect could arise from an underlying quark substructure [6]. The employed step

function is viewed as a crude way of incorporating a momentum scale at which a qualitative change in the behavior of the self-energy can be effected, thus interpolating between the Fermi gas and MFT description of the medium as a function of momentum transfer, and is not intended to reflect a particular dynamical mechanism. Momentum dependent self-energy functions have been used previously [7] in a similar context to investigate the role of final-state interactions with the nuclear matter medium, but have not been applied to the large momentum transfer data which is presently of interest.

Calculations of total  $(e,e')$  cross sections based on MFT in nuclear matter yield good fits to the data in the vicinity of the quasielastic peak at the lower momentum transfers [8,9], however the separated longitudinal and transverse response functions offer only qualitative agreement. In particular, the MFT calculation of the longitudinal response in nuclear matter, when applied to data, consistently overestimates the energy transfer region above the quasielastic peak, but reproduces the data below the peak reasonably well. This behavior is still evident in the present calculations, and is presumably a symptom of the approximate treatment of the final-state interaction of the struck particle with the finite nucleus by MFT in nuclear matter. The nuclear matter description is nevertheless sufficient for the investigation of the qualitative features to be discussed here. The benefit of using a simple model is that the observed effects can be directly correlated to changes in the introduced scale. More sophisticated models [10–14] have recently been applied to the description of response functions at momentum transfers near 0.5 GeV. Improvements to the longitudinal response by comparison with the data have been achieved, for example, through the study of final-state interactions [10] and many-body correlation effects [12] based on the nuclear optical potential [15] approach of Ref. [16], and the relativistic random-phase approximation [11] to the Walecka model [4]. The application of these more sophisticated treatments to the data at a momentum transfer near 1.0 GeV is clearly needed to corroborate the preliminary results presented here, however it is expected that the conclusions of this naive approach will survive.

The longitudinal response in a uniform medium can be written as [8,17]

$$R_L(\mathbf{q}, \omega) = -\frac{V}{(2\pi)^3} \int \frac{d^3p}{E(\mathbf{p})E(\mathbf{p}+\mathbf{q})} \Theta(\mu - \epsilon(\mathbf{p})) \Theta(\epsilon(\mathbf{p}+\mathbf{q}) - \mu) \times \delta(\omega - \epsilon(\mathbf{p}+\mathbf{q}) + \epsilon(\mathbf{p})) [T_1 + T_2 + T_3], \quad (1)$$

where reference to particle type (neutron or proton) has been suppressed. Here the single-particle eigenenergy  $\epsilon$  and the energy  $E$  are obtained from the Dirac equation

$$[\alpha \cdot \mathbf{k} + \beta M + (\Sigma_0 - \beta \Sigma_S) \Theta(\Lambda - |\mathbf{k}|)] U(\mathbf{k}, \sigma) = \epsilon(\mathbf{k}) U(\mathbf{k}, \sigma), \quad (2)$$

and for particle states are given by  $\epsilon(\mathbf{k}) = \Sigma_0 \Theta(\Lambda - |\mathbf{k}|) + E(\mathbf{k})$  and  $E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + [M - \Sigma_S \Theta(\Lambda - |\mathbf{k}|)]^2}$ . The chemical potential  $\mu$  is given the value of the single-particle eigenenergy at the Fermi momentum  $k_F$ . The quantities  $T_i$  are defined as

$$\begin{aligned} T_1 &= -F_1^2(q^2) [E(\mathbf{p})E(\mathbf{p}+\mathbf{q}) + M(\mathbf{p})M(\mathbf{p}+\mathbf{q}) + \mathbf{p} \cdot \mathbf{q} + \mathbf{p}^2] \\ T_2 &= 2F_1(q^2)F_2(q^2) [M(\mathbf{p})(\mathbf{p} \cdot \mathbf{q} + \mathbf{q}^2) - M(\mathbf{p}+\mathbf{q})\mathbf{p} \cdot \mathbf{q}] \\ T_3 &= F_2^2(q^2) [\mathbf{q}^2 (M(\mathbf{p})M(\mathbf{p}+\mathbf{q}) - E(\mathbf{p})E(\mathbf{p}+\mathbf{q}) - \mathbf{p} \cdot \mathbf{q} - \mathbf{p}^2) \\ &\quad + 2\mathbf{p} \cdot \mathbf{q} (\mathbf{p} \cdot \mathbf{q} + \mathbf{q}^2)], \end{aligned} \quad (3)$$

where  $M(\mathbf{p}) \equiv M - \Sigma_S \Theta(\Lambda - |\mathbf{p}|)$ , and the form factors  $F_1$  and  $F_2$  are obtained by fitting dipole forms to the free Sachs form factors  $G_E$  and  $G_M$  [18]. For  $^{56}\text{Fe}$ , the self-energy functions,  $\Sigma_S$  and  $\Sigma_0$ , are given the values 0.35 and 0.29 GeV respectively, and  $k_F = 0.26$  GeV [4]. From Eq.(2), it is evident that the medium effects provided by the self-energy functions are suppressed for solutions with momentum  $|\mathbf{k}|$  greater than  $\Lambda$ .

The additional momentum dependence introduced by the step function in Eq.(2) could in general imply the existence of medium modifications in the photon-nucleon vertex [19]. However, without explicit knowledge of the underlying dynamics responsible for this additional momentum dependence, modifications made to the portions of the vertex that are relevant to the response functions are largely arbitrary. It is well known, for example, that gauge invariance alone is not a sufficient constraint to uniquely determine the vertex [20]. In fact, aside from the condition provided by the Ward identity at zero momentum, gauge invariance only restricts the contribution to the vertex that is longitudinal to the four-momentum

transfer,  $q_\nu$ , which does not contribute to electron scattering. Medium modifications to the vertex will therefore not be considered in the qualitative discussion offered here, but will instead be left for a future investigation.

In this description of the longitudinal response, the position of the quasielastic peak is largely dictated by the argument of the energy-conserving delta function in Eq.(1), and therefore by the single-particle energy eigenvalues. The magnitude of the initial single-particle momentum,  $|\mathbf{p}|$ , is restricted by the chemical potential to be less than the Fermi momentum,  $k_F$ . The magnitude of the final single-particle momentum,  $|\mathbf{p} + \mathbf{q}|$ , is then restricted to lie between  $|\mathbf{q}| - k_F$  and  $|\mathbf{q}| + k_F$ . This allows three qualitatively distinct regions of interest with respect to the scale  $\Lambda$  to be identified:

$$\begin{aligned}
 \text{I.} \quad & \Lambda < k_F, \\
 \text{II.} \quad & k_F < \Lambda < |\mathbf{q}| - k_F, \\
 \text{III.} \quad & |\mathbf{q}| + k_F < \Lambda.
 \end{aligned}
 \tag{4}$$

The successful application of MFT to the description of nuclei [21] implies that values of  $\Lambda$  in region I are not physically sensible since these would require modifications of the single-particle energies below the Fermi level. This also excludes the free Fermi gas description, which is obtained for  $\Lambda = 0$ . The MFT description in nuclear matter is obtained for values of  $\Lambda$  in region III. There, both the initial and final single-particle energies include the medium effects provided by the constant self-energy functions. For values of  $\Lambda$  in region II, the initial energy,  $\epsilon(\mathbf{p})$ , is that of a particle in the nuclear matter medium, while the final energy,  $\epsilon(\mathbf{p} + \mathbf{q})$ , is that of a free particle. That is, the particle in the final state does not interact with the medium. Values of  $\Lambda$  in the region between  $|\mathbf{q}| - k_F$  and  $|\mathbf{q}| + k_F$  produce a family of curves which lie between those of region II and region III, and represent a suppression of medium effects for single-particle final states with momentum greater than  $\Lambda$ .

The reproduction of the MFT results for  $|\mathbf{q}| = 0.55$  GeV requires that  $\Lambda$  is in region III or greater than 0.81 GeV. Conversely, for  $|\mathbf{q}| = 1.14$  GeV, values of  $\Lambda$  greater than

$|\mathbf{q}| - k_F = 0.88$  GeV are ruled out by the data since these lead to a significant shift in the position of the quasielastic peak toward higher energy transfer, and hence away from the data, due to the medium modification of the single-particle final state energy. The implication is that any chance of fitting both data sets requires  $\Lambda$  to be somewhere between 0.8 and 0.9 GeV. Shown in Fig.2 (a) and (b) are the longitudinal response curves calculated from Eq.(1) with  $\Lambda = 0.84$  GeV, and  $|\mathbf{q}| = 1.14$  and 0.55 GeV respectively. Although the fit to the data is not quantitatively correct at energy transfers above the quasielastic peak, where it has been suggested [14] that the reported experimental uncertainties are largely underestimated, it is now qualitatively the same for both the low and high momentum transfers. We are therefore lead to conjecture that the interaction of the struck particle with the medium is suppressed in the higher momentum transfer case. This behavior persists in the local density approximation to MFT for the finite nucleus where the fit to the data is improved, for example the position of the peak more closely coincides with the data; however it is not sufficiently different (qualitatively) from the nuclear matter case to warrant reporting here.

At larger momentum transfers, the longitudinal response is effectively describing charge density correlations over small distances. From the present analysis it appears that the medium has less effect on these short range density correlations than at lower momentum transfers where the distances probed are sufficiently large that the influence of the medium on density correlations is significant. More data, and a more sophisticated treatment are required to make a conclusive statement. Nevertheless, this analysis suggests that data with momentum transfers  $|\mathbf{q}|$  above 1.14 GeV should display behavior similar to that at 1.14 GeV, since these exceed the scale at which the struck particle is influenced by the medium. Further, the data for momentum transfers between 0.5 GeV and 1.0 GeV should display a momentum dependence which is transitional in character; particularly in the vicinity of the scale  $\Lambda$  where an enhancement of the response may occur.

In summary, the longitudinal response data examined here display a dependence upon the transferred momentum,  $|\mathbf{q}|$ . Such an effect has been noted previously [22] from a purely

theoretical basis. In the present investigation, the momentum dependence is attributed to the influence of the medium on the particle in the final state. This was demonstrated here by comparing two calculations of the longitudinal response, which are limiting cases with respect to the inclusion of medium effects on the particle in the final state, with response data at large and intermediate momentum transfers. From this analysis of the data, it is conjectured that the medium effects on the particle in the final state at a momentum transfer of 1.14 GeV are suppressed relative to those at a momentum transfer at 0.55 GeV. This is consistent with the recent  $y$ -scaling analysis [23], where it has been shown in a non-relativistic framework that for large momentum transfer the longitudinal response approaches the result obtained in the plane wave impulse approximation; indicating a suppression of final state interactions. Using a crude mechanism for suppressing medium effects for momenta above a scale  $\Lambda$  in the nucleon self-energy, it is estimated here that this suppression should become apparent as the momentum transfer approaches  $\Lambda \sim 0.8 - 0.9$  GeV.

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## FIGURES

FIG. 1. The longitudinal response for  $^{56}\text{Fe}$  at momentum transfers (a)  $|\mathbf{q}| = 1.14$  GeV (data without Coulomb corrections from Ref. [1]) and (b)  $|\mathbf{q}| = 0.55$  GeV (data from Ref. [3]) is plotted as a function of energy transfer,  $\omega$ . Calculations are shown for the free relativistic Fermi gas model (dotted line) and mean field theory in nuclear matter (solid line).

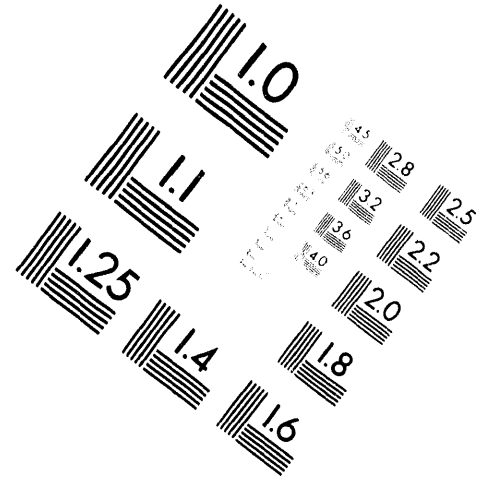
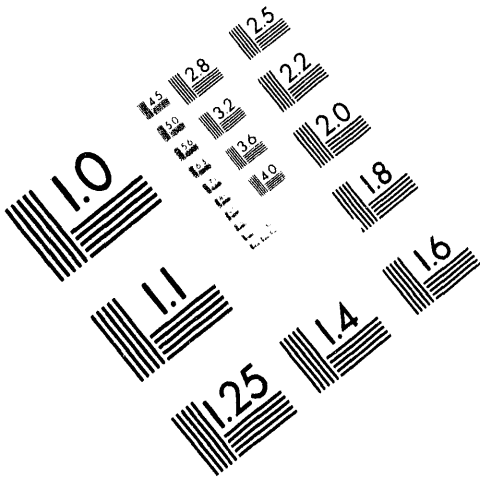
FIG. 2. The longitudinal response for  $^{56}\text{Fe}$  at momentum transfers (a)  $|\mathbf{q}| = 1.14$  GeV (data without Coulomb corrections from Ref. [1]) and (b)  $|\mathbf{q}| = 0.55$  GeV (data from Ref. [3]) is plotted as a function of energy transfer,  $\omega$ . The calculation shown is for  $\Lambda = 0.84$  GeV in the model proposed in the text.



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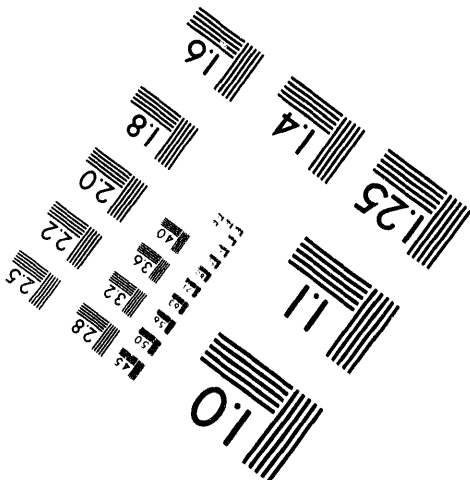
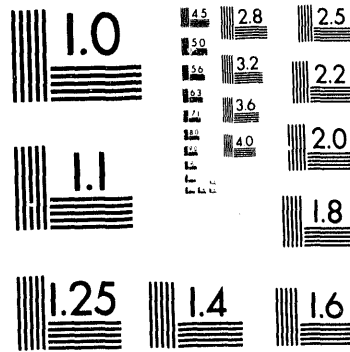
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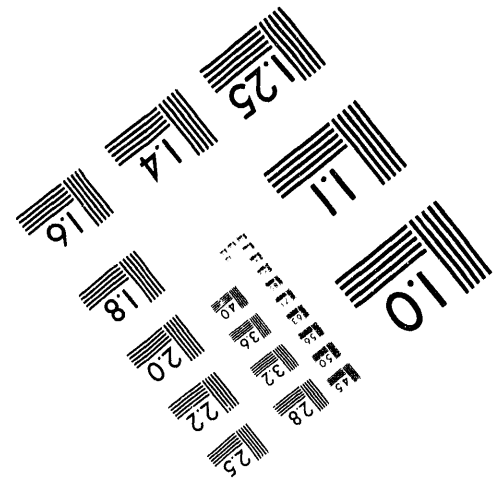
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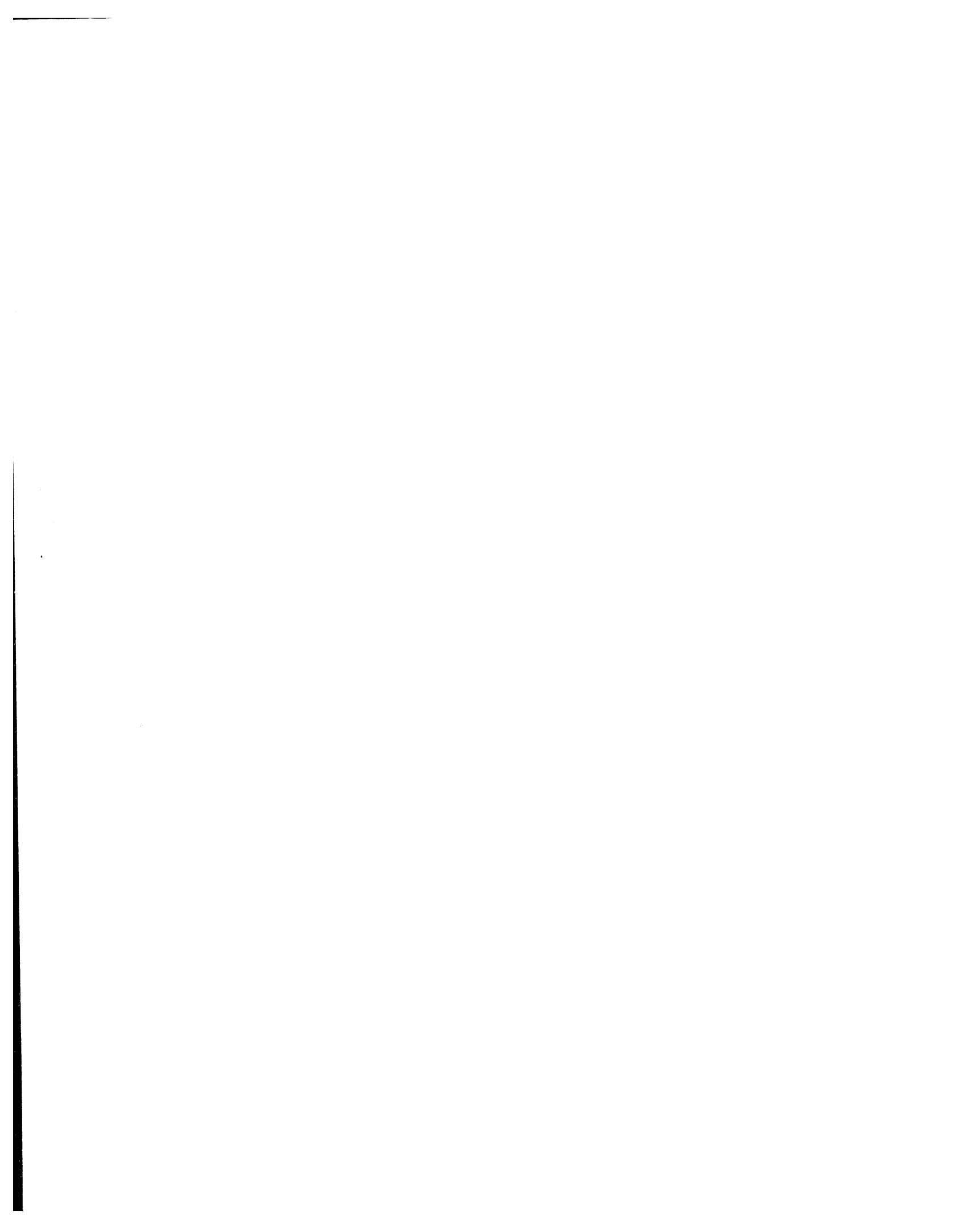


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