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Design of a Miniature Explosive Isentropic Compression Experiment

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Abstract

The purpose of this design study is to adapt the High Explosive Pulsed Power Isentropic Compression Experiment (HEPP-ICE) to milligram quantities of materials at stresses of ~ 100 GPa. For this miniature application we assume that a parallel plate stripline of ~ 2.5 mm width is needed to compress the samples. In any parallel plate load, the rising currents flow preferentially along the outside edges of the load where the specific impedance is a minimum [1]. Therefore, the peak current must be between 1 and 2 MA to reach a stress of 100 GPa in the center of a 2.5 mm wide parallel plate load; these are small relative to typical HEPP-ICE currents. We show that a capacitor bank alone exceeds the requirements of this miniature ICE experiment and a flux compression generator (FCG) is not necessary.

The proposed circuit will comprise one half of the 2.4-MJ bank, i.e., the 6-mF, 20-kV, 1.2 MJ capacitor bank used in the original HEPP-ICE circuit. Explosive opening and closing switches will still be required because the rise time of the capacitor circuit would be of the order of $30\ \mu\text{s}$ without them. For isentropic loading in these small samples, stress rise times of $\sim 200\ \text{ns}$ are required.

Introduction

The task of this study was to determine the feasibility of performing HEPP-ICE on milligram quantities of metals at Los Alamos. A companion report gives more details of the study [2]. The sample dimensions determine the required rise time of the current in the load that is shown to be $\sim 200\ \text{ns}$. From this it is found that, given the maximum size of the samples that are anticipated, a *magnetic flux compressor* is *not* required in a miniature HEPP-ICE circuit. Based on that finding, the HEPP circuit is designed around an existing opening switch technology that is used to deliver the correct current profile to the load.

This required current rise time, 200 ns, is fast for a typical HEPP experiment and innovative techniques have been designed to obtain it. An explosive opening switch is used to produce a ~ 3.5 -MA peak current with a rise time of $\sim 500\ \text{ns}$. However, only ~ 1 MA is needed to achieve the necessary pressures in the sizes of sample considered here. Consequently, the rise of the current to 1 MA takes 200 ns; after that time, even though the current keeps rising, acoustic relief waves in the sample terminate the pressure rise and hence the experiment.

The performance of the HEPP Miniature ICE experiment has been modeled in some detail using an electromechanical hybrid simulation. Analytic and partial differential equation solving software have been combined to model the magnetic loading of the samples.

These results were integrated into a SPICE based circuit code to couple the intimate interaction of the electrical circuit with the dynamic impedance of the load. The results of a simulation of the miniature HEPP-ICE experiment show that isentropic EOS data can be acquired at pressures of ~ 1 Mbar in tungsten samples of ~ 2 to 3 mm diameter and 0.5 to 1 mm thickness.

Description of HEPP-ICE

The ICE experiment uses rapidly rising magnetic fields to compress materials to these high stresses over periods of several hundred nanoseconds. By observation of the material response to the stresses a complete set of data is acquired in one experiment, i.e., continuously from zero up to the peak stress. High quality isentropic EOS data have been obtained for many materials. The ICE work was first performed on the Z-machine at the Sandia National Laboratory [3]. Recent work at LANL employed an explosive version of the ICE method, which uses a simple and compact explosive apparatus [4]. The HEPP-ICE technique has been shown to provide higher stresses than other methods and the inaccuracies in stress may be as small as 0.2%. In this study, the exceptionally small sample size offers some advantages, also some challenges, compared to conventional HEPP-ICE experiments.

The basic compression mechanism for ICE is the isentropic magnetic loading of two or more identical samples of different thicknesses, situated on the rear surfaces of a pair of parallel conductors; the complete configuration is called the load. These samples may be separate from the conductors or, in the case of HEPP-ICE, may sometimes be integrated into the conductors as one solid piece, i.e., if the samples are good conductors. In typical HEPP-ICE experiments, the current rise times are 500 ns to 1 μ s, although rise times of several microseconds might ultimately be used in the larger HEPP-ICE systems [4]. As will be seen later, the rise times for the proposed milligram-sized experiments would need be < 300 ns. Two samples, with a difference in thickness of h , are compressed by identical magnetic forces. Laser velocimetry techniques [5, 6] are used to measure the surface velocity profiles, $U_{fs}(t)$, at the rear surfaces, Figure 1.

Using *Lagrangian* wave analysis [7], the wave speed $c_L(u)$ is calculated for the waves to travel a distance h at each value of $u(t)$ in time as shown. The result is a continuous plot of c_L as a function of the particle velocity u ; $c_L(u) = h / \Delta t(u)$. From the Rankine-Hugoniot equations [8, 9] we calculate the change in stress, $d\sigma$, corresponding to each small step in particle velocity, du , ascending or descending a ramp wave, i.e., $d\sigma = \dot{m}du$; the mass flow per unit time per unit area is \dot{m} . In Lagrangian coordinates, $\dot{m} = \rho_0 c_L$ and $d\sigma = \rho_0 C_L(u)du$. The total stress as a function of time is then obtained by integration.

In practice the samples are not aligned as in the left of Figure 1, but are situated on opposite sides of a slit separating two parallel conductors. The inside surfaces of the conductors share a common magnetic field, P_B , i.e., the magnetic loading on the two samples is *exactly* matched; exact field matching is an essential requirement of the technique. In this analysis for milligram-sized samples, the sample thicknesses will be hundreds of microns and the slit microns wide.

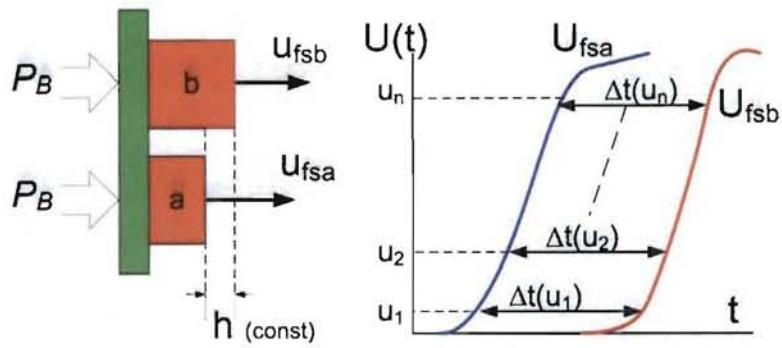


Figure 1. Left: B-forces (P_B) compress samples a and b. Surface velocities U_{fsa} and U_{fsb} are measured on the rear (outside) surfaces. Right: Surface velocities $U_{fsa}(t)$ and $U_{fsb}(t)$.

Relief waves and their effect on the compression wave

When the opposing currents start to flow on the inside surfaces of the load, identical compression waves are launched from the inside surface of each conductor into the sample and they travel at the speed of the compression waves; that wave speed is pressure dependent. At pressures in the vicinity of 100 GPa the Lagrangian wave velocity is ~ 6 km/s in tungsten. When those waves reach the back free surface, i.e., an interface with air or a vacuum, they are reflected such that the pressure at the free surface is always zero. These negative going pressure reflections (relief waves) sweep back into the sample, as they do they oppose the original positive pressure, eventually returning it to zero. When those waves arrive back at the inside surface the electrodes are torn apart and the experiment is over.

For the proposed experiments the goal was to limit the combined sample masses to a maximum mass m_{max} . To minimize the effects of the lateral rarefactions, thus maintaining a useful region of one-dimensional flatness, the thicknesses of the cylinders must be limited. To facilitate the use of optical diagnostics which have typical spot sizes of ~ 1 mm diameter, the central flat region must have a radius of $r_{min} = 1/2$ mm. Consequently for a cylinder of outer radius r and thickness x the minimum mass of the cylinder becomes $m_{max} = \rho_0 \times \pi(x + r_{min})^2 x$ where ρ_0 is the density. From these considerations a maximum thickness of 500 μm to 600 μm was chosen. For these thicknesses the planar relief waves traverse twice the thickness at a velocity of 6 km/s, which limits the duration of the experiment to 166 to 200 ns.

Therefore, to attain a high pressure, the current must rise fast enough to reach the peak before the relief waves arrive. However, if the rise time is too short the advancing compression wave may steepen into a shock wave thus destroying the isentropic nature of the experiment. The optimum rise time depends on the thickness of the samples, the wave velocity in the samples, and the relationship between wave velocity and pressure for the sample. The movement of the conductor boundaries and the changes in temperature and conductivity of the conductors must eventually be modeled with an 3-D MHD-hydrocode. For now, approximations have been used which have proved to be of good accuracy.

The standard and miniature HEPP-ICE circuits

The typical HEPP-ICE circuit for load currents of 5 to 10 MA comprises a 6-mF, 20-kV, 1.2 MJ capacitor bank that provides a seed current of \sim 2 MA to a 4x1, 4-in \times 5-in plate flux compression generator (FCG) [10-15]. The FCG transfers up to 12 MA into a storage inductor of \sim 25 nH and an explosively-formed fuse (EFF) opening switch [16-19]. Three parallel explosively-driven polyimide closing switches then close to transfer current to the load at the appropriate time. This circuit is capable of delivering 7 to 10 MA with a rate of rise, dl/dt , of $\sim 3 \times 10^{13}$ A/s into ICE loads of 1 to 2 cm width with the required rise times and will produce isentropic compression at pressures in the range of 0 to 250 GPa. For this application we assume that a load of \sim 2.5 mm width is needed to compress the samples. To reach a pressure of 100 GPa a current density of $J = 400$ MA/m is required, and at a width of 2.5 mm the peak current must therefore be 1.0 MA. This current is small relative to typical HEPP currents. Our 2.4-MJ capacitor bank is capable of producing currents up to 4.5 MA. Consequently, this capacitor bank exceeds the requirements of the mini-ICE experiment and *a flux compressor will not be necessary*.

As a flux compression generator is not necessary, the HEPP-ICE circuit may be simplified as shown in Figure 2. The new circuit could comprise one half of the 2.4-MJ bank, i.e., a 6-mF, 20-kV, 1.2 MJ capacitor bank. It is anticipated that a current of \sim 3 MA will be required which will be transferred to an EFF opening switch. Just one explosively-driven closing switch would be required to transfer current to the load at the appropriate time. An explosive EFF opening switch, storage inductor and closing switch would still required because the rise time of the capacitor circuit would be of the order of 30 μ s without these components.

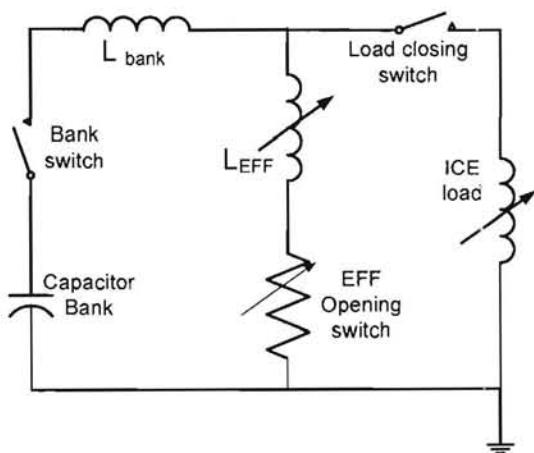


Figure 2. Proposed miniature HEPP-ICE circuit without a flux compressor.

Initially all closing switches would be open and the EFF opening switch would be closed. The sequence of events would be that once the capacitor bank had been charged to the required voltage the capacitor bank closing switch would be closed. Current would then flow from the bank through the inductances L_{bank} and L_{EFF} of the circuit and the EFF opening switch. At the time of peak current the EFF would be initiated and its resistance would rise over a few microseconds thereby increasing the voltage that appears at the load closing switch. When the load switch voltage has reached the desired value the load switch would be closed and current will flow to the load.

Relief waves reduce pressure rise time

The arrival of relief waves from the outside surface terminates the rise in pressure in the load. So if the load is overdriven with a larger than necessary current the relief waves reduce effective rise time.

An EFF opening switch design for Mini-ICE

The original EFF design for ICE [20] was much larger than necessary for the miniature HEPP-ICE experiment and a new opening switch was designed. Smaller switches are easy to manufacture and a relatively small 40-cm wide (12-cm diameter) switch capable of producing 100 kV suffices. The standard EFF design comprises a Teflon die, an explosive drive system, an aluminum foil, return conductor and insulation. A series of 12.7 mm deep, 6.0 mm wide grooves are cut across the die with a center-to-center spacing of 7.5 mm. The dimensions of the teeth in the die pattern, the metal type and thickness are not changed with the scale of the switch, only the number of teeth and the width of the conductors are changed.

The rise time of the load current, i_{load} , is primarily a function of the resistances and inductances of the load and EFF circuit. If i_{bank} is the bank current at the moment of load switch closure then $i_{load} \approx i_{bank} (1 - e^{-t/\tau})$ where $\tau = (L_{eff} + L_{load}) / R_{eff}(t) \approx \tau_{eff}(t)$ [2]. The dominant time constant is that of the EFF switch, $\tau_{eff}(t) \propto 1 / R_p(t)$ and is independent of the EFF dimensions; it only depends on $R_p(t)$ which is the standard resistivity per pattern. This is very convenient because as $R_p(t)$ is a function of time, the rise time can be controlled by the timing of the EFF switch in relation to the closure of the load switch. For a typical EFF the maximum resistance would be 3.28 mΩ.m/pattern which corresponds to a minimum characteristic time $\tau_{eff}(t) = 64$ ns.

SPICE analysis of circuit

For experimental design and subsequent data analysis, it is important to have good predictive models of the system performance. For the HEPP-ICE circuit this is not straightforward. The capacitor bank and closing switches may be modeled by simple circuit parameters, but the other circuit components are dynamic. The explosive components and load are time dependent, and the load is also current dependent, so accurate dynamic models had to be developed for each. The EFF resistance is a dynamic function of time and an accurate model had to be developed for it. The EFF model was a combination of an experimental resistance versus time relationship, $R(t)$, and a calculated inductance model for $L(t)$ and $dL(t)/dt$.

Load model for non-uniform current distribution

To determine the uniformity of current flow in the load conductors, the load section was modeled to solve for the magnetic field distribution as a function of time. This had to be done by assuming that the conductors were in a fixed position, because the FlexPDE software that was used made no provision for dynamic boundaries. It is nonetheless useful to do these calculations before hydrodynamic flow is added to the model. The movement of boundaries was accommodated by repeating the calculations for different load separations. In addition, it was assumed that the conductivity of the conductors was constant, which is incorrect because the current densities are high enough to melt and perhaps ablate the surfaces. The movement of the boundaries and the change in temperature and state of the conductors will eventually be modeled with an MHD-hydrocode such as MACH2 [21].

Two-dimensional magnetic field calculations were performed with FlexPDE, a partial differential equation (PDE) problem solver [22]. The results are shown in Figure 3 for a stepped load at 200 ns, when relief waves end the experiment. In this simulation the temperature of the

tungsten conductor was arbitrarily set to 1000°C. At 200 ns the total load current was 1.42 MA and the calculated pressure was 110 GPa in the center of the load.

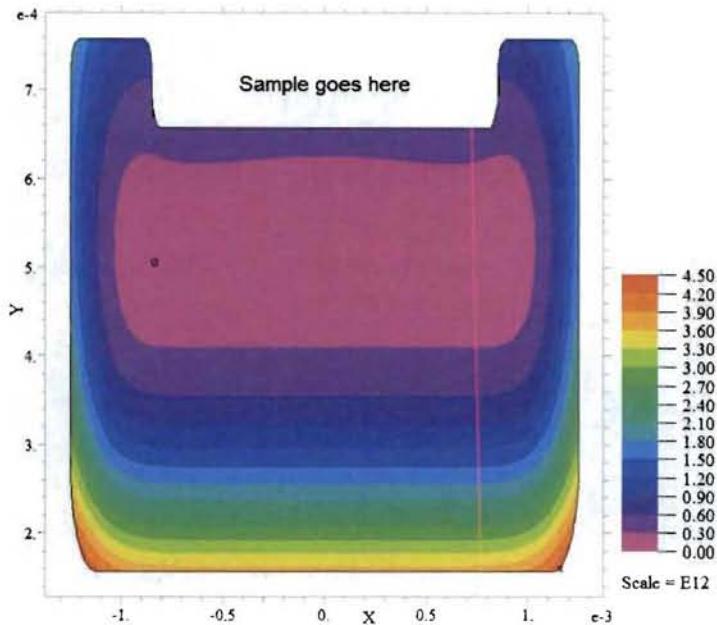


Figure 3. The aerial load current densities (A/m^2) at 200 ns. The X and Y dimensions are not to scale. Sample goes in the top, the slit as at the bottom.

The sample in the top of the figure will be surrounded by a confining material to mitigate against lateral reflections. In cross-section therefore the load will appear to be stepped on its outside surface. The steps not only mitigate against lateral reflections but partially screen the sample from the prevailing magnetic field. The steps shown in Figure 3 were arbitrarily chosen to be 0.1 mm wide and 0.4 mm high.

Transient pressure load model

It was found that the transient pressures in the central flat region normal to the surface, P_y , could be accurately fitted with a simple logarithmic function, $P_y = \frac{1}{2} \mu_0 (0.11853 - 0.21641 \times \ln \frac{h}{W}) J^2$, where J is the linear current density. The ratio of h/W rarely exceeds $\frac{1}{2}$ before the EOS phase of the experiment is over. In the calculations that follow, each tungsten electrode will have moved by 180 μm by the end of the EOS stage of the experiment, so $h/W = 0.36/2.5$; the pressure is then reduced to $0.54 \hat{P}$. It is clear that this logarithmic reduction in pressure, as the electrodes repel each other, ultimately limits the peak pressure attained in an HEPP-ICE experiment. This pressure expression was incorporated into the SPICE model for the load as a quasi-2D approximation and the calculated peak pressures were then found to be close to the experimental values obtained in previous HEPP-ICE experiments. All these separate models have been incorporated into one SPICE circuit, shown in Figure 4, which provides the rapid and accurate calculations of ICE circuit performance.

In the figure C1, L_b and R_b are the capacitance, inductance and resistance of the capacitor bank, its cables and connections between the bank and the experiment; L_s is the storage inductance; R_{diff} is the diffusion resistance of the primary circuit, i.e., of all parts of the circuit not involving the load.

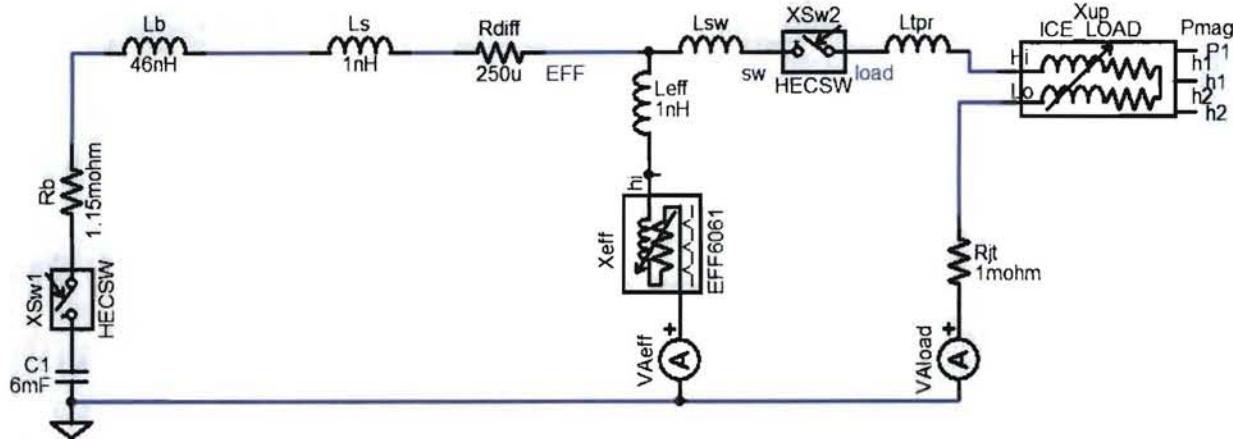


Figure 4. The complete SPICE circuit simulation for the miniature-HEPP-ICE experiment. X_{up} is the load simulation circuits, h_1 and h_2 are the load displacements and P_1 is the common stress; X_{eff} is the EFF opening switch; HECSW is an explosively-loaded closing switch.

Transmission line analog to load plates

The wave propagation and reflection in the expanding electrodes were modeled as an electromechanical analog, where mechanical particle velocity was equated to electrical current and mechanical pressure to electrical voltage. In the analog, each load with a free surface at the rear face is treated as a short-circuited transmission line. The electrical impedance of the transmission line Z was equated to the acoustic wave impedance of the load. The wave impedance is the product of mechanical Lagrangian wave speed c_L of the pressure disturbance in the plates and the initial density of the plates, ρ_0 , i.e., $Z = \rho_0 c_L$. The delay time of each transmission line was set to its length divided by c_L . The wave velocity c_L is a function of pressure and thus increases with time during the ICE experiment. It was difficult to model such a *dynamic* transmission line in SPICE, because the code tended to become unstable. After many attempts the most stable solution was found using a 64-staged lumped RCL circuit model for the transmission line where the capacitance per stage was a function of the applied voltage and the inductance per stage was fixed.

The load inductance $L(t)$ of the actual mini-ICE experiment was calculated according to $L(t) = \mu_0 K I \times (d(t)/W)$ where K is a function of W/d derived from an analytic solution [23] and I is the length of the conductors. $K = 1$ when W/d is large and $K = 1/2$ when $W/d = 1$.

SPICE simulation of miniature ICE experiment

The circuit calculation in Figure 5 is for two tungsten samples, 500 and 600 μm thick. The point X marks the time when the relief waves arrive back at the inside surface of the thinnest sample, h_2 . After this time, the surface velocity data cannot be used for EOS analysis because the internal stresses in the two samples are no longer matched. The peak pressure at that time is 116 GPa and the time is 185 ns after the start of load current flow.

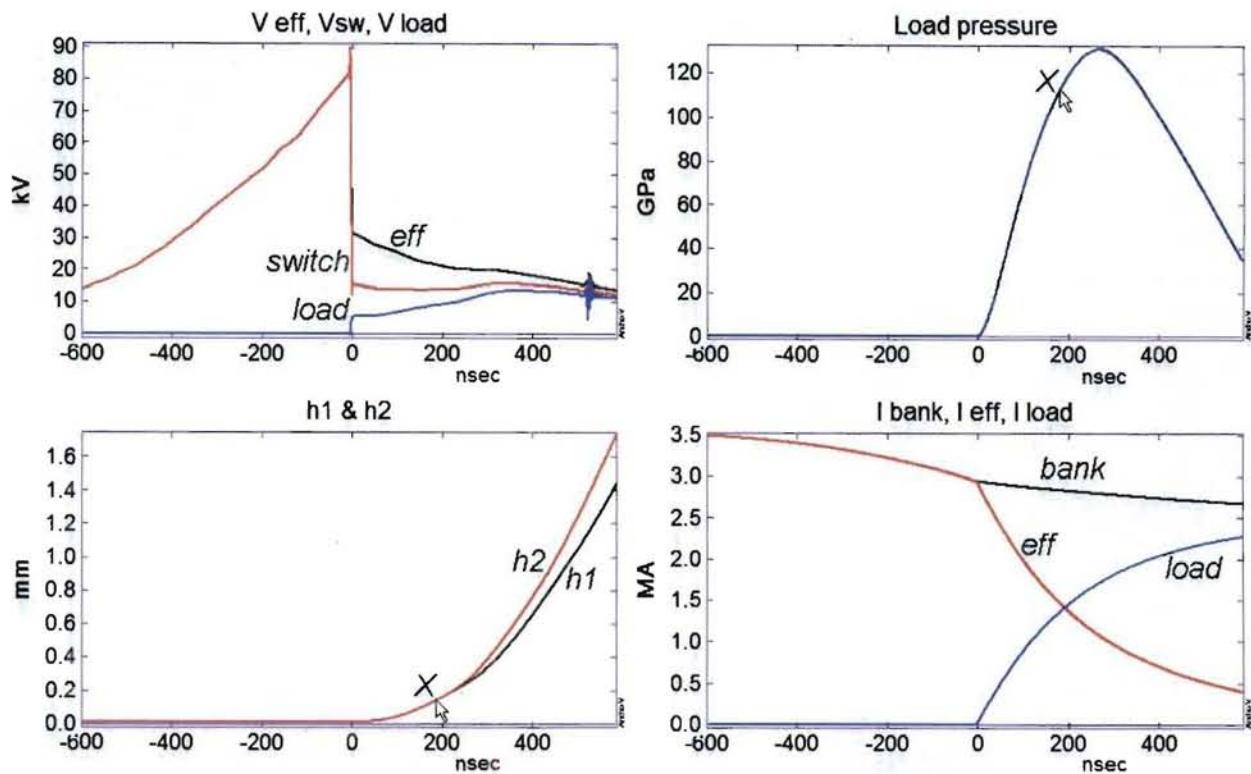


Figure 5. The calculation of the miniature ICE load response. Top left: the EFF, switch and load voltages. Top right: load pressure. Bottom left: sample displacements. X is the time when relief waves arrive in the smaller sample (h2) to end data collection and terminate the EOS experiment. Bottom right: bank, EFF and load currents. The bank current started flowing at $-30\mu\text{s}$ on this time scale.

In the calculations the sinusoidal peak current in the EFF was 3.5 MA. Note that the rise time of the load current to its peak is close to 500 ns but the arrival of the relief waves terminate the pressure rise in the sample thereby shortening the effective rise time of the pressure to 185 ns.

Conclusions

It was found that samples as small as 1.2 mm in diameter can be used to produce useful EOS data. Consequently, a total sample mass of ~ 30 mg may be possible with a 1.4-mm combined thickness. The sample dimensions determine the required rise time of the current in the load which was shown to be approximately 200 ns. From this it is found that a *magnetic flux compressor* is *not* required in a miniature HEPP-ICE circuit. The required current rise time, 200 ns, is fast for a typical HEPP experiment and innovative techniques have been designed to obtain it. An explosive opening switch is used to produce a ~ 3.5 -MA peak current with a rise time of ~ 500 ns.

However, only ~ 1 MA is needed to achieve the necessary pressures in the sizes of sample considered here. Consequently, the rise of the current to 1 MA takes 200 ns; after that time acoustic relief waves terminate the pressure rise and hence the experiment. The performance of a suitable miniature HEPP-ICE experiment has been estimated with a hybrid electromechanical simulation using a combination of analytic and computational techniques. The results of the simulation show that isentropic EOS data can be acquired at pressures of ~ 1 Mbar in tungsten samples of ~ 2 to 3 mm diameter and 0.5 to 1 mm thickness.

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