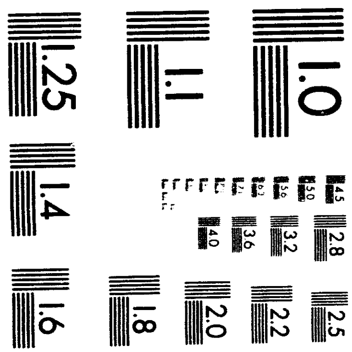
1.25
1.1
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DOE/ER/13846-5

MATHEMATICAL MODELS OF HYSTERESIS

Progress Report

January 1993 - December 1993

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September 1993

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY
UNDER GRANT NUMBER DEFG0588ER13846

MATHEMATICAL MODELS OF HYSTERESIS

Progress Report

The research undertaken under the grant No. DEFG0588ER13846 during the past year of the project has been in complete compliance with the work statement of the grant. The main results obtained during this period can be briefly summarized as follows:

1. Further development of vector Preisach-type models of hysteresis.

Here, our research has been focussed on the development of new generalized vector Preisach models which circumvent deficiencies of the classical vector Preisach model. Similar to the classical vector Preisach model, the developed generalized vector Preisach models are constructed as superpositions of scalar Preisach models continuously distributed along all possible directions. However, these scalar Preisach models are driven not by input projections (as in the case of the classical vector model), but by "generalized" input projections. These "generalized" input projections are mappings of the vector input realized by some function g. This function is not specified in advance but rather has to be determined through the identification procedure. The presence of additional unspecified function g opens the opportunity to match experimental data obtained from both "scalar" and "rotational" experiments. This is the most important additional feature of the generalized vector Preisach models as compared to the classical vector Preisach model. We have developed a mathematical machinery for the solution of the identification problem for the generalized vector Preisach models of hysteresis.

2. Modeling of rotational hysteretic losses.

It is well-known that in the case of isotropic hysteresis a uniformly rotating magnetic field results in uniformly rotating magnetization. This magnetization lags behind the magnetic field due to hysteretic losses. Depending on the physical nature of hysteresis, these losses may decrease to zero or even continuously increase as the magnitude of the uniformly rotating magnetic field is increased. It is the generally held opinion that the former case is realized for magnetic hysteresis, while there is some credible evidence that the latter case is true for superconducting hysteresis. Successful Preisach models of vector hysteresis should be able to accommodate this broad spectrum of behavior of rotational hysteretic losses. We have demonstrated that this is true for the generalized isotropic vector Preisach models of hysteresis. Namely, we have shown that, with appropriate choice of function g, the generalized vector models can replicate various asymptotic behavior of rotational hysteretic losses.

3. Experimental testing of generalized vector Preisach models of hysteresis.

The experimental testing of generalized isotropic vector Preisach models of hysteresis has been performed for an isotropic magnetic tape material Ampex-641 ($\gamma - Fe_2O_3$). This has been accomplished by using a vibrating sample magnetometer equipped with orthogonal pairs of pickup coils. The performed testing has demonstrated that the generalized vector hysteresis models have superior ability to mimic the correlation between mutually orthogonal components of output and input than the classical vector Preisach model. It is important to note that the correlation between orthogonal components of input and output has long been regarded as an important "testing" property for any vector hysteresis model in the area of magnetics.

4. Development of Preisach-type models for aftereffect.

It is well-known that the physical origin of hysteresis is due to the multiplicity of metastable states. At equilibrium, large deviations of random thermal perturbations may cause a hysteretic system to move gradually from higher to lower energy metastable states. This phenomenon is generally referred to in the literature as "aftereffect," "viscosity," or "creep". Traditionally, the modeling of hysteresis and aftereffect has been pursued along two quite distinct lines. In phenomenological modeling of hysteresis the Preisach approach has been prominent, while the aftereffect has been studied by using thermal activation type models. It is desirable to develop the uniform approach to the modeling of both hysteresis and aftereffect. Recently, it was suggested to use the Preisach model driven by stochastic inputs as model for aftereffect. However, the stochastic inputs were modeled by discrete time i.i.d. (independent identically distributed) random processes. During the past year, we have further extended this approach by modeling the stochastic inputs by continuous time diffusion processes. From the mathematical point of view, it makes the problem much more complicated. We have shown that these difficulties can be largely overcome by using the mathematical machinery of the "exit problem" for diffusion processes. By using this machinery, we have developed an analytical technique for the calculation of time evolutions of the expected value of the output of the Preisach model driven by a diffusion process.

5. Analytical investigation of penetration of electromagnetic fields into superconductors with gradual resistive transitions.

Models of superconducting hysteresis are based on analytical study of penetration of electromagnetic fields into hard superconductors. In critical state-type models, this study has been carried out under the assumption of ideal (sharp) resistive transition. However, actual resistive transitions are gradual and they are usually described by the following "power law"

$$E = \left(\frac{j}{k}\right)^n, (n > 1),$$

where E is an electric field and j is a current density.

The exponent "n" is a measure of the sharpness of the resistive transition and it may vary in the range 4-1000. At first, the power law was regarded only as a reasonable empirical description of the resistive transition. However, recently there has been a considerable research effort focused on the theoretical justification of power law. As a result, models based on flux creep, Josephson-junction coupling between grains, sausaging and spatial distribution of critical current have been proposed. In our research work, power law (1) was used as a constitutive equation for superconductors. By using this equation, the penetration of electromagnetic fields into superconductors with gradual resistive transition has been analytically investigated. This investigation requires the solution of nonlinear partial differential equations of parabolic type. It has been shown that the exact analytical solutions can be found in the case of circular polarization of electromagnetic fields due to the high degree of symmetry of the problem under consideration. In the limiting case of sharp (ideal) transition, the obtained solutions are reduced to those previously asserted by C.P. Bean.

6. Computation of magnetic fields in hysteretic media.

Several attempts were made in the past to utilize the Stoner-Wohlfarth model in the computation of magnetic fields in hysteretic media by using the integral equation technique. Then, it was shown that the vector Preisach models had several advantages over the Stoner-Wohlfarth model. Among those advantages are the ability to describe nonsymmetrical minor loops, the existence of well-defined identification procedures, and the relative numerical simplicity. This prompted the idea of using the vector Preisach model in computations of magnetostatic fields in media with hysteresis. This idea was realized and the solution of the 2-D magnetostatic problem in hysteretic media was attempted. Recently, more accurate generalized vector Preisach-type models have been developed. For this reason, we have extended the integral equation approach to solve 3-D magnetostatic problems in hysteretic media by utilizing those vector Preisach models. The very nature of the hysteresis phenomenon suggests that the magnetization at any instant of time is dependent on the past history of magnetic field variations. For this reason, a time stepping approach was used to trace the evolution of the magnetic field at each point of media. At each time step, the problem was formulated in terms of integral equations and magnetization was computed iteratively until convergence was achieved.

7. Development of new techniques for the calculation of 3-D eddy current problems.

We have developed a new iterative method for the solution of 3-D eddy current problems by using $\vec{A}-V$-formulation. This technique has two main advantages. First, the method has a **global** convergence, which means that it converges for any choice of initial guess. This makes the technique quite robust. Second, numerical implementation of this method requires successive solutions of the Laplace and Poisson equations and, consequently, it can be carried out with minimal new software developments. This technique also has some limitations. First, it is applicable for nonmagnetic conductors. Second, this iteration method converges fast only if the skin depth is comparable with the geometric dimensions of the conductor. Nevertheless, this method may find many practical applications. It is important to note that the mathematical development of the technique brings some new and interesting insights into the nature of eddy current problems. The value of these insights may well extend far beyond the utility of the iterative technique itself. For example, some efficient (i.e., easily computable) estimates (inequalities) for eddy current losses have been derived. These estimates represent a useful by-product of the mathematical machinery developed for the substantiation of the iterative technique. These estimates allow one to obtain some useful information concerning eddy current losses without resorting to laborious eddy current computations.

We have also developed a new technique for the calculation of eddy currents in conductors with small skin depths. This technique is based on impedance boundary conditions. These boundary conditions have been represented in terms of magnetic field. This has led to a scalar magnetic potential formulation of 3-D eddy current problems with small skin depths. The scalar potential formulation has been then reduced to a weak Galerkin form. The finite element discretization of this form has resulted in two (volume and **surface**) "stiffness" matrices.

The detailed discussion of many results described above can be found in our papers already published or submitted for publication. Copies of these papers are attached to this report.

The funds supplied for the last year of the project have been completely utilized to carry out the research summarized above.

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