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A Simulation Study of Linear Coupling Effects and Their Correction in RHIC

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RHIC PROJECT

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1. Introduction

This paper describes a possible skew quadrupole correction system for linear coupling for the RHIC92 lattice. A simulation study has been done for the correction system. Results are given for the performance of the correction system, and the required strength of the skew quadrupole correctors.

An important effect of linear coupling in RHIC is to shift the tune ν_x, ν_y, sometimes called tune splitting. Most of this tune splitting can be corrected with a two family skew quadrupole correction system. For RHIC92, the same 2 family correction system will work for all likely choices of β^*. This was not the case for the RHIC91 lattice where different families of correctors were needed for different β^*.

The tune splitting described above which is corrected with a 2 family correction system is driven primarily by the $\nu_x - \nu_y$ harmonic of the skew quadrupole field given by the field multipole a_1. There are several other effects of linear coupling present which are driven primarily by the $\nu_x + \nu_y$ harmonics of the skew quadrupole field, a_1. These include the following

1. A higher order residual tune shift that remains after correction with the 2 family correction system. This tune shift is roughly quadratic in a_1.
2. Possible large changes in the beta functions.
3. Possible increase in the beam size at injection due to the beta function distortion and the emittance distortion at injection.

The simulation study computes the magnitude of two of these effects, the higher order residual tune shift and the change in the beta functions.

2. The 2 Family Correction System

The 2 family correction system is based on canceling the driving term for the nearby difference resonance, $\nu_x - \nu_y = p$, p being an integer. For the $\nu_x - \nu_y = 0$ resonance the driving term may be written as

$$\begin{aligned} \Delta\nu &= \frac{1}{4\pi\rho}\int ds\,(\beta_x\beta_y)^{\frac{1}{2}}\,a_1 \exp\left[i\overline{\nu}\,(\theta_x - \theta_y)\right] \\ \overline{\nu} &= (\nu_x + \nu_y)/2 \\ \theta_x &= \psi_x/\nu_x\ ,\ \theta_y = \psi_y/\nu_y \end{aligned} \tag{2.1}$$

One needs two families of a_1 correctors to correct both the real and imaginary parts of $\Delta\nu$. The phase of the exponent in (2.1) is nearly $\psi_x - \psi_y$, so the two families should be located at places where $\psi_x - \psi_y$ differ by $\pi/2$. One obvious place to put the a_1 correctors is near the high beta quadrupoles Q2 and Q3, where they are most effective. It will be seen from Fig. 1 that all the high beta quadrupoles have $\psi_x - \psi_y \simeq 0$, and thus another family of a_1 correctors is needed at a location where $\psi_x - \psi_y \simeq \pi/2$.

Fig. 1 plots $(\psi_x - \psi_y)/2\pi$ against the path distance s for a RHIC92 lattice with six $\beta^* = 10$ insertions and also for six $\beta^* = 2$ insertions. Looking at the $\beta^* = 10$ case, one sees that $\psi_x - \psi_y \simeq 0$ at the high beta quadrupoles Q2,Q3, while a_1 correctors near Q4 and Q5 have $\psi_x - \psi_y \simeq \pi/2$. It is proposed that one family of a_1 correctors be located near each Q2 or Q3, which will be called the Q23 family and one family be located near each Q4 or Q5, which will be called the Q45 family. For the $\beta^* = 2$ case, the phases are not as perfectly chosen but these two families will be adequate in this case too, although about 20% less effective.

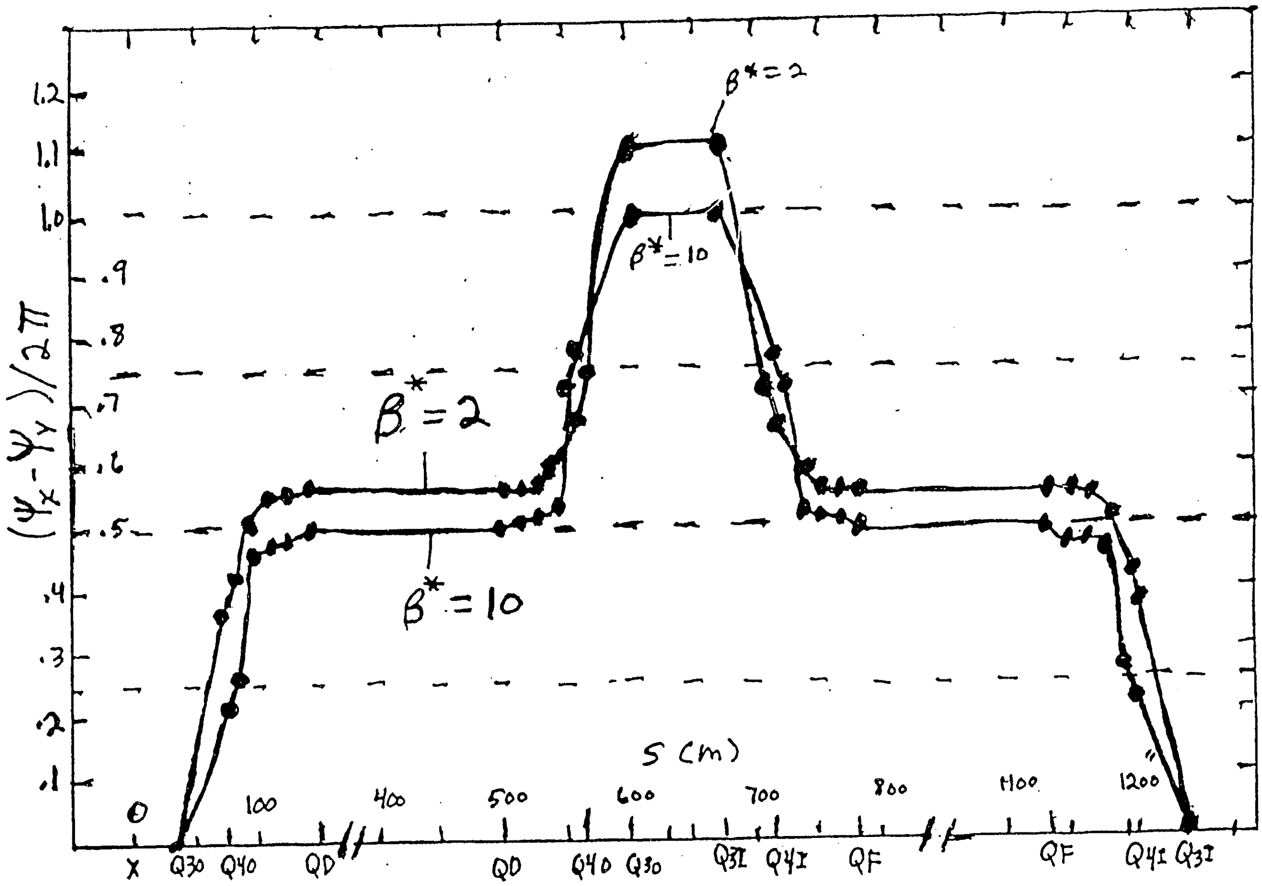

Figure 1: Plot of $(\psi_x - \psi_y)/2\pi$ against s for the RHIC92 lattice with 6 $\beta^* = 10$ and 6 $\beta^* = 2$ insertions. Each dot on the curves indicates the location of a quadrupole.

Fig. 2 plots $(\psi_x - \psi_y)/2\pi$ against s for $\beta^* = 1, 2, 10$ and 16. From this plot, it seems likely that the proposed 2 families will work for the entire range of $\beta^* = 1$ to $\beta^* = 16$. The $\psi_x - \psi_y$ for $\beta^* = 1$ nearly overlaps the curve for $\beta^* = 2$ except for the indicated points for $\beta^* = 1$.

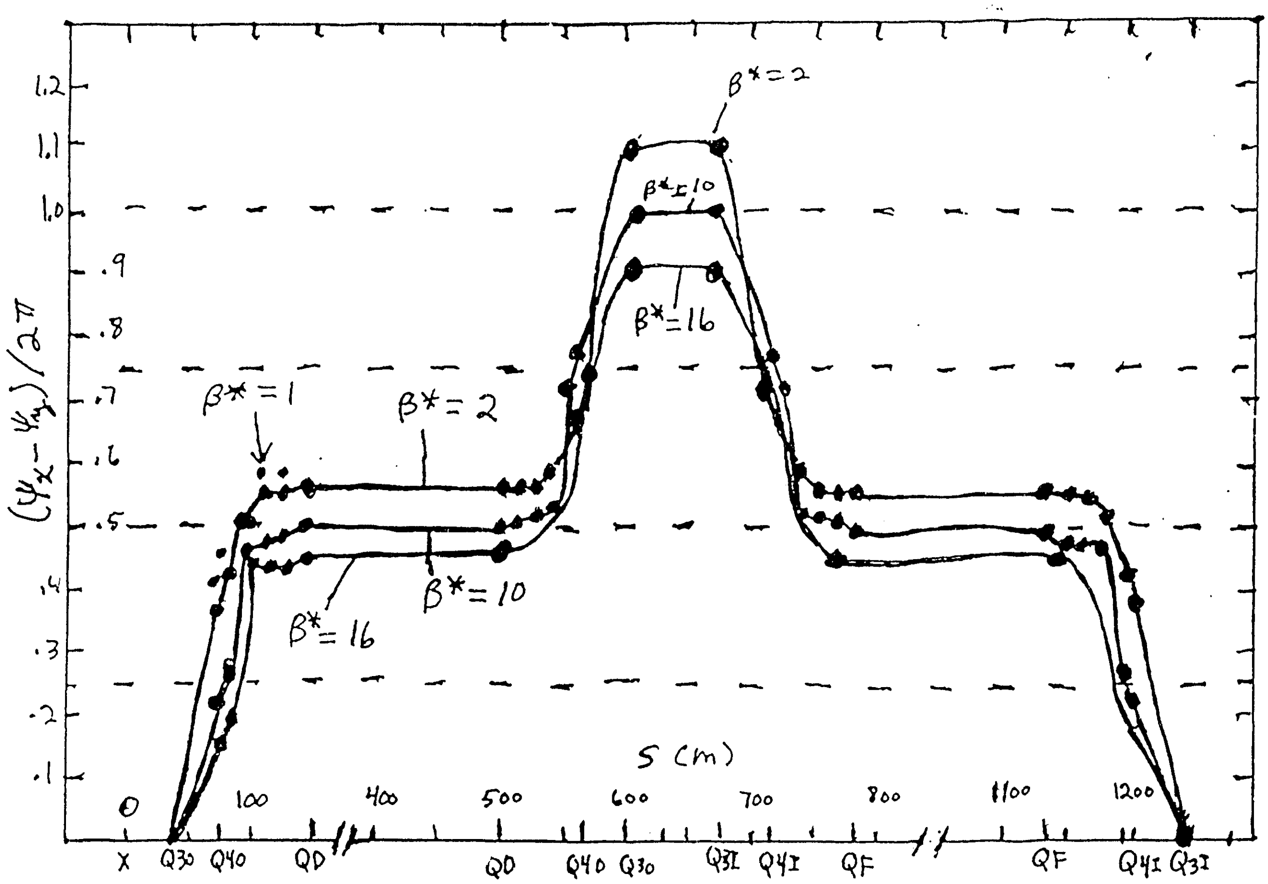

Figure 2: Plot of $(\psi_x - \psi_y)/2\pi$ against s for RHIC92 lattices with $\beta^* = 1, 2, 10, 16$. The $\beta^* = 1$ curve nearly overlaps the $\beta^* = 2$ curve except for the indicated points for $\beta^* = 1$.

3. Simulation Study Results

For this simulation study the random a_1 and b_1 present in the magnets have a magnitude given at the end of this section. If the random a_1 and b_1 can be reduced then the effects, the tune shift and beta function shifts, can be scaled in an appropriate way. In this study the random a_1 and the random b_1 are both assumed to be present. However, the tune shift due to random b_1 is corrected using the QF and QD quadrupoles. Thus the tune shifts shown below are due to the random a_1 errors and the a_1 correctors. The presence of the random b_1 errors does affect the shift in the beta functions.

Table 1 gives the results for the tune, ν_1, ν_2, and the maximum beta functions at QF and QD, β_1, β_2 for ten different distributions of the random a_1 and b_1. Results are shown before the 2 family correction system is applied, the uncorrected case, and after the correction is applied, the corrected case. The correctors are set to cancel the driving terms $\Delta\nu$ given by Eq. (2.1). The RHIC92 lattice has 6 $\beta^* = 2$ insertions.

Table 1. Results for the correction of the tune splitting for a RHIC92 lattice with six $\beta^* = 2$ insertions using a 2 family correction system set to make $\Delta\nu = 0$. The unperturbed tune is 28.826, 28.821. β_1 is the maximum β_1 at a QF and β_2 is the maximum β_2 at a QD. The maximum unperturbed beta function is 50 m.

Field Error	— uncorrected —					— corrected —				
	ν_1	ν_2	$\|\nu_1 - \nu_2\|$ /10^{-3}	β_1 (m)	β_2 (m)	ν_1	ν_2	$\|\nu_1 - \nu_2\|$ /10^{-3}	β_1 (m)	β_2 (m)
1	.854	.792	62	67	65	.828	.819	9	62	66
2	.860	.791	69	78	69	.829	.822	7	63	58
3	.835	.814	21	61	62	.829	.819	10	64	63
4	.868	.801	67	103	94	.839	.830	9	79	94
5	.840	.791	49	88	80	.829	.820	9	62	65
6	.892	.807	85	127	101	.857	.836	21	66	68
7	.906	.779	127	120	94	.872	.831	41	102	72
8	.941	.709	232	92	105	.832	.817	15	65	59
9	—		unstable	—		.876	.856	20	94	86
10	.782	.870	88	69	75	.834	.821	13	63	72

Table 1 shows that one can get tune splittings as large as $|\nu_1 - \nu_2| = 232 \times 10^{-3}$ before correction. One error distribution leads to unstable motion before correction, although this error distribution would not be unstable for the $\beta^* = 10$ lattice. After correction, with the correctors set to cancel the driving term $\Delta\nu$, there remains a residual tune shift with the largest $|\nu_1 - \nu_2| = 41 \times 10^{-3}$. Previous studies[1] indicate that this residual tune shift is roughly quadratic in a_1 if ν_x, ν_y are close to the difference resonance.

Table 1 also shows large shifts for the beta functions, as much as $\Delta\beta/\beta = 100\%$ after correction. It has been shown that the beta function shift[2] and the higher order residual tune shift[3] are driven primarily by the harmonics of a_1 close to $\nu_x + \nu_y$. Part of the beta function shift is due to random b_1 which can cause a maximum $\Delta\beta/\beta$ of about 20%.[4]

Similar results have also been computed for a RHIC lattice with 6 $\beta^* = 10$ insertions. In this case the effects are smaller, and the largest residual tune splitting found is $|\nu_1 - \nu_2| = 14 \times 10^{-3}$, and the largest beta function found at QF or QD is $\beta_1 = 61$ at a QF after correction.

In Table 1, the 2 family correction system was set to cancel the driving term $\Delta\nu$, Eq. (2.1). In operating accelerators, the correction system is usually set to minimize the tune splitting, as $\Delta\nu$ is not known. Table 2 gives the results for ν_1, ν_2 and β_1, β_2 when the 2 family correction is set to minimize $|\nu_1 - \nu_2|$. The lattice is a RHIC92 lattice with 6 $\beta^* = 2$ insertions. Table 2 gives the results only after correction.

Table 2. Results for the correction of the tune splitting for a RHIC92 lattice with six $\beta^* = 2$ insertions using a 2 family correction system set to minimize $|\nu_1 - \nu_2|$. Results are shown after correction.

Field Error	ν_1	ν_2	$\|\nu_1 - \nu_2\|/10^{-3}$	β_1	β_2
1	.824	.823	1	59	59
2	.828	.824	4	56	55
3	.827	.821	6	54	55
4	.836	.833	3	72	71
5	.825	.821	3	65	63
6	.856	.837	19	58	64
7	.856	.838	18	64	63
8	.831	.818	13	63	58
9	.876	.856	20	93	86
10	.829	.826	3	74	67

In Table 2, the results for the tune splitting is somewhat smaller than found in Table 1. The largest tune splitting found after correction is $|\nu_1 - \nu_2| = 20 \times 10^{-3}$. The largest beta function shift after correction is $\Delta\beta/\beta = 86\%$. The results are smaller but still appreciable.

For a RHIC92 lattice with 6 $\beta^* = 10$ insertions, and the correctors set to minimize the tune splitting, the largest residual tune splitting found is $|\nu_1 - \nu_2| = 8 \times 10^{-3}$ and the largest beta function at QF or QD after correction is $\beta_1 = 59$ at a QF.

Required Correction Strengths

In the above simulation, the largest corrector strengths needed are the following

$$\int ds\ B_0\ a_1 = 15 \text{ kG}, \quad \text{family Q45}$$

$$\int ds\ B_0\ a_1 = 6 \text{ kG}, \quad \text{family Q23}$$

Random a_1 and b_1 Errors Assumed in Study

The random quadrupole errors are due to a number of sources that include construction errors in the magnet coils, effective length errors in the quadrupoles, and rotational errors in the positioning of the quadrupoles.

a_1 and b_1 are defined so that the field due to a_1 and b_1 on the median plane is given by

$$B_y = B_0 b_1 x$$

$$B_x = B_0 a_1 x$$

where B_0 is the main dipole field.

The rms random a_1, b_1 used in this study are given in the following table.

Source	$a_1/10^{-5}$ (cm^{-1})	$b_1/10^{-5}$ (cm^{-1})
Dipole coil error	16.8	8.4
Quadrupole coil error	15	15
Quadrupole effective length	—	40
Quadrupole rotation error	40	—

The assumed effective length error in the quadrupoles is $\Delta L/L = 2 \times 10^{-3}$ rms. The assumed rotational error in the quadrupoles is $\Delta\theta = 1 \times 10^{-3}$ rad rms.

4. References

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