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CRITICALITY ASSESSMENT OF TRU BURIAL GROUND CULVERTS

by

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Criticality Assessment of TRU Burial Ground Culverts (U)

The above WSRC-RP, which follows this cover sheet, includes a Table of Contents, Main Text, and Appendicies A-C. The report, which reviews detailed calculations and measurements, is primarily a reference document for the Culvert Criticality Research Team (R.C. Hochel, K.J. Hofstetter, R.A. Sigg, and W.G. Winn). However, the general information of the report should be of interest to all listed on the distribution.

Criticality Assessment of TRU Burial Ground Culverts (U)

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INTRODUCTION

An effort to assess the criticality risks of ^{239}Pu in TRU Burial Ground Culverts has been underway for several years. The concern arose from discrepancies in two types of monitors that have been used to assay the ^{239}Pu waste prior to storage in 55-gallon drums that are placed in the culverts. One type is the solid waste monitor (SWM), which is based on gamma-ray measurements; the other is the neutron coincidence monitor, which is based on neutron measurements. The NCC was put into routine service after 1985 and has generally yielded higher ^{239}Pu assays than the SWM. Culverts with pre-1986 waste only had SWM assays of ^{239}Pu; thus, it was questioned whether their actual ^{239}Pu loadings could be high enough to pose criticality concerns.

Studies to characterize the culvert criticality potential have included appraisal of NCC vs SWM [refs 1,2], neutron measurements atop the culverts [refs 3,4], gamma-ray measurements atop the culverts [refs 5,6], and probabilistic risk analyses [refs 2,7,8,9]. Overall, these studies have implied that the culverts are critically safe; however, their results have not been examined collectively.

The present report uses the collective information of the preceding studies to arrive at a more complete assessment of the culvert criticality aspects. A conservative k_{eff} is estimated for an individual suspicious culvert and a PRA is evaluated for its "worst" drum. These two pieces of information form the basis of the appraisal, but other evidence is also included as support.

SUMMARY

This collective review of the data indicates that the culverts are critically safe. Neutron measurements atop the culverts differ from the SWM projections due to the error in the SWM value and/or subcritical multiplication. Assuming no error in SWM yields the largest possible multiplication; the corresponding highest culvert k_{eff} was 0.904, which is safely subcritical. The neutron measurements could not distinguish individual drums, but PRA appraisals predict that the worst-case individual drum has a probability of 6.85×10^{-8} of going critical.

Other information also supports the criticality safety of the culverts. With no multiplication assumed, the largest culvert inventory is 2488 g, which is in compliance with a loading limit of 2800 g. An analysis with a combination of equal factors for SWM error and multiplication yields a maximum culvert loading of 1573 g and a maximum k_{eff} of 0.745, which are both smaller than the preceding estimates. Also, by removing some of the conservatisms of the PRA analysis, the criticality probability of the worst-case individual drum becomes 8.6×10^{-10}, which is in reasonable agreement with a value of 5×10^{-10} that was estimated for a generic worst-case drum [ref 2]. The neutron measurements atop the culvert actually agree better with the SWM rather than the higher NCC projections, implying that earlier recorded SWM values were reasonably accurate and that the present assay policy of using (SWM+NCC)/2 is conservative. Finally, waste loading policies and gamma measurements favor even lower estimates for the above criticality parameters.

ESTIMATION OF CULVERT Keff

Neutron measurements atop a culvert yielded the ratio of measured/projected neutron rates, which may be interpreted as the product of a subcritical multiplication factor

$$M = 1/(1-k_{eff}) \qquad (1)$$

and a ^{239}Pu mass correction factor

$$f = {}^{239}Pu(actual)/{}^{239}Pu(SWM). \qquad (2)$$

Thus, in general, the measurements are governed by the following relationship

$$(meas/proj) = f\ M. \qquad (3)$$

The earlier analysis [refs 3,4] assumed M = 1 to estimate conservatively high ^{239}Pu(actual). By contrast, the following discussion effectively assumes f = 1 to estimate a conservatively high k_{eff}.

Neutron measurements were performed on 118 worst-case culverts of a total of 211 suspect culverts [refs 3,4]. Prior to these measurements, a culvert was calibrated with neutron detectors at the Fab Lab in 773-A [ref 3]. Both fast and slow neutron detectors, centered atop this culvert, were calibrated with a known ^{239}Pu source. The calibrations (count rate/source mass) were performed as a function of source location and drum moderator loading, so that the recorded SWM mass loading data could project the neutron rate expected for a suspect culvert. The projected rate was generally anticipated to be lower than the measured rate, since the SWM masses were thought to be low on average.

For the conservative k_{eff} analysis, we essentially set f = 1; however, for cases that have SWM less than the minimum critical mass of 500 g, an f = 500/SWM is used to permit a potentially critical case. Using Equation 3, M is calculated as:

$$M = (\text{meas/proj}) / f = (\text{meas/proj}) / \text{MAX}(1 \text{ or } 500/\text{SWM}), \qquad (4)$$

where MAX(X or Y) is the larger of X and Y. From this, the k_{eff} is calculated from a rearrangement of Equation 1, viz

$$k_{eff} = 1 - 1/M \qquad (5)$$

The k_{eff} for 29 measured culverts are given in Table 1. The first 25 cases have been selected from the largest (meas/proj) measurements. The last 4 cases attempt to summarize the remaining cases, as they represent the maximum criticality parameters (underlined in the table). A raw and refined k_{eff} are given in the table. The raw value is determined directly from Equation 4. The refined value is based on Method 5B for the measurements [ref 4]; it uses an empirical correction for any ^{238}Pu neutron rates (lowers M) and uses a 3-sigma upper limit (raises M) for the final M used to calculate k_{eff}. Of all the cases shown, only two have raw or refined Keff > 0.9, and even these are comparable to k_{eff} levels used in developing criticality loading limits. The refined values are considered more appropriate, from which a worst k_{eff} = 0.904 is adopted.

PRA FOR INDIVIDUAL DRUMS

A PRA approach was used to appraise individual drums of a culvert, because the culvert neutron measurements could not discriminate between individual drums. The probability of a drum being critical is defined as

$$P(C) = \int_{500}^{\infty} p(C|m)\ f(m)\ dm, \qquad (6)$$

where p(C|m) is the probability of a criticality for ^{239}Pu mass m, and f(m)dm is the incremental probability for having mass m in the drum. The above integral includes all masses above 500 g, the minimum critical mass, but in general it is known that f(m) = 0 above some upper-limit mass U, so that the following relation is used

$$P(C) = \int_{500}^{U} p(C|m)\ f(m)\ dm, \qquad (7)$$

Accordingly, this study selects appropriate $p(C|m)$, U, and $f(m)$ to effect a reasonable and conservative model for estimating P(C).

A $p(C|m)$ was modeled using PRA concepts developed by S.C. Chay [ref 2], but it incorporates additional conservatisms to address the possibility of lumped fuel criticalities and a lessening effect of poisons at higher mass loadings [refs 10,11,12]. Overall the model incorporates the effects of fissile mass, fuel and moderator density, geometrical configuration, and poisons. The modelled $p(C|m)$, derived in detail in Appendix A, is

$$p(C|m) = 8.05 \times 10^{-7} \; (m/500)^2 \; (m/500 - 1) \; [0.00253 + 0.064(m/500 - 1)] \; (2 - 500/m) \qquad (8)$$

where m is the ^{239}Pu mass in grams.

A U = 5000 g is assumed to be reasonable as an upper-limit mass. The largest SWM drum value was recorded as 187.04 g and even if this were a single cut, the limit U would require an NCC/SWM discrepancy factor of 5000/187.04 = 26.7 as compared to a maximum observed factor of 13. This fact alone supports the choice of U = 5000 g, but even further support results from the strong likelyhood that a drum comprises multiple cuts (typically 10), which in summation would yield NCC/SWM well below 13.

The $f(m) = f_c(m)$ for a cut is the log-Normal distribution, which is defined by

$$f_c(m)dm = \frac{1}{\sqrt{2\pi}\,\sigma} \exp(-[\ln(m) - \ln(c)]^2 / 2\sigma^2) \, d\ln(m) \qquad (9)$$

where $\sigma = 0.74472$ is the log-normal sigma, and c = NCC is the median, as determined from Dr. Chay's correlation [ref 2]

$$\ln(NCC) = \ln(SWM)*1.1358 - 0.2803 \quad \pm \quad (\sigma = 0.74472) \qquad (10)$$

Obviously, this model is only appropriate for a drum if it has just one cut. Typically drums have about 10 cuts. For the same ^{239}Pu drum loading, a drum with one cut will have more σ-uncertainty than a drum with several cuts, because of the averaging effect of the cut sum. Thus, the single cut model has larger fluctuations, and $f_c(m)$ does not tail off with m as rapidly as with the $f(m)$ for a multi-cut model.

A f(m) for a multi-cut model could be best developed using Monte Carlo methods that incorporate the cut mass distribution and the correlation of Equations 9 and 10, as discussed in Appendix B. Because such a multi-cut f(m) is difficult to develop, a more conservative two-cut f(m) was developed using the following 10 cases:

cut-1	cut-2
(1) SWM	(0) SWM
(17/18) SWM	(1/18) SWM
(16/18) SWM	(2/18) SWM
|	|
(9/18) SWM	(9/18) SWM

Equation 10 yields the NCC and σ values for each pair of cuts, and the two are combined to yield the sum

$$NCC(sum) = NCC(cut_1) + NCC(cut_2) \qquad (11)$$

and its error, which is deduced from

$$(err)^2 = (err_1)^2 + (err_2)^2 , \qquad (12)$$

where $err_i = NCC(cut_i)\,(\exp(0.74472) - 1)$ per transforming the log-normal excursions to linear ones. Because the model still uses the log-normal distribution, err is transformed back to the appropriate σ as

$$\sigma = \sigma(sum) = \ln(1 + err/NCC(sum)\ ) \qquad (13)$$

The P(C) from the 10 cut pairs were averaged two ways. One average weights the pairs evenly; the other has relative weightings of 1,2,...,10 from the (1)-(0) SWM-pair to the (9/18)-(9/18) SWM-pair, which attempts to reflect the multiplicity of combinations in forming cut_1 and cut_2 from subcuts. The second or "weighted two-cut" model was adopted for the f(m) in this work.

Using the above p(C|m), U, and f(m) yields drum criticality probab~~ility P(C)~~ summarized in Table 1. For the worst drum (SWM = 187.04), the P(C) is quite low at 6.9×10^{-8}. (Even if the more conservative $U = \infty$ and the "one-cut" model were used, the corresponding P(C) is only 5.4×10^{-7}). Appendix B describes the detailed calculations.

ADDITIONAL SUPPORTING EVIDENCE

Mass Estimates. The neutron culvert measurements [refs 3,4] included analyses whereby M = 1 in Equation 3, so that meas/proj predicted the culvert ^{239}Pu. In this work, a statistical method [Method #9, ref 4] was preferred and indicated that the largest culvert loading of those measured would have only 0.1% probability of exceeding 2488 g. A total of 118 of 211 suspect culverts were measured, and these include the worst cases by far. A recent reappraisal [ref 7] for ^{239}Pu culvert loadings indicates a safe limit of 2800 g (or 200 g per drum). Thus, these neutron measurements imply that the culverts are safe.

Combined Mass and Keff Estimates. In the preceding discussion either f or M of Equation 3 has been set to 1 to yield either a conservative estimates of mass or k_{eff}. In a more realistic treatment, both f and M will be different from 1. As an example case, we examine results for

$$f = M = \sqrt{(meas/proj)} \quad (14)$$

With this formalism, both a mass and k_{eff} may be predicted. Such predictions are in Appendix C, which includes cases that identify the corresponding maximum ^{239}Pu and k_{eff}. The raw estimates yield a maximum ^{239}Pu of 2197 g and a maximum k_{eff} of 0.838, both of which are lower than the earlier estimates. The adopted refined approach, which includes a correction for the ^{238}Pu neutron rate, yields even lower values: a maximum ^{239}Pu of 1573 g and a maximum k_{eff} of 0.745. These examples illustrate that the more realistic analyses, which address both f and M simultaneously, yield lower mass and k_{eff} estimates.

Gamma-Ray Culvert Measurements. Gamma-ray measurements atop some of the culverts have identified neutron sources other than ^{239}Pu. Corrections for these sources can only reduce the (meas/proj) for the neutron rates, yielding lower mass and k_{eff} estimates. The analyses of these gamma measurements are being reviewed for documentation [ref 6].

SWM Measurement Accuracy. The culvert neutron measurements imply that the SWM values are more accurate than the NCC values. Thus, the pre-1986 SWM inventories may be reasonably accurate, and their corresponding culvert loadings have already been appraised as safe. In the neutron appraisals, detectors were calibrated for culvert geometries using a well-characterized ^{239}Pu source [ref 3]. In addition to the suspect culverts, thirty-six check culverts with both ~~SWM and~~ NCC assays were measured with the neutron detectors [ref 4]. Eighteen of these culverts contained only ^{239}Pu and no other neutron emitters. The average (meas/proj) for these culverts was 0.95 ± 0.11 for SWM projections and 0.74 ± 0.09 for NCC projections. These results imply that the SWM is reasonably accurate on average and that the NCC reads high. A recent study of FB-Line waste also concludes that the NCC readings overestimate the amount of ^{239}Pu [ref 13]. The present inventory records now use (SWM+NCC)/2, which is a conservatively high average.

PRA Drum Analysis. The preceding basic PRA for drums includes various conservatisms that might be removed to yield a more realistic lower P(C):

A U of 3000 g has been argued as a more realistic, but this would only decrease P(C) by about 25%.

Use of a multicut model for f(m) could cause a reduction by about a factor of 3.

Lumped fuel criticality and uniform fuel cases were cases examined, where the probabilities for the uniform densities were made more conservative by a factor of 2 to address the lumped fuel cases.

Less poison effect for larger m was addressed by raising its critical probability at m=500 from 0.05 to 0.5 and modelling it to asymptotically increase toward 1 with increasing m. (e.g. at 2500 g it has increased to 0.9).

By taking credit for these items, the P(C) is reduced by a factor of (1/.75)x 3x 2x 10 = 80, yielding a worst drum P(C) of 8.6 x 10^{-10}, which is in reasonable agreement with Dr. Chay's estimate of 5 x 10^{-10} for a generic worst drum [ref 2]. For the generic worst drum, an average f(m) distribution based on drums loaded with single cuts was developed from recent cut data [ref 14].

Drum Loading Aspects. The drum PRA used conservatisms that are unlikely in typical cases [ref 15]. The moderator (polyethylene) is dispersed, having a density of about 1/5 that of water; it is unlikely that this moderator should condense to form the water-like moderator used in the calculation. Visual inspections are likely to prevent acceptance of cuts greater than 1000 g ^{239}Pu; thus, the extreme fluctuations projected by the NCC vs SWM correlation (Equations 9 and 10) are less probable.

CONCLUSIONS

The present examination indicates that the 211 burial culverts with suspect levels of ^{239}Pu are critically safe. The conclusion is based on conservative estimates that predict culvert $k_{eff} < 0.91$ and drum $P(C) < 7 \times 10^{-8}$. Additional information supports this conservatism, illustrating that values of $k_{eff} < 0.75$ and $P(C) < 9 \times 10^{-10}$ might be more realistic. The k_{eff} estimates are based on neutron measurements atop 118 of the culverts; the other 93 culverts have much lower SWM inventories and thus are appraised to be less critical than many of the 118 culverts studied directly. The drum P(C) estimates address all cases directly.

The above conclusion is also supported by other considerations. The largest culvert ^{239}Pu loading is conservatively estimated to be less than 2500 g, but a more realistic maximum is considered to be 1600 g. Both of these ^{239}Pu estimates are below a PRA-limit loading of 2800 g [ref 7]. Gamma measurements on the culverts indicate that some of the criticality estimates can be lowered due to backgrounds from other neutron sources. The earlier SWM assays for the suspect culverts may be reasonably accurate, based on the calibrated neutron culvert assays in the field. These earlier SWM-based loadings were in compliance with criticality limits; corresponding NCC-based loadings would be conservatively higher. Finally, aspects of the drum assay/loading procedures indicate that the moderator and mass treatments are conservative in the PRA.

ACKNOWLEDGEMENTS

S.C. Chay contributed very useful insights for developing the p(C|m) model used in this work. In addition, he has reviewed the overall PRA methodology [refs 10,11,12] and concurs that it is realistic and conservative. Discussions with D.R. Finch led to the incorporation of conservative factors to address lumped fuel effects and the decreasing effect of poisons for higher fissile mass. R.C. Hochel, K.J. Hofstetter, and R.A. Sigg have also reviewed the methodology and have made suggestions for its presentation in the present report.

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Table 1. Culvert Keff and Max Drum Criticality Probability P(C) using derived P(C|m)

Notes:

- o Meas/Proj from neutron measurements [refs 3,4].
- o SWM is gamma assay of Pu-239 grams in COBRA files.
- o Keff = 1 - 1/M for culvert where
  Raw uses M = (meas/proj)/MAX(1 or 500/SWM).
  Refined uses Mass est 5B [ref 4]/MAX(500 or SWM).
- o P(C) = weighted two-cut model with U = 5000 g.
- o Extreme parameters underlined in last 4 cases.

Meas/	Culvert		Culvert Keff		Max Drum	
Proj	Number	SWM	Raw	Refined	SWM	P(C)
38.305	507	355.056	0.963	0.870	179.71	5.57E-08
14.940	324	339.472	0.901	0.904	176.20	5.01E-08
12.195	399	342.056	0.880	0.784	50.67	2.19E-11
10.341	481	343.477	0.859	0.673	74.28	2.91E-10
10.275	412	154.470	0.685	0.0	86.86	7.95E-10
10.229	332	288.188	0.831	0.635	133.94	1.08E-08
9.831	529	322.100	0.842	0.842	162.02	3.17E-08
7.307	392	351.124	0.805	0.351	174.38	4.74E-08
7.067	516	333.290	0.788	0.752	174.70	4.78E-08
6.997	527	267.410	0.733	0.107	183.29	6.19E-08
6.943	515	325.730	0.779	0.721	170.18	4.15E-08
6.082	456	344.396	0.716	0.248	104.08	2.44E-09
5.857	370	216.070	0.605	0.0	64.80	1.18E-10
5.687	409	182.690	0.519	0.0	91.59	1.11E-09
5.587	518	320.040	0.721	0.324	163.99	3.39E-08
5.404	402	146.709	0.371	0.0	43.02	5.79E-12
5.330	550	88.345	0.0	0.608	44.697	9.08E-12
5.248	401	84.780	0.0	0.0	47.54	1.41E-11
5.118	404	131.873	0.259	0.0	131.15	9.62E-09
5.127	420	309.360	0.685	0.0	137.39	1.26E-08
4.644	405	44.970	0.259	0.0	50.99	2.28E-11
4.500	417	184.890	0.398	0.0	61.21	8.03E-11
4.359	521	241.97	0.526	0.0	183.26	6.19E-08
4.010	482	341.354	0.635	0.0	71.346	2.24E-10
4.003	526	245.400	0.490	0.219	175.98	4.98E-08
3.596	384	195.46	0.291	0.0	101.42	2.08E-09
2.715	513	688.95	0.632	0.773	144.82	1.70E-08
1.803	493	340.105	0.187	0.0	187.04	6.90E-08
0.609	549	1221.550	0.0	0.393	131.95	9.96E-09

APPENDIX A. Derivation of p(C|m) Model

Derivation of p(C|m)/ General

The derivation of p(C|m) is given in detail below. The result is plotted in Figure A-1 and shown to exceed an earlier "two-point" p(C|m) model [refs 11,12] in the range of m = 530 to 1500. The earlier model was based on two values derived from the PRA conducted by S.C. Chay [ref 2]. The present p(C|m) diverges to much lower values for higher m, and consequently it yields lower P(C) values.

Derivation of p(C|m) / Detailed Components

The derivation of p(C|m) uses the same approach as presented by Dr. Chay [ref 2], but the final expression illustrates the mass dependence explicitly. The p(C|m) depends on the density of ^{239}Pu, its configuration, and poisons. For a given mass m, the present derivation develops a probability function g(D)dD for the density D and then calculates

$$p(C|m) = \int_{D_{min}}^{D_{max}} p(K)\ p(P)\ g(D)dD \qquad \text{(A-1)}$$

where p(K) is the density-dependent configuration probability, p(P) is the density-dependent poison probability, and the possible critical densities lie in the range of D_{min} to D_{max}.

<u>g(D)</u>. Dr. Chay [ref 2] considers that the density probability is proportional to volume V containing the mass. Thus, the differential probability dh may be written as

$$dh = a\ VdV \qquad \text{(A-2)}$$

where a is a constant. Integrating over the entire drum volume V_0, the mass must be contained somewhere so that

$$\int_{0}^{V_0} dh = (1/2)\ a\ (V_0)^2 = 1 \qquad \text{(A-3)}$$

which yields $a = 2/(V_0)^2$. Thus, with a change of variables to density D = m/V, we may write

$$dh = 2/(V_0)^2\ \ VdV = 2\ (m/V_0)^2\ D^{-3}\ dD \qquad \text{(A-4)}$$

The dh must be multiplied by some additional probabilities to obtain dg = g(D)dD. A uniform density is required in the Chay treatment, and he estimates that the probability p(Uf) of this is about 1/20 for a mass of 500 g. Being a bit more conservative to address a D.R. Finch's concern for possible lumped fuel criticalities, the present development assumes 1/10 and writes a general formula as

$$p(Uf) = 0.1\ (V_{500}/V) = 0.1\ D/D_{500} \qquad \text{(A-5)}$$

where V_{500} is the critical volume (16.7L) for 500 g and D_{500} is its corresponding density (30g/L). This p(Uf) models the fact that it is less probable to have uniformity within a larger volume. The dh must also be multiplied by the probability p(Md) that enough moderator is present in the drum. The present work assumes p(Md) is 0.15, following Hochel and Chay [refs 2,8]. In sum, we write

$$\begin{aligned} dg = g(D)dD &= p(Uf)\ p(Md)\ dh \\ &= (0.1D/D_{500})\ (0.15)\ \ 2\ (m/V_0)^2\ D^{-3}\ dD \\ &= 0.03\ (m/V_0)^2/D_{500}\ D^{-2}\ dD \qquad \text{(A-6)} \end{aligned}$$

<u>p(K)</u>. The configuration probability assumes that the ^{239}Pu and moderator both take spherical shapes and that they overlap. For the 500 g mass, Chay assumes the spherical shape probabilities p(Sp) are 1/20 for for each the Pu and moderator. These probabilities should decrease for larger volumes, as more parts need to be assembled properly; thus, p(Sp) for the general case is modeled as

$$p(Sp) = 0.05\ (V_{500}/V) = 0.05\ (D/D_{500}),\ \text{Pu shape} \qquad \text{(A-7)}$$
$$\text{" \quad " \quad " \quad " \quad "}\ ,\ \text{moderator shape}$$

For the present, we write the spherical overlap condition as p(So), so that

$$\begin{aligned} p(K) &= p(Sp)\ p(Sp)\ p(So) \qquad \text{(A-8)} \\ &= 0.0025\ (D/D_{500})^2\ p(So) \end{aligned}$$

<u>p(P)</u>. For the present the general notation p(P) will be used for the probalistic effect of the poisons.

Derivation of p(C|m) / Final Form

Using the detailed components (Equations A-6 and A-8) of the preceding section, the p(C|m) expression of Equation A-1 may be written as

$$p(C|m) = \int_{D_{min}}^{D_{max}} 0.000075\ (m/V_0)^2\ (D_{500})^{-3}\ p(So)\ p(P)\ dD$$

$$= 0.000075\ (m/V_0)^2\ (D_{500})^{-3} \int_{D_{min}}^{D_{max}} p(So)\ p(P)\ dD \qquad \text{(A-9)}$$

$$= 0.000075\ (m/V_0)^2\ (D_{500})^{-3}\ (D_{max} - D_{min})\ \langle p(So)\rangle\ \langle p(P)\rangle$$

where the last three factors are determined as

$$(D_{max} - D_{min}) = 100\ (m/500 - 1)$$

$$\langle p(So)\rangle = 0.00253 + 0.32(m-500)/2500) \qquad \text{(A-10)}$$

$$\langle p(P)\rangle = 0.5\ (2 - 500/m)$$

Here the expression for $D_{max} - D_{min}$ was determined empirically by its examination as a function of m, as deduced from Figure A-2. The effective average <p(So)> for the integral was modeled using the p(So) = 0.00253 Chay calculated at m = 500 and the p(So) = 0.32 Hochel estimates for m = 3000; the <p(So)> is assumed to vary linearly with mass m so that it includes these two values. Due to discussions with D.R. Finch, the effective average <p(P)> was conservatively increased from an earlier value of 0.05 to 0.5 for m = 500; the model yields 0.5 at m=500 and asymptotically rises to 1.0 as m increases.

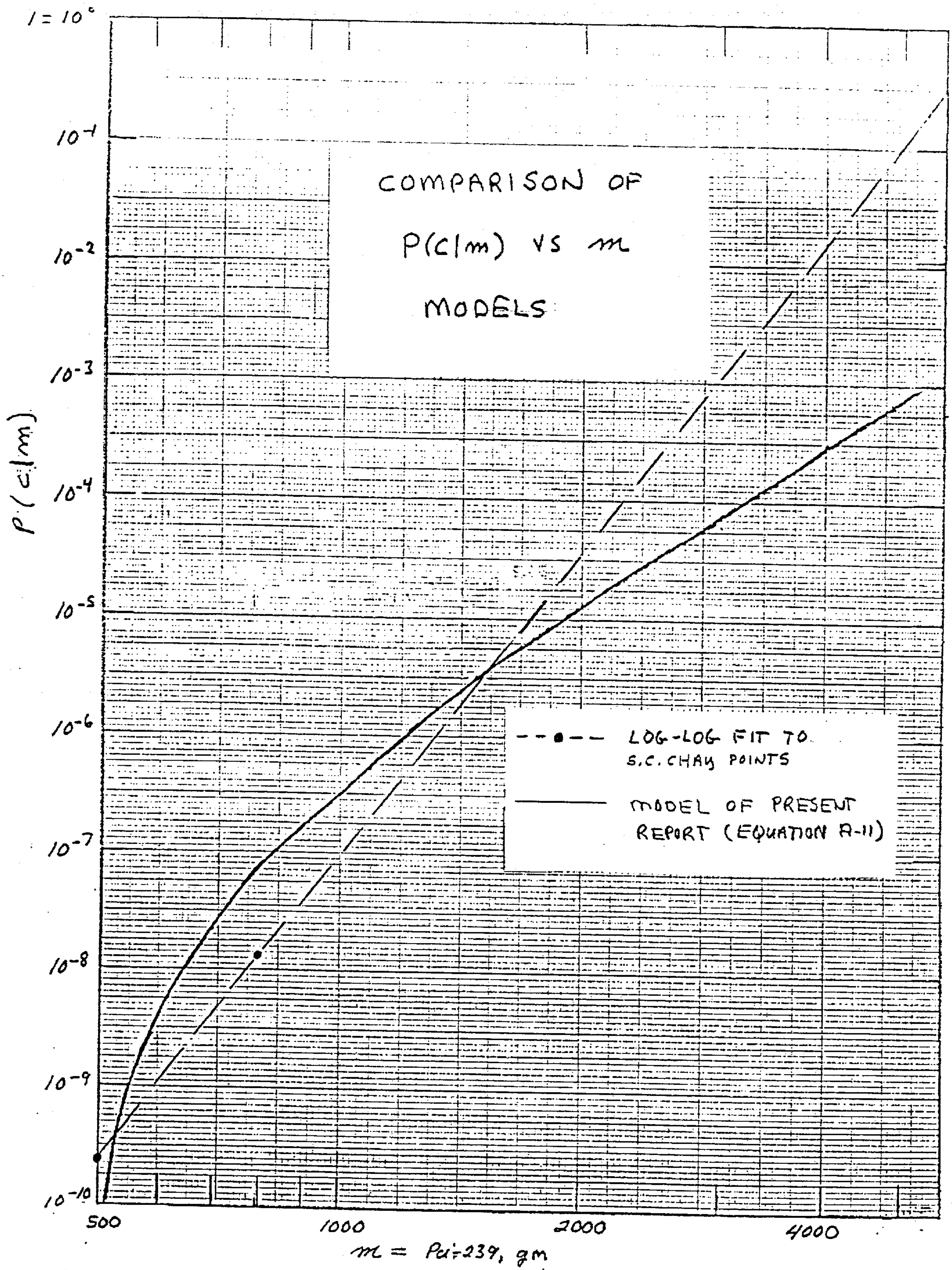
Using the constants V_0 = 208 L, D_{500} = 30 g/L, and the three factors above, p(C|m) may be expressed solely as a function of m, viz

$$p(C|m) = 8.05 \times 10^{-7}\ (m/500)^2\ (m/500 - 1)\ [0.00253 + 0.064(m/500 - 1)]\ (2 - 500/m) \qquad \text{(A-11)}$$

This function is plotted in Figure 1. The BASIC code CULPBFM4 (Appendix B) recasts Equation A-11 as

$$p(C|m) = 20.4 \times 10^{-10}\ (X - 3X^2 + X^3) + 515 \times 10^{-10}\ (-X + 4X^2 - 5X^3 + 2X^4) \qquad \text{(A-12)}$$

where X = m/500


COMPARISON OF
P(C|m) VS m
MODELS
LOG-LOG FIT TO
S.C. CHAY POINTS
MODEL OF PRESENT
REPORT (EQUATION A-11)
P (c|m)
1 = 10°
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
500
1000
2000
4000
m = Pu-239, gm

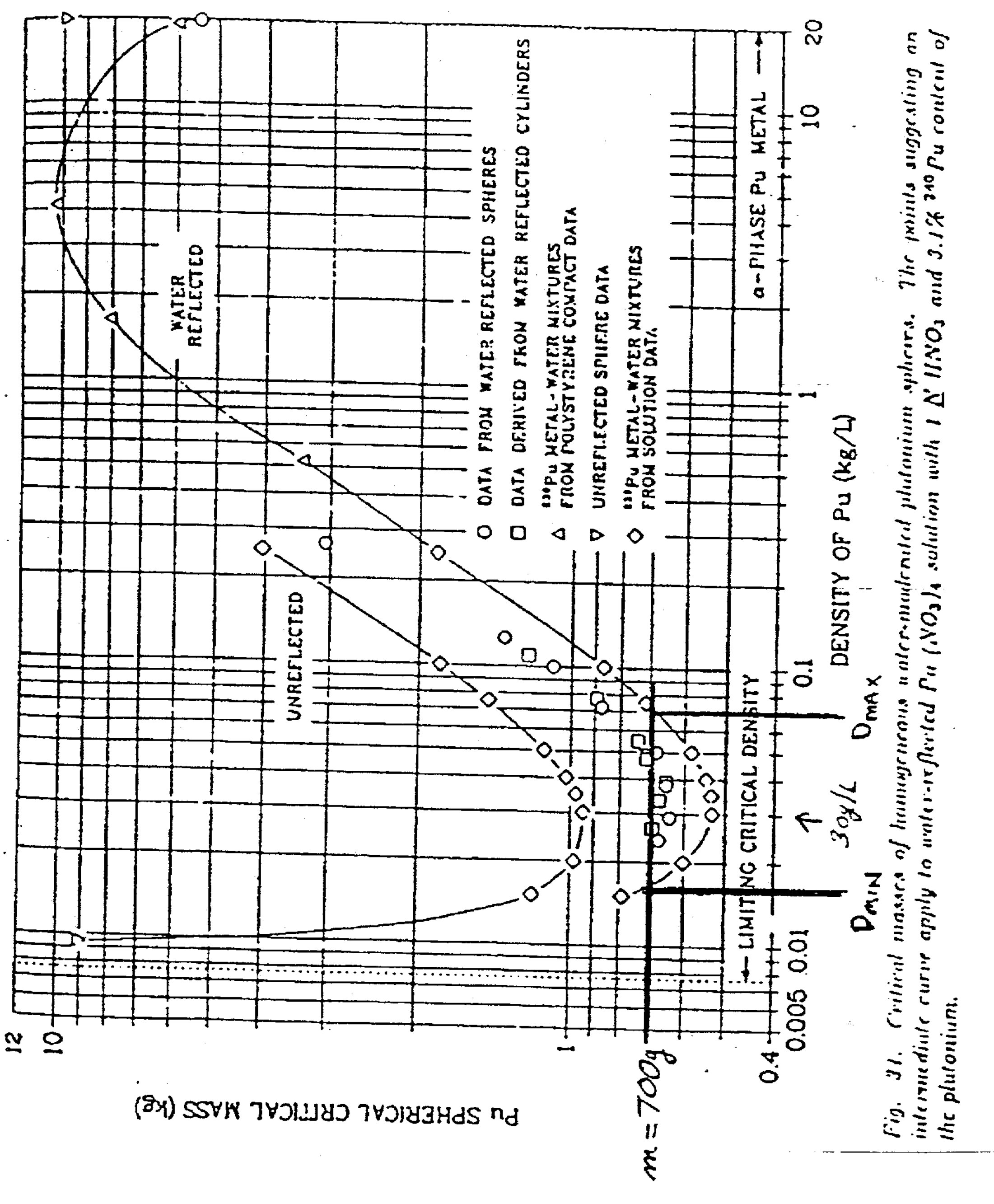


FIGURE A-2. - Critical masses of homogeneous water-moderated Pu spheres (Reproduced from Paxton and Pruvost).

APPENDIX B. Detailed Calculation of Drum P(C)

Code Development

The BASIC code CULPBFM4 was developed to calculate the drum P(C) values. The code listing is given in Figure B-1. It incorporates the $p(C|m)$ developed in Appendix A and the $f(m)$ presented in the main text. In essence, the calculation can be written as

$$P(C) = \int_{500}^{U} p(C|m)\ f(m)\ dm,$$

$$= \int_{500}^{U} \sum_{j=1}^{4} a_j m^j \sum_{k=1}^{10} w_k\ f_{ck}(m)\ dm \qquad \text{(B-1)}$$

where the $a_j m^j$ are the polynomial terms of $p(C|m)$, and w_k is the weight assigned to the "two-cut" probability function $f_{ck}(m)$. As described in the main text, the "two-cut" probability has the form of $f_c(m)$ given by Equation 9, but its centroid c is given by Equation 11 and its σ is derived from Equations 12 and 13, viz

$$c_k = NCC(sum) = NCC(cut_1) + NCC(cut_2)$$

$$\sigma_k = \sigma(sum) = \ln(1 + err/NCC(sum)\ ) \qquad \text{(B-2)}$$

$$\text{where} \quad (err)^2 = (err_1)^2 + (err_2)^2$$

$$err_j = NCC(cut_j)\ (\exp(0.74472) - 1)$$

At this point Equation B-1 may be written as

$$P(C) = \sum_{j=1}^{4} \sum_{k=1}^{10} a_j\ w_k \int_{500}^{U} m^j\ f_{ck}(m)\ dm \qquad \text{(B-3)}$$

where each integral is given by

$$\int_{500}^{U} m^j \, f_{ck}(m) \, dm =$$

$$= \int_{500}^{U} m^j \, \frac{1}{\sqrt{2\pi}\sigma_k} \, \exp(-[\ln(m) - \ln(c_k)]^2 / 2\sigma_k^2) \, d\ln(m)$$

(B-4)

$$= (c_k)^j \exp(j^2\sigma_k^2/2) \times$$

$$\int_{500}^{U} \frac{1}{\sqrt{2\pi}\sigma_k} \exp(-[\ln(m) - \{\ln(c_k)+j\sigma^2\}]^2 / 2\sigma_k^2) \, d\ln(m).$$

The final expression of Equation B-4 is obtained by writing m^j as exp(jln(m)) and rearranging the terms in the resulting exp(...) by completing the square for ln(m), as detailed earlier [ref 4, Appendix A]. The integral of the final expression is that of a log-Normal distribution with centroid of $\ln(c_k+j\sigma_k^2)$ and sigma of σ_k, and standard numerical data for these integrals are available from many sources.

The code essentially calculates the expression in Equation B-3 per the formalism of Equation B-4. Recall that the a_j are the coefficients of the m^j for the polynomial p(C|m), and that the w_k are the weights for the "two-cut" model. The code yields P(C) as a function of U for evenly distributed w_k and for linearly increasing w_k with decreasing cut size, as described in the main text. An example calculation is given in Figure B-2.

Multiple Cut Model/ Monte Carlo Approach

A more accurate (less conservative) model for the cut effects would allow for multiple cuts in a drum, rather than just two as in the present treatment. The Monte Carlo approach described below could develop the proper f(m), in event it is ever desirable to demonstrate a lower p(C).

The Monte Carlo approach proceeds as follows. For a given SWM drum loading, a distribution of cut loadings would be selected with the following steps:

(1) Random selection of cut_1 from cut mass distribution below drum SWM, followed by random selection of cut_2 from cut mass distribution below SWM - SWM(cut_1), etc until all selected cuts sum to the SWM mass.

(2) For SWM(cut_1), SWM(cut_2), etc select a random NCC(cut_1), NCC(cut_2), etc using NCC vs SWM correlations (Equations 9 and 10 of main text).

(3) Sum the randomly selected NCC(cut_j) values as the drum NCC.

(4) Repeat above steps until a sufficient number of drum NCC values exist to define an f(m).

Dr. Chay pointed out that such an f(m) would not be expected to lower the P(C) by more than a factor of 1000, because the single cut possibility itself would be expected to be randomly selected in about 1/1000 of the cases.

Figure B-1. CULPBFM4 Code

The BASIC code CULPBFM4 is listed on the following pages, along with an example output. An outline of the code structure is given below:

CODE OUTLINE: FUNCTION (LINES)

Input Data and Initilization (10-95)

P(C) calculation as function of U (99-999)

j-loop [code j=N] (99-180)

U-loop values [code U => ULK] (100-170)
k-loop [code k => I] (106-158)
Log-Normal subroutine calcs (107-130)
Equation B-4 calcs [U,j,k] (150-154)

j-loop => P2A(j,U) = $\sum_{k=1}^{10} w_k \int_{500}^{U} m^j\ f_{ck}(m)\ dm$
or P2B(j,U)
[A uniform, B weighted] (156-159)

Calc P(C) = $\sum_{j=1}^{4} a_j$ P2X(j,U) with X = A or B (200-250)

Subroutines (1000-3299)

Gaussian Integrals [Inputs from Log-Normal] (1000-1100)

2-cut values per Equation B-2 (3000-3299)

Data files (9000-9040)

Integral Gaussian data (9000-9040)

```
10 REM CULPBFM4.BAS
20 REM This calculates Culvert Probabilities for P(crit|m) = model
40 DIM G(100),P2A(4,10),P2B(4,10)
50 FOR I=0 TO 42:READ G(I):NEXT I
55 CLS
56 PRINT"Results for P(C|M) model":PRINT
60 INPUT"Drum PHA load/ one cut";PHA
62 REM INPUT"LN Sigma";SIGMA
64 LL=500:REM INPUT"Linear Lower Limit";LL
65 REM INPUT"Linear Upper Limit";UL
66 REM INPUT"P(M>limit)";PL
67 PCM = 1E-10
68 PRINT "P(criticality|";LL;")";PCM:PRINT:PRINT
90  PRINT      "  ULM           P2A                P2B    "
95 PRINT
99 FOR N=1 TO 4:PRINT "N=";N:PRINT
100 FOR ULK = 1 TO 10
101 UL=500*ULK
104 P2TA=0:P2TB=0
106 FOR I = 0 TO 9
107 GOSUB 3000:CENT=YS:SIGMA=LNSTD
109 LIM1 = LOG(LL)
110 AVG = LOG(CENT)+N*(SIGMA)^2
115 GOSUB 1000:PLL#=P#
119 IF N=0 THEN PUL#=0:GOTO 150
120 LIM1 = LOG(UL)
125 GOSUB 1000:PUL#=P#
130 LIM1 = LOG(UL)
131 REM LIM1 = LOG(UL)
135 REM AVG = LOG(CENT)
140 REM GOSUB 1000: PREST#= P#
150 PROB = ((CENT/LL)^N)*EXP( .5*(N*SIGMA)^2)*(PLL#-PUL#):REM + PREST#
152 REM IF N = 0 THEN PROB1 = PROB
154 PROB2 = PROB
156 P2 = PCM*PROB2:P2TA=P2TA+P2:P2TB=P2TB+P2*(10-I)
158 NEXT I
159 P2A = P2TA/10:P2B=P2TB/55:P2A(N,ULK)=P2A:P2B(N,ULK)=P2B
160 PRINT USING" ####    ##.####^^^^     ##.####^^^^ ";UL,P2A,P2B
165 REM IF N<> 0 THEN PRINT (LOG(LL)-AVG)/SIGMA,(LOG(UL)-AVG)/SIGMA,PLL,PUL
170 NEXT ULK
180 NEXT N
190 PRINT:PRINT:PRINT
194 CLS
195 PRINT"Model results for PHA = ";PHA:PRINT
196 PRINT       "  U        P(C)_a          P(C)_b"
197  PRINT
200 FOR ULK = 1 TO 10
210 UL = 500*ULK
220 P2A = 20.4*(P2A(1,ULK) - 3*P2A(2,ULK) + 2*P2A(3,ULK) ) +
          515*(-P2A(1,ULK) + 4*P2A(2,ULK) - 5*P2A(3,ULK) + 2*P2A(4,ULK))
230 P2B = 20.4*(P2B(1,ULK) - 3*P2B(2,ULK) + 2*P2B(3,ULK) ) +
          515*(-P2B(1,ULK) + 4*P2B(2,ULK) - 5*P2B(3,ULK) + 2*P2B(4,ULK))
240 PRINT USING" ####    ##.####^^^^     ##.####^^^^ ";UL,P2A,P2B
250 NEXT ULK
999 END
```

```
1000 REM Calculate probabilit[illegible]
1005 FLAGX=1
1010 X = (LIM1-AVG)/SIGMA
1011 REM PRINT X
1015 IF X<0 THEN FLAGX=-1:X=ABS(X)
1020 XN=INT(10*X):IF XN>41 THEN P#=(1/SQR(2*3.1416)/(1*X))*EXP(-.5*X^2):IF FLAGX
=-1 THEN P#=1-P#
1021 REM PRINT X,XN
1025 IF XN>41 THEN 1100
1030 P# = G(XN) + (G(XN+1)-G(XN))*(X-XN/10)/.1:IF FLAGX=-1 THEN P#=1-P#
1040 P# = 1-P#
1100 RETURN
3000 REM sub to run through two mass sums
3090 Z = PHA:DZ=Z/18
3110 Z1 = Z/2+I*DZ:Z2 = Z/2-I*DZ
3120 IF Z1 > 0 THEN LNY1 = 1.1358*LOG(Z1)-.2803
3130 IF Z2 > 0 THEN LNY2 = 1.1358*LOG(Z2)-.2803
3140 IF Z1 > 0 THEN Y1 = EXP(LNY1) ELSE Y1 = 0
3150 IF Z2 > 0 THEN Y2 = EXP(LNY2) ELSE Y2 = 0
3220 DY1 = Y1*(EXP(.74472)-1)
3230 DY2 = Y2*(EXP(.74472)-1)
3240 YS = Y1+Y2:DYS=SQR(DY1^2+DY2^2)
3250 LNSTD = LOG(1 + DYS/YS)
3285 REM PRINT Z1,Z2,YS,LNSTD
3299 RETURN
9000 REM area of gaussian data
9001 REM line 90N0 and .M  for x = N.M  (See Cramier tables)
9002 REM M=  0       1       2       3       4       5       6       7       8       9
9005 DATA .50000,.53983,.57926,.61791,.65542,.69146,.72575,.75804,.78814,.81594
9010 DATA .84134,.86433,.88493,.90320,.91924,.93319,.94520,.95543,.96407,.97128
9020 DATA .97725,.98214,.98610,.98928,.99180,.99379,.99534,.99653,.99744,.99813
9030 DATA .99865,.99903,.99931,.99952,.99966,.99977,.99984,.99989,.99993,.99995
9040 DATA .99997,.99999,1.0000
```

Example Output:

P(C)_a = 2-cut uniform model
P(C)_b = 2-cut weighted model

Model results for PHA = 187.04

U	P(C)_a	P(C)_b
500	0.0000E+00	0.0000E+00
1000	5.3633E-09	4.6169E-09
1500	2.3549E-08	1.8088E-08
2000	4.5602E-08	3.2115E-08
2500	6.5870E-08	4.3427E-08
3000	8.2804E-08	5.1942E-08
3500	9.6465E-08	5.8241E-08
4000	1.0728E-07	6.2895E-08
4500	1.1588E-07	6.6381E-08
5000	1.2262E-07	6.8998E-08

APPENDIX C. Simultaneous Mass and Keff Estimates

Results for ^{239}Pu mass and k_{eff} estimates using the condition f = M are tabulated in Table C-1. Monotonically ordered fM SWM [presented as mass estimates <M=1> in ref 4] are listed in the table to allow quick identification of the maximum ^{239}Pu and k_{eff}.

The search for maximum ^{239}Pu is conducted as follows. The cases of interest have fM > f > 1. In the table, cases of ^{239}Pu = f SWM are calculated beginning with the largest fM SWM and then in decreasing monotonic order. The maximum ^{239}Pu is continually noted as the calculations proceed. When the maximum ^{239}Pu exceeds the remaining fM SWM values, the search is completed as this is the absolute maximum of all cases studied. For example, in the refined data, the maximum tabulated ^{239}Pu of 1573 g exceeds all ^{239}Pu which have fM SWM ≤ 1529 g; thus, it is the absolute maximum.

The search for maximum k_{eff} is more straight forward, as it corresponds to the maximum fM alone (no SWM factor). It is readily identified using Table 1.

TABLE C-1. Simultaneous Mass and Keff Estimates

All values calculated such that $f = M = \sqrt{fM}$ per

$$^{239}Pu = f \; SWM = \sqrt{fM} \; SWM$$

$$k_{eff} = 1 - 1/M = 1 - 1/\sqrt{fM}$$

Data from Raw Method [Method #1, ref 4]

Culvert	SWM g	fM SWM g	^{239}Pu g	k_{eff}
507	355.056	13600	2197	0.838
324	339.472	5072	1312	0.741
399	342.056	4171	1194	0.713
481	343.477	3552	1105	0.689
529	322.100	3167	1010	0.681
332	288.188	2948	922	0.687
392	351.124	2566	949	0.630
516	333.290	2355	886	0.624
515	325.730	2262	858	0.620
456	344.396	2095	849	0.595

Data from Refined Method [Method #5B, ref 4]

Culvert	SWM g	fM SWM g	^{239}Pu g	k_{eff}
324	339.472	5524	1332	0.745
507	355.056	3875	1172	0.697
529	322.100	3168	994	0.676
552	797.840	3101	1573	0.492
513	688.950	3033	1448	0.523
558	760.166	2599	1406	0.459
528	972.150	2442	1541	0.369
399	342.056	2313	889	0.615
543	861.775	2025	1321	0.348
549	1221.550	2016	1569	0.221
516	333.290	2015	820	0.593
515	325.730	1788	762	0.573
551	1131.993	1779	1419	0.202
555	805.730	1745	1186	0.320
554	955.320	1732	1286	0.257
545	677.780	1714	1078	0.371
553	976.340	1707	1291	0.244
544	811.310	1614	1144	0.291
481	343.477	1529	724	0.526