Co-f-921122--8

ANL-HEP-CP-92-109
November 1992

ANL-HEP-CP--92-109

DE93 004853

RIGOROUS QCD PREDICTIONS FOR DECAYS OF P-WAVE QUARKONIA*

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ABSTRACT

We present a new factorization theorem for the decay rates of P-wave states of heavy quarkonia. Infrared logarithms that had appeared in previous perturbative calculations of P-wave decays are absorbed into a quantity that is related to the amplitude for the heavy quark and antiquark to be in a relative color-octet S-wave state. We predict all of the light-hadronic and electromagnetic decays rates of the χ_c and h_c states in terms of two phenomenological parameters.

The annihilation of the heavy $Q\overline{Q}$ pair in quarkonium is a short-distance process, occurring when the Q and $\overline{Q}$ are within $O(1/M_Q)$ of each other. If all of the interactions associated with the annihilation were to occur at distance scales set by M_Q, then one could, because of asymptotic freedom, calculate the decay rate in a perturbation series in $\alpha_s(M_Q)$. Unfortunately, quark-gluon interactions invalidate this simple scenario, since the associated infrared (IR) and collinear singularities imply the presence of long-range interactions. Nevertheless, for the annihilation of S-wave states a relatively simple picture emerges. If one neglects the relative velocity $\vec{v}$ of the Q and $\overline{Q}$, keeping the leading term in an expansion in powers of v/c, then the annihilation cross section for a meson m with total spin S factors into a short-distance piece times a long-distance piece:

$$\Gamma\left(m(^{2S+1}\mathrm{S}) \to X\right) \;=\; G_1(m)\,\hat{\Gamma}_1\left(Q\bar{Q}(^{2S+1}\mathrm{S}) \to X\right). \tag{1}$$

The quantity $\hat{\Gamma}_1$ is the short distance piece. It is the (on-shell) parton-level annihilation cross section, and asymptotic freedom allows its computation in perturbative QCD. The perturbation series for the annihilation of an on-shell $Q\overline{Q}$ in an S wave is well behaved: final-state IR and collinear divergences cancel according to the KLN theorem, and initial-state IR divergences cancel because the meson is a color singlet. G_1 is the long-distance piece, which contains all of the nonperturbative effects. It is analogous to a parton distribution. G_1 is proportional to the probability to find the Q and $\overline{Q}$ at the same point:

$$G_1(m) \approx (3/2\pi)|R_{mS}(0)|^2/M_Q^2, \tag{2}$$

*Talk presented by G. Bodwin at DPF92, Fermilab, Batavia, Illinois, November 10–14, 1992 where $R_{mS}(0)$ is the nonrelativistic radial wave function at the origin.

One might guess that a factorization formula similar to Eq. (1) would hold for P-wave decays as well. However, in this case the situation is more complicated. Owing to the angular dependence of the wave function, the leading term in the v/c expansion vanishes upon integration over the angular orientation of $\vec{v}$. Consequently, the annihilation cross section for P waves is suppressed by v^2/c^2 relative to S-wave annihilation. The first subleading term in the v/c expansion yields, in perturbation theory, a contribution that is proportional to the *derivative* of the wave function at the origin.

Now, the first subleading term in the v/c expansion measures the color currents of the Q and $\overline{Q}$, rather than their color charges. Thus, for the subleading term, infrared divergences need not cancel, even though the quarkonium has no net color charge. In fact, the perturbation series for $Q\overline{Q}$ annihilation on shell in a P wave contains a logarithmic IR divergence. This is a clear signal that long distance effects are present and that the use of perturbative QCD is not valid. Past (nonrigorous) treatments of P-wave decays have invoked the confinement scale or the binding energy as an IR cutoff—but with no fundamental justification.

The structure of these IR divergences has a simple physical interpretation. The divergences arise when the P-wave color-singlet state converts to an S-wave color-octet state through the emission of a soft gluon. Then the Q and $\overline{Q}$ annihilate from the S-wave state. The soft-gluon transition costs a factor v^2/c^2. However, S-wave annihilation is enhanced by a factor c^2/v^2 relative to P-wave annihilation, so the color-octet process is competitive with direct color-singlet P-wave annihilation.

By taking into account the fact that the IR divergences are associated with the color-octet mechanism, one can write a new factorization theorem for P-wave decays:[1]

$$\Gamma\left(m(^{2S+1}\mathrm{P}) \to X\right) \;=\; H_1(m)\,\hat{\Gamma}_1\left(Q\bar{Q}(^{2S+1}\mathrm{P}) \to X\right) \;+\; H_8(m)\,\hat{\Gamma}_8\left(Q\bar{Q}(^{2S+1}\mathrm{S}) \to X\right). \quad (3)$$

The first term in Eq. (3) corresponds to the naive (color-singlet) factorization picture. The second term is new and gives the contribution of the color-octet mechanism. The $\hat{\Gamma}$'s are the parton-level cross sections for on-shell $Q\overline{Q}$ annihilation, except that the IR divergent part of $\hat{\Gamma}_1$ is extracted and put into H_8. The precise way in which this is done is the factorization prescription. H_1 is proportional to the derivative of the P-wave color-singlet $Q\bar{Q}$ wave function at the origin:

$$H_1(m) \;\approx\; (9/2\pi)|R'_{mP}(0)|^2/M_Q^4. \quad (4)$$

H_8 is related to the amplitude to find the Q and $\overline{Q}$ in a relative color-octet S-wave state. Since H_8 contains information about the $Q\overline{Q}g$ Fock state, it is not simply expressible in terms of the nonrelativistic $Q\overline{Q}$ wave function. In perturbation theory

$$H_8(m) \;\approx\; \frac{16}{27\beta_0}\ln\left(\frac{\alpha_s(\epsilon_m)}{\alpha_s(M_Q)}\right) H_1(m) \;\sim\; \frac{16}{27\pi}\alpha_s \ln\left(\frac{M_Q}{\epsilon_m}\right) H_1(m), \quad (5)$$

where ϵ_m is the binding energy. The logarithm signals that Eq. (5) is not trustworthy as an estimate of H_8. However, by comparing it with the IR divergent parts of previous calculations of P-wave decay[2], one can extract the $\hat{\Gamma}$'s. Both H_1 and H_8 have precise definitions in terms of operator matrix elements, so they can, in principle, be measured in lattice simulations, or they can simply be treated as phenomenological parameters.

In general, H_1 and H_8 depend on the total angular momentum J and the total spin S of the P-wave state. However, if one describes the heavy $Q\overline{Q}$ system in terms of a low-energy effective Lagrangian, then the spin-dependent terms are suppressed by powers of $1/M_Q$, that is, by powers of v/c. Thus, to leading order in v/c, we can take H_1 and H_8 to be independent of S and J. Corrections to this give terms of order v^2/c^2 in the decay rates, where $v^2/c^2 \approx 20\%$ for charmonium.

The factorization formula Eq. (3) combined with the leading-order expressions for the $\hat{\Gamma}$'s gives

$$H_1 = \frac{45}{16\pi}\frac{\Gamma(\chi_{c2} \to \mathrm{LH}) - \Gamma(\chi_{c1} \to \mathrm{LH})}{\alpha_s^2(M_c)}, \qquad H_8 = \frac{1}{\pi}\frac{\Gamma(\chi_{c1} \to \mathrm{LH})}{\alpha_s^2(M_c)}, \tag{6}$$

where "LH" denotes light hadrons. Using the Particle Data Group[3] values for the branching ratios and the recent E760 data[4] for the χ_{c1} and χ_{c2} total widths, we find that $H_1 = 15.3 \pm 6.6$ MeV and $H_8 = 3.2 \pm 1.4$ MeV . The quoted error is the experimental error, which includes the uncertainty in α_s, combined in quadrature with our estimate of the theoretical uncertainty, which includes v^2/c^2 corrections and higher-order perturbative QCD corrections. Given H_1 and H_8 and the leading-order expressions for the $\hat{\Gamma}$'s, we can use Eq. (3) to compute the partial widths for the decays of the χ_{c0} and h_c into light hadrons, the decay of h_c into γ plus light hadrons, and the decays of χ_{c0} and χ_{c2} into two γ's. There is also a simple relationship between the radiative decay rates of the P-states[5], which is correct to leading order in v/c:

$$\Gamma(^1\mathrm{P}_1 \to \gamma\, ^1\mathrm{S}_0)/E_\gamma^3(11) \simeq \Gamma(^3\mathrm{P}_J \to \gamma\, ^3\mathrm{S}_1)/E_\gamma^3(3J), \quad J = 0, 1, 2, \tag{7}$$

where $E_\gamma(11)$ and $E_\gamma(3J)$ are the energies of the γ's in the singlet and triplet decays, respectively. Eq. (7) is well satisfied for the χ_{c1} and χ_{c2}. We use it to obtain predictions for the radiative decay widths of the χ_{c0} and h_c. Our predictions for the χ_{c0} are

$$\begin{aligned} &\Gamma(\chi_{c0}) = (5 \pm 2)\ \mathrm{MeV}, && B(\chi_{c0} \to \mathrm{LH}) = (98 \pm 1)\%, \\ &B(\chi_{c0} \to \gamma + J/\psi) = (2 \pm 1)\%, && B(\chi_{c0} \to \gamma\gamma) = (7 \pm 4) \times 10^{-4}. \end{aligned} \tag{8}$$

The predictions for the χ_{c0} total width and branching fraction into $\gamma + J/\psi$ differ significantly from the accepted values of 14 ± 5 MeV and $(0.66 \pm 0.18)\%$, respectively. More precise data on the χ_{c0} would provide useful tests of the QCD predictions. Our prediction for the branching fraction of the χ_{c2} into two γ's is

$$B(\chi_{c2} \to \gamma\gamma) = (4 \pm 2) \times 10^{-4}. \tag{9}$$

This is somewhat smaller than the old Particle Data Group[3] value of $(11 \pm 6) \times 10^{-4}$. However, a new E760 measurement[6] yields a branching fraction into two γ's of $(1.7 \pm 0.6) \times 10^{-4}$. Our predictions for the h_c are

$$\begin{aligned} &\Gamma(h_c) = (1.0 \pm 0.2)\ \mathrm{MeV}, && B(h_c \to \mathrm{LH}) = (52 \pm 11)\%, \\ &B(h_c \to \eta_c + \gamma) = (46 \pm 11)\%, && B(h_c \to \gamma + \mathrm{LH}) = (2 \pm 1)\%. \end{aligned} \tag{10}$$

The prediction for the h_c total width is consistent with the upper bound of 1.1 MeV obtained recently by the E760 collaboration.[7] We predict a significant rate for h_c into γ plus light hadrons. The hard γ recoiling against a jet plus soft hadrons could be a distinctive signature for this decay. A large component of the error in all of these predictions arises from the theoretical uncertainty. It could be reduced considerably by making a complete next-to-leading-order calculation of the $\hat{\Gamma}$'s.

One can treat quarkonium production processes using techniques that are very similar to those that we have described for quarkonium decay processes. Possible applications include photoproduction, leptoproduction, and hadron-hadron production. The application to production of χ_c states in B-meson decay can be found in Ref. 8. The nonperturbative quantities G_1 and H_1, which appeared in quarkonium decay, appear in the color-singlet production processes as well. However, color-octet production involves a new nonperturbative quantity H_8'. Whereas H_8 is analogous to a parton distribution function, H_8' is analogous to a fragmentation function. The two quantities are related by crossing, but that relationship is a simple one only in lowest-order perturbation theory. Consequently, one must determine H_8' from experiment or extract it from a lattice calculation.

This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Contract W-31-109-ENG-38 and under Grant DE-FG02-91-ER40684, and by the National Science Foundation.

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