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Research on Trust-region Algorithms for Nonlinear Programming

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Progress Report

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1 Project Description

This grant is for research for developing and testing effective global, and robust algorithms for general nonlinear programming problems. The progress of our research this year has been especially dizzying in the area of interior point methods for handling inequality constraints. This work has resulted in publications for the linear programming problem so far, but we anticipate developments in nonlinear programming. In the general area, our research has been steady and of excellent quality. This research can be divided into five subareas which we will now list and briefly describe.

1. Interior-Point Methods for Linear Programming.

   The research in this area was particularly successful. Much understanding was gained and several very important open question were answered. It is satisfying that this research represents contributions across the entire spectrum of the algorithmic aspects of interior point methods.

   Some of the reports listed below represent very theoretical issues while others represent implementation or design issues.

   (a) A. El-Bakry, R.A. Tapia and Y. Zhang, "A study of Indicators for Identifying Zero Variables in Interior-point Methods," Rice University Department of Mathematical Sciences Technical Report TR91-15, June 1991.

   (b) A.S. El-Bakry, R.A. Tapia and Y. Zhang, "Numerical Comparisons of Local Convergence Strategies for Interior-point Methods in Linear Programming," Rice University Department of Mathematical Sciences Technical Report TR91-18, July 1991.

   (c) C.M. Samuelsen and R.A. Tapia, "The Dikin-Karmarkar Principle for Steepest Descent: Avoiding Short Steps," Rice University Department of Mathematical Sciences Technical Report TR91-20, July 1991.

   (d) Y. Ye, R.A. Tapia and Y. Zhang, "A Superlinearly Convergent $O(\sqrt{n}L)$- Iteration Algorithm for Linear Programming," Rice University Department of Mathematical Sciences Technical Report TR91-22, July 1991.

(e) J. Ji, F. Potra, R.A. Tapia and Y. Zhang, "An Interior-Point Method with Polynomial Complexity and Superlinear Convergence for Linear Complementarity Problems," Rice University Department of Mathematical Sciences Technical Report TR91-23, July 1991.

(f) Y. Zhang, R.A. Tapia and Y. Ye, "On the Convergence of the Iteration Sequence in Primal-Dual Interior-Point Methods," Rice University Department of Mathematical Sciences Technical Report TR91-24, August 1991.

(g) Y. Ye, O. Güler, R.A. Tapia and Y. Zhang, "A Quadratically Convergent $0(\sqrt{n}L)$-Iteration Algorithm for Linear Programming," Rice University Department of Mathematical Sciences Technical Report TR91-26, August 1991.

(h) R.A. Tapia and Y. Zhang, "Superlinear and Quadratic Convergence of Primal-Dual Interior-Point Methods for Linear Programming Revisited," Rice University Department of Mathematical Sciences Technical Report TR91-27, August 1991.

(i) Y. Zhang and R.A. Tapia, "On the Convergence of Interior-point Methods to the Center of the Solution Set in Linear Programming," Rice University Department of Mathematical Sciences Technical Report TR91-30, September 1991.

2. Trust-region SQP Newton's Method for General Nonlinear Programming Problems.

The main thrust of the work in this area continues the development of trust-region algorithms for nonlinear programming along the lines of the well-known Celis-Dennis-Tapia trust region-method. This latter algorithm was developed and implemented under DOE sponsorship. A new production type implementation has been undertaken under separate funding.

The second approach has been a new trust-region method for unconstrained problems. The approach taken is that the Hessian is approximated by PTP^T, where P is orthogonal and fixed for several iterations, and $T = T^T$ is tridiagonal and is updated at each iteration by a sparse

symmetric Broyden method. The paper contains a strong global convergence analysis and very good numerical results.

(a) (with N. Echebest, M.T. Guardarucci, J.M. Martínez, H.D. Scolnik, and C. Vacchino) A Curvilinear Search Using Tridiagonal Secant Updates for Unconstrained Optimization, *Siam Journal on Optimization* 1 (1991), pp. 333-357.

(b) "An Algorithm Based on a Convenient Trust-Region Subproblem for Nonlinear Programming," by J.E. Dennis, J.M. Martinez, R.A. Tapia and K.A. Williamson, in draft form.

(c) "A Robust Trust-Region Algorithm for Nonlinear Programming, by K.A. Williamson," Mathematical Science TR 90-22.

(d) "Solution of a Trust-Region Subproblem for Nonlinear Programming: Minimizing a Non-convex Quadratic Subject to Two Quadratic Constraints," by K.A. Williamson, J.E. Dennis, and J.M. Martinez, Mathematical Science TR 90-23.

3. Trust Region SQP Newton's Method for Large Sparse Nonlinear Programming Problems with Applications to Oil Reservoir Management.

   The research in this area is currently being performed by Cristina Maciel and Dr. Michael Lewis. under the supervision of J.E. Dennis. Last year in this work, the concentration was on the solution of the SQP subproblem. The novel idea here is to couple the conjugate reduced gradient method with the Steihaug-Toint dogleg method as a means of solving the large sparse SQP subproblem. The theory seems quite interesting and promising, as does the algorithm. This year we have spent most of our time incorporating the progress being made in choosing the penalty term in the augmented Lagrangian merit function and in designing methods that would not require factorizations of any piece of the constraint Jacobian. This is because of our interest in problems whose constraints are discretized equations of fluid flow.

4. A Unified Approach to Global Convergence of Trust-Region Methods for Nonsmooth Optimization.

   The research in this area was the subject of Shou-Bai Li's Ph.D. dissertation under the direction of Dennis and Tapia. This work unifies

the convergence theory developed by various authors and isolates essentially what components or ingredients in the theory are necessary for convergence. This work was reported on last year, our progress this year has consisted of refining the work for publication. It has been provisionally accepted by *Mathematical Programming* with an extremely complementary referee's report. This research is described in the following report: "A Unified Approach to Global Convergence of Trust-Region Methods for Nonsmooth Optimization", by J.E. Dennis, Shou-Bai Li and R.A. Tapia, to appear in *Mathematical Programming*

5. SQP Augmented Lagrangian BFGS Algorithm for Constrained Optimization. The main effort this year in this area has been to revise the following paper which has been accepted for publication: "An SQP Augmented Lagrangian BFGS Algorithm for Constrained Optimization," by R.H. Byrd, R.A. Tapia, and Yin Zhang, in press *Siam Journal on Optimization* 2 (1992)

   In this work the authors build an effective BFGS secant method for constrained optimization based on the BFGS secant updates proposed by Tapia in "On Secant Updates for Use in General Constrained Optimization," Math. Comp. 51 (1988) pp. 181-202.

   This latter work was also developed under the sponsorship of DOE. The BFGS secant method is globalized using a line search strategy. Impressive numerical results are obtained. In fact these results are superior to those obtained for the Powell damped BFGS secant method for constrained optimization — the algorithm of choice for the last 10 years or so. A very elegant convergence analysis is given. This theory when specialized to unconstrained optimization gives a new and important result for the BFGS secant method.

2 Graduate Students Supported

The following graduate students received either full or partial support from the DOE contract and were directed by either Dennis or Tapia, or both:

Natalia Alexandrov
Shou-Bai Li, Ph.D. received
Eva Lee
Cristina Maciel
Amr El Bakry, Ph.D. received
Cathy Samuelsen, Ph.D. thesis accepted
Marcela Rosemblun
Kurt Overley, Ph.D. received

3 Unexpended Balance

There will be no unexpended funds.

4 Modifications

We are pleased with the research progress made this year and do not wish to make any modifications in the scope of the research.

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