DOE/ER/40291--26
Conf-9409277--3

SUNY BING 10/15/94

Tau as a Probe for New Physics

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The usage of polarimetry and spin-correlation tests to determine the *complete Lorenz structure* of the tau lepton's charged and neutral-current couplings is reviewed. The emphasis is on tests for "something" in a $(V-A)+$ "something" structure in $J^{Charged}{}_{Lepton}$ current, so as to bound the scales Λ for "new physics" such as arising from tau weak magnetism, weak electricity, and/or second-class currents. Tests for T and for CP violation are discussed.

1. TESTS FOR "ADDITIONAL STRUCTURE" TO THAT OF THE STANDARD MODEL

There are many exciting directions in which the tau leptons, $\tau^{\pm}, \nu_\tau, \bar{\nu}_\tau$, can be used to probe for new physics beyond that in the standard model. Here we will focus on and review the usage of polarimetry and spin- correlation tests [1–6] to determine the *complete Lorentz structure* of the tau lepton's charged and neutral-current couplings. Tests for time-reversal violation and for CP violation are thereby included.

For charged-current couplings, the emphasis will be on [7–10]

- Tests for "something" in a

  (V-A) + "something"

  structure in the $J^{Charged}{}_{Lepton}$ current.

- Tests for a non-CKM-type leptonic CP violation in $\tau \to \rho\nu$ and $\tau \to a_1\nu$ decays.

For neutral-current couplings, the emphasis will be on

- Tests for "additional structure" in both the weak and the pure electromagnetic $J^{Neutral}{}_{Lepton}$ currents.

- Complete tests for possible CP and/or T violations in Z^o, or $\gamma^* \to \tau^-\tau^+$.

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Our discussion of neutral-current couplings will be brief for they have already been discussed at this conference by W. Marciano [11] and A. Stahl [12].

In searching for new physics, it is very important to carefully and systematically search for *symmetry violations*, such as an unexpected Lorentz structure in tau couplings, a leptonic CP or T violation, a finite ν_τ mass, a violation of "lepton number" conservation or of L_τ, L_μ, or L_e conservation. Such violations may show up by the observation of an otherwise forbidden decay mode, or though polarimetry and spin-correlation tests.

A healthy "working attitude" is to remember that we will only see the *symmetry violations* which the observables allow. Hence, if a violation exists, until we uncover the right observable it won't be seen. If it's a small effect (for some reason), it still won't be seen. However, ***if it isn't fundamentally forbidden, it will make itself seen** somewhere, sometime*!

2. "ADDITIONAL STRUCTURE" IN CHARGED-CURRENT COUPLINGS?

2.1. The simplest S2SC function

The idea is to search for "additional structure" in $J^{Charged}{}_{Lepton}$ by generalizing the τ spin-correlation function $I(E_\rho, E_B)$ by including the ρ polarimetry information [4] that is available from the $\rho^{ch} \to \pi^{ch}\pi^o$decay distribution [8,9].

Invited talk at *Third Workshop on Tau Lepton Physics,* Montreux, Switzerland, 19-22 Sept. '94.

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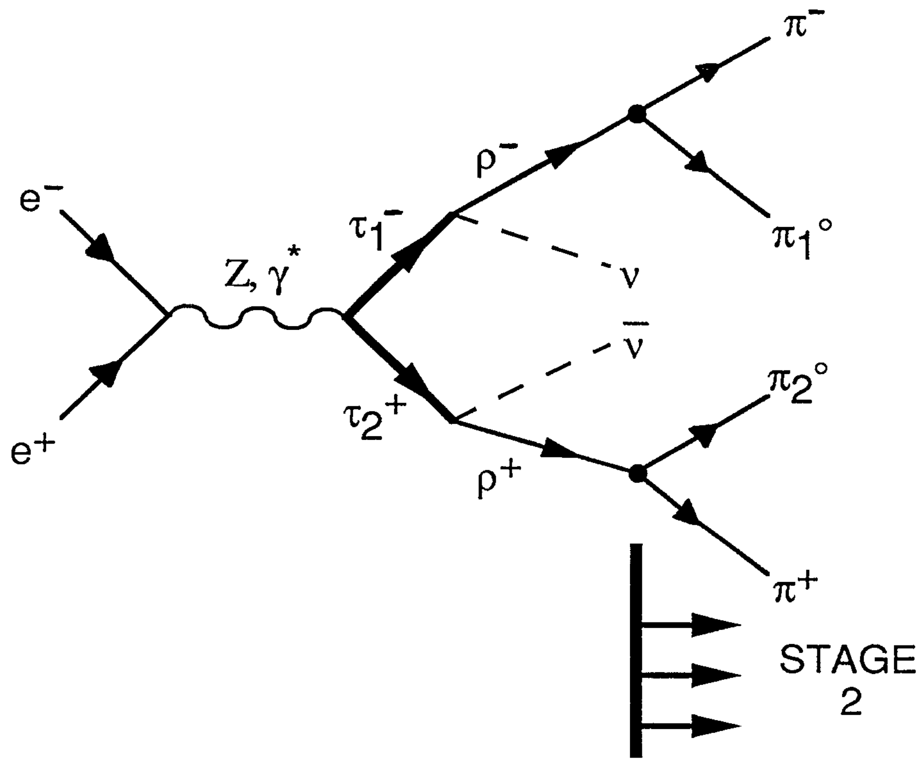

Figure 1. Illustration of the two stage structure of the stage-two spin- correlation (S2SC) function. ρ polarimetry information from the next stage of decays $\rho^{ch} \rightarrow \pi^{ch}\pi^0$ is included in the helicity amplitude treatment of the production-decay sequence $e^-e^+ \rightarrow Z^0, \gamma^* \rightarrow \tau_1^- \tau_2^+ \rightarrow (\rho^-\nu)(\rho^+\bar{\nu})$. Compare Fig. 3.

The symbol $B = \rho, \pi, l$. Since this adds on spin-correlation information from the next stage of decays in the decay sequence, we call such an energy- angular distribution a stage-two spin-correlation (S2SC) function. Compare Fig.1.

The simplest useful S2SC is for the CP-symmetric decay sequence Z^o, or $\gamma^* \rightarrow \tau^-\tau^+ \rightarrow (\rho^-\nu_\tau)(\rho^+\bar{\nu}_\tau)$ followed by both $\rho^\mp \rightarrow \pi^\mp\pi^o$.

$$
\begin{aligned}
I(E_\rho, E_{\bar{\rho}}, \tilde{\theta}_1, \tilde{\theta}_2) = & |T(+-)|^2 \rho_{++}\bar{\rho}_{--} \\
& + |T(-+)|^2 \rho_{--}\bar{\rho}_{++} + |T(++)|^2 \rho_{++}\bar{\rho}_{++} \\
& + |T(--)|^2 \rho_{--}\bar{\rho}_{--}
\end{aligned}
$$

If we think in terms of probabilities, the quantum-mechanical structure of this expression for I_4 is almost intuitively obvious, since the $T(\lambda_{\tau^-}, \lambda_{\tau^+})$ helicity amplitudes describe the production of the $(\tau^-\tau^+)$ pair via Z^oor $\gamma^* \rightarrow \tau^-\tau^+$. So for instance in the 1st term, the factor $|T(+,-)|^2$ ="Probability to produce a τ^- with $\lambda_{\tau^-} = \frac{1}{2}$ and a $\tau^+$with $\lambda_{\tau^+} = -\frac{1}{2}$ " is simply multiplied by the product of the decay probablity, ρ_{++}, for the positive helicity $\tau^- \rightarrow \rho^-\nu \rightarrow (\pi^-\pi^o)\,\nu$ times the decay probablity, $\bar{\rho}_{--}$, for the negative helicity $\tau^+ \rightarrow \rho^+\bar{\nu} \rightarrow (\pi^+\pi^o)\,\bar{\nu}$.

What are the kinematic variables? The 1st stage of the decay sequence $\tau^-\tau^+ \rightarrow (\rho^-\nu_\tau)(\rho^+\bar{\nu}_\tau)$ is described by the 3 variables

$$
\begin{aligned}
\theta_1^\tau &\equiv E_\rho \\
\theta_2^\tau &\equiv E_{\bar{\rho}} \\
\cos\phi &\equiv \cos\psi
\end{aligned}
$$

where the helicity variables $\theta_1^\tau, \theta_2^\tau, \cos\phi$ are defined as in Fig.2.

These are equivalent to the Z^o, or γ^* center-of-mass variables, $E_\rho, E_{\bar{\rho}}, \cos\psi$. Here ψ ="opening $\angle$ between the ρ^- and ρ^+momenta in the Z/γ^* cm".

When the Lorentz "boost" to one of the ρ rest frames is directly from the Z/γ^* cm frame, the 2nd stage of the decay sequence is described by the usual 2 spherical angles, see Fig.3, for the π^{ch} momentum direction in that ρ rest frame:

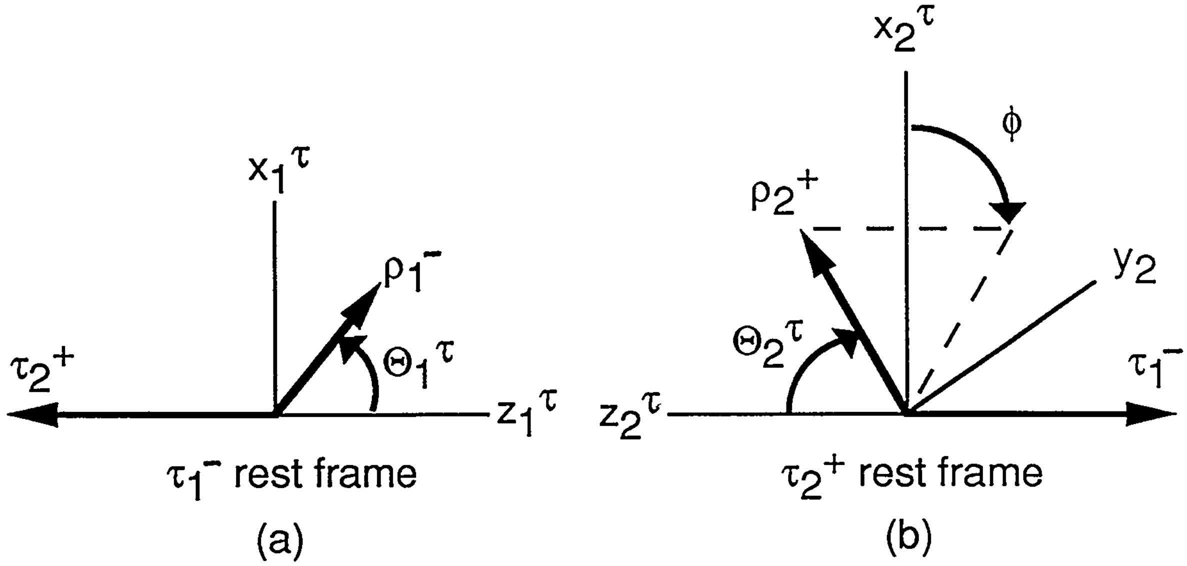

Figure 2. For "stage 1", the three angles $\theta_1^\tau, \theta_2^\tau$, and ϕ describe the sequential decay Z^0 or $\gamma^* \to \tau_1^- \tau_2^+$ with $\tau_1^- \to \rho_1^- \nu$ and $\tau_2^+ \to \rho_2^+ \bar{\nu}$. From (a) a boost along the negative z_1^τ axis transforms the kinematics from the τ_1^- rest frame to the Z^0/γ^* rest frame and, if boosted further, to the τ_2^+ rest frame in (b).

For $\rho_1^- \to \pi_1^- \pi_1^o$: $\tilde{\theta}_1, \tilde{\phi}_1$ For $\rho_2^+ \to \pi_2^+ \pi_2^o$: $\tilde{\theta}_2, \tilde{\phi}_2$

What are the dynamical variables? More precisely the $T(\lambda_{\tau^-}, \lambda_{\tau^+})$ helicity amplitudes describing the production of the $(\tau^- \tau^+)$ pair via Z^oor $\gamma^* \to \tau^- \tau^+$, are defined by the associated matrix element

$$\langle \Theta_\tau, \Phi_\tau | J = 1, M \rangle = D^{1*}_{M\lambda}(\Phi_\tau, \Theta_\tau, -\Phi_\tau) T(\lambda_1, \lambda_2)$$

The composite decay density matrix elements are given by (for only ν_L couplings)

$$\begin{aligned} \rho_{hh} = \\ & (1 + h\cos\theta_1^\tau)\left[\cos^2\omega_1 \cos^2\tilde{\theta}_1 + \tfrac{1}{2}\sin^2\omega_1 \sin^2\tilde{\theta}_1\right] \\ & + \tfrac{r_a^2}{2}(1 - h\cos\theta_1^\tau)\left[\sin^2\omega_1\cos^2\tilde{\theta}_1 \right. \\ & \left. + \tfrac{1}{2}(1+\cos^2\omega_1)\sin^2\tilde{\theta}_1\right] \\ & + h\tfrac{r_a}{\sqrt{2}}\cos\beta_a \sin\theta_1^\tau \sin 2\omega_1 \left[\cos^2\tilde{\theta}_1 - \tfrac{1}{2}\sin^2\tilde{\theta}_1\right] \end{aligned}$$

They are, however, simply the decay probability for a τ_1^- with helicity $\frac{h}{2}$ to decay $\tau^- \to \rho^- \nu \to (\pi^- \pi^o)\nu$ since

$$\frac{dN}{d(\cos\theta_1^\tau)\, d\left(\cos\tilde{\theta}_1\right)} = \rho_{hh}\left(\theta_1^\tau, \tilde{\theta}_1\right) \tag{1}$$

For the CP-conjugate decay of the τ_2^+ ,

$$\bar{\rho}_{hh} = \rho_{-h,-h}\,(subscripts 1 \to 2, a \to b) \tag{2}$$

It is more fun to discuss the remaining quantities (β_a, r_a, ω_1) in ρ_{hh} in answer to the question "What is the pure ν_L's $I(E_\rho, E_{\bar{\rho}}, \tilde{\theta}_1, \tilde{\theta}_2)$ good for? The answer is that one obtains 2 tests for "non-CKM-type leptonic CP violation". This is explained in the next section.

However, there is even a simpler interesting application of I_4: At this conference, there has been considerable discussion about different methods for the determination of the mass of the τ neutrino. The full I_4 spin-correlation function including ν_R and ν_L couplings (see Sec. 2.3) depends on the ratio $\left(\frac{m_\nu}{m_\tau}\right)^2$. So before using any I_4 for other purposes where we will set $m_\nu = 0$, it is important to check the sensitivity to $m_\nu \neq 0$. The present experimental limit $m_\nu = 24MeV$ corresponds to

$$\delta\left((\frac{m_\nu}{m_\tau})^2\right) = 2\frac{m_\nu}{m_\tau}\delta(\frac{m_\nu}{m_\tau}) = 6 \cdot 10^{-4} \tag{3}$$

Table 1 gives the sensitivity of the full I_4 to $\left(\frac{m_\nu}{m_\tau}\right)^2$.

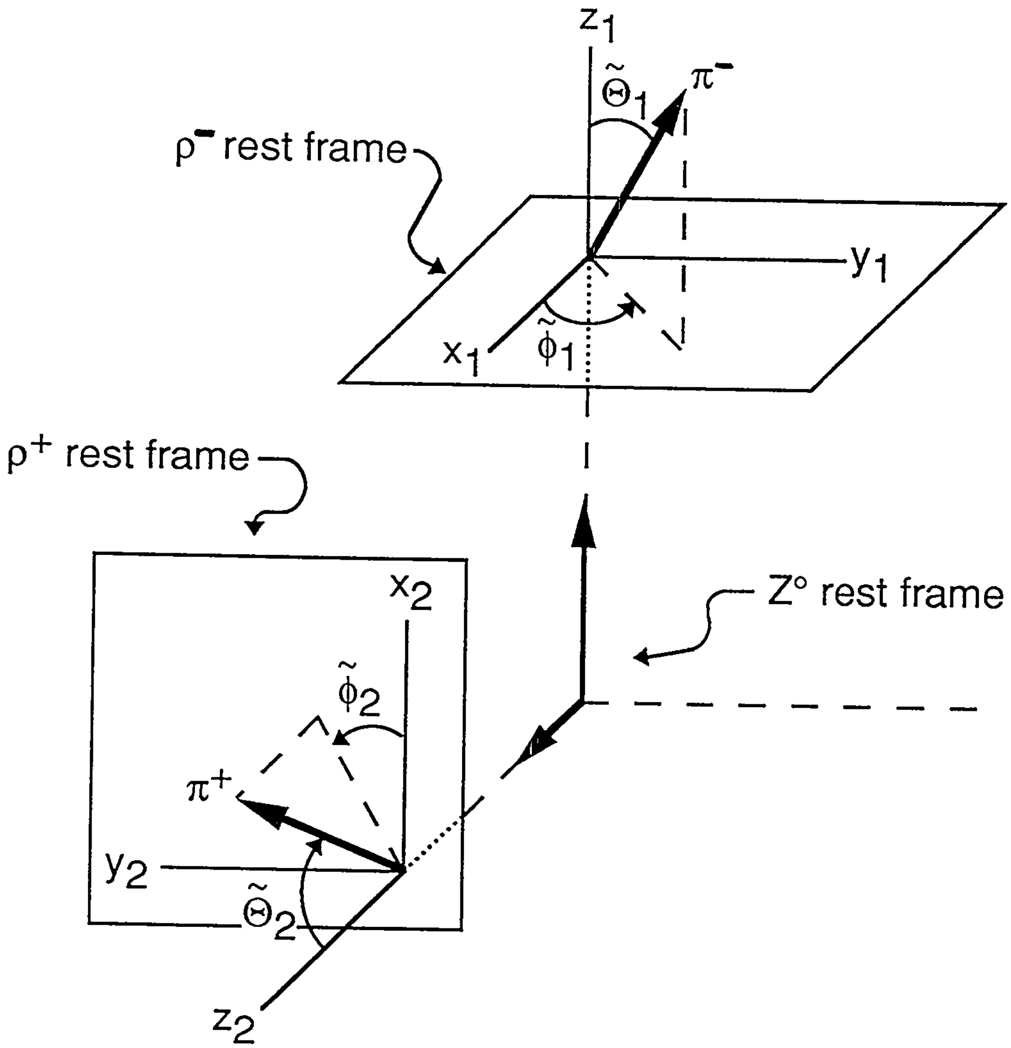

Figure 3. For "stage 2", the spherical angles $\tilde{\theta}_1, \tilde{\phi}_1$ specify the π^- momentum in $\rho_1^- \to \pi^-\pi^0$ decay in the ρ_1^- rest frame when the boost is directly from the Z^0 or γ^* cm frame. Similarly, $\tilde{\theta}_2, \tilde{\phi}_2$ specify the π^+ momentum in $\rho_2^+ \to \pi^+\pi^0$. The $\rho^-\rho^+$ production half-plane specifies the positive x_1 and x_2 axes.

Table 1
Sensitivity of $I(E_1, E_2, \tilde{\theta}_1, \tilde{\theta}_2)$ to $m_\nu \neq 0$:

	Pure $V-A$ coupling	
	At M_Z	10, or 4 GeV
$\{\rho^-, \rho^+\}$ mode:		
	$70 \cdot 10^{-4}$	$13 \cdot 10^{-4}$
$\{a_1^-, a_1^+\}$ mode:		
	$18 \cdot 10^{-4}$	$3 \cdot 10^{-4}$

All tables in this paper list only the ideal statistical errors [5], and assume respectively $10^7 Z^o$events and 10^7 $(\tau^-\tau^+)$pairs.

At $10GeV$, or $4GeV$ the numbers correspond to 613,387 events for $\{\rho^-, \rho^+\}$or $B(\tau \to \rho\nu)$ = 24.6% , and 324,000 events for $\{a_1^-, a_1^+\}$. Throughout this paper for the a_1 mode we sum the charged plus neutral pion a_1 final states so $B(\tau \to a_1{}^{ch+neu}\nu) = 18\%$, and use $m_{a_1} = 1.275GeV$. Note that Table 1 shows the expected substantial improvement in the limit on m_ν upon using the greater a_1 mass, instead of the ρ mass. When systematic errors are included, the numbers shown in Table 1 mean that multi-pion modes must be used. For the analysis in Table 1, we did not need form-factor assumptions based on current algebra type arguments; perhaps eventually form-factor independent measurements will be performed for the multi-pion τ decay modes.

2.2. Tests for non-CKM-type leptonic CP violation

It is simple to see from the kinematics why two CP tests are possible: First we again start with the classic definitions for the necessary helicity amplitudes in each decaying particle's rest frame. In the τ^- rest frame, the matrix element for $\tau^- \to \rho^- \nu$ is

$$\langle \theta_1^\tau, \phi_1^\tau, \lambda_\rho, \lambda_\nu | \frac{1}{2}, \lambda_1 \rangle = D^{\frac{1}{2}*}_{\lambda_1,\mu}(\phi_1^\tau, \theta_1^\tau, 0) A(\lambda_\rho, \lambda_\nu)$$

where $\mu = \lambda_\rho - \lambda_\nu$. Similarly, in the ρ^- rest frame

$$\langle \tilde{\theta}_a, \tilde{\phi}_a | \lambda_\rho \rangle = D^1_{\lambda_\rho,0}(\tilde{\phi}_a, \tilde{\theta}_a, 0) c \qquad (4)$$

where c is a constant factor. And, for the CP-conjugate process, $\tau^+ \to \rho^+ \bar{\nu} \to (\pi^+ \pi^o) \bar{\nu}$, in the τ^+ rest frame

$$\langle \theta_2^\tau, \phi_2^\tau, \lambda_{\bar{\rho}}, \lambda_{\bar{\nu}} | \frac{1}{2}, \lambda_2 \rangle = D^{\frac{1}{2}*}_{\lambda_2,\bar{\mu}}(\phi_2^\tau, \theta_2^\tau, 0) B(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}})$$

with $\bar{\mu} = \lambda_{\bar{\rho}} - \lambda_{\bar{\nu}}$. In the ρ^+ rest frame

$$\langle \tilde{\theta}_b, \tilde{\phi}_b | \lambda_{\bar{\rho}} \rangle = D^1_{\lambda_{\bar{\rho}},0}(\tilde{\phi}_b, \tilde{\theta}_b, 0) \bar{c} \qquad (5)$$

Assuming a L-handed ν_τ, $\tau^- \to \rho^- \nu$ depends on

$$A\left(-1, -\tfrac{1}{2}\right) = |A\left(-1, -\tfrac{1}{2}\right)| e^{\iota \phi^a_{-1}}$$
$$A\left(0, -\tfrac{1}{2}\right) = |A\left(0, -\tfrac{1}{2}\right)| e^{\iota \phi^a_0}$$

and $A\left(1, -\frac{1}{2}\right) = 0$ by rotational invariance. In this section we will neglect the R-handed ν_τ amplitudes. Note that the R-handed ν_τ amplitudes for pure $(V-A)$ are of order

$$\frac{A\left(1, \frac{1}{2}\right)}{A\left(-1, -\frac{1}{2}\right)} \simeq \frac{m_\nu m_\tau}{m_\tau^2 - m_\rho^2}$$
$$\frac{A\left(0, \frac{1}{2}\right)}{A\left(0, -\frac{1}{2}\right)} \simeq \frac{m_\nu m_\rho^2}{m_\tau (m_\tau^2 - m_\rho^2)}$$

Similarly for the CP-conjugate process, assuming a R-handed $\bar{\nu}_\tau$, $\tau^+ \to \rho^+ \bar{\nu}$ depends on

$$B\left(1, \tfrac{1}{2}\right) = |B\left(1, \tfrac{1}{2}\right)| e^{\iota \phi^b_1}$$
$$B\left(0, \tfrac{1}{2}\right) = |B\left(0, \tfrac{1}{2}\right)| e^{\iota \phi^b_0}$$

and $B\left(-1, \frac{1}{2}\right) = 0$.

By CP invariance, the 2 tests for non-CKM-type leptonic CP violation are:

$$\beta_a = \beta_b \qquad \textbf{first test}$$

where $\beta_a = \phi^a_{-1} - \phi^a_0$, $\beta_b = \phi^b_1 - \phi^b_0$,and

$$r_a = r_b \qquad \textbf{second test}$$

where

$$r_a = \frac{|A\left(-1, -\frac{1}{2}\right)|}{|A\left(0, -\frac{1}{2}\right)|}, r_b = \frac{|B\left(1, \frac{1}{2}\right)|}{|B\left(0, \frac{1}{2}\right)|}$$

This easily follows since in the helicity formalism the symmetries for $\tau^- \to \rho^- \nu$ and $\tau^+ \to \rho^+ \bar{\nu}$ are:

Invariance	*Relation*
P	$A(-\lambda_\rho, -\lambda_\nu) = A(\lambda_\rho, \lambda_\nu)$
	$B(-\lambda_{\bar{\rho}}, -\lambda_{\bar{\nu}}) = B(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}})$
C	$B(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}}) = A(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}})$
CP	$B(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}}) = A(-\lambda_{\bar{\rho}}, -\lambda_{\bar{\nu}})$
$\tilde{T}_{FS}$	$A^*(\lambda_\rho, \lambda_\nu) = A(\lambda_\rho, \lambda_\nu)$
	$B^*(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}}) = B(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}})$
$CP\tilde{T}_{FS}$	$B^*(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}}) = A(-\lambda_{\bar{\rho}}, -\lambda_{\bar{\nu}})$

There is a basic theorem that measurement of a non-real helicity amplitude implies a violation of $\tilde{T}_{FS}$ invariance when a first-order perturbation in an "effective" hermitian Hamiltonian is reliable. So $\tilde{T}_{FS}$ invariance is expected to be violated when there are significant final-state interactions; and it is to be distinguished from canonical T invariance which requires interchanging "final" and "initial" states, i.e. actual time-reversed reactions are required.

In the standard lepton model with a pure $(V-A)$ coupling, the values of the these polar parameters are $\beta_a = 0, r_a = \frac{\sqrt{2} m_\rho}{E_\rho + q_\rho} \simeq \sqrt{2} m_\rho / m_\tau \simeq 0.613$.

These 2 tests should be compared with the classic CP test for the equality of the partial widths of CP-conjugate reactions $A_\Gamma \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$. Where, e.g. for $\tau \to \rho\nu$ decay, $\Gamma = \Gamma(\tau^- \to \rho^- \nu)$ and $\bar{\Gamma} = \bar{\Gamma}(\tau^+ \to \rho^+ \bar{\nu})$. So, A_Γ tests for CKM-type CP violation and for $r_a/r_b \neq 1$, but it is <u>not</u> sensitive to $\beta_a \neq \beta_b$.

In contrast, I_4's two tests for $r_a/r_b \neq 1$ and $\beta_a \neq \beta_b$ are tests for a non-CKM-type

Table 2
2 Tests for "Non-CKM-Type Leptonic CP Violation" in $\tau \to \rho\nu$ decay (ideal statistical errors):

E_{cm}	$\sigma(r_a)$ $CP\tilde{T}_{FS}, CP$	$\sigma(\tilde{\beta}) \simeq \sigma(\beta_a)$ CP $\tilde{T}_{FS}$	$\sigma(\beta')$ $CP\tilde{T}_{FS}, CP$
M_Z	0.6%	$\sim 1.9^o$	$\sim 3^o$
$10GeV$	0.1%	$\sim 0.4^o$	$\sim 0.7^o$
$4GeV$	0.1%	$\sim 0.9^o$	$\sim 1.1^o$

Note $\tilde{\beta} = \beta_a - \beta_b$ and $\beta' = \beta_a + \beta_b$.

of leptonic CP violation. Any overall leptonic CKM-type phases in any mixture of V and A couplings will equally effect the $A\left(-1, -\frac{1}{2}\right)$and $A\left(0, -\frac{1}{2}\right)$amplitudes and so will cancel out in r_a and in β_a. (However, for S, P, T and T_5 couplings, CKM-phases are in general observable by S2SC functions.) Quantitatively, for $A_{amp} = A + \delta a, \bar{A}_{amp} = A$, then $A_\Gamma = \delta a/A \geq (1-4\%)$ since for τ 2-body modes the denominator is known to $1-4\%$. Whereas, as shown in Table 2 for $\tau^- \to \rho^-\nu$, $\delta r_a/r_a = (0.1-1\%)$.

Table 3
Comparison of discrete symmetry requirements for moduli ratios and phase differences:

$CP\tilde{T}_{FS}$	CP	$\tilde{T}_{FS}$
$r_a = r_b$	$r_a = r_b$ same	No prediction
$\beta_a = -\beta_b$, or $\beta_a + \beta_b = 2\pi$	$\beta_a = \beta_b$ opposite	All β's vanish

Table 3 shows that the CP and $CP\tilde{T}_{FS}$ predictions for the phase relation between β_a and β_b are opposite. So this provides a method for distinguishing between a new physics effect due to an unusual CP-violating final state interaction and one with a different mechanism of CP violation.

In ρ_{hh}, the ω_1 parameter is only a function of $\tilde{\theta}_1$ (i.e. of E_ρ) since

$$\sin\omega_1 = m_\rho \beta\gamma \sin\theta_1^\tau / p_1$$

$$\cos\omega_1 = \frac{M}{4m_\tau^2 p_1}\left(m_\tau^2 - m_\rho^2 + \left[m_\tau^2 + m_\rho^2\right]\beta\cos\theta_1^\tau\right)$$

where $M = E_{cm}$, $\gamma = M/(2m_\tau)$. Physically ω_1 characterizes the Wigner rotation angle which occurs in going from the τ^- rest frame's description of ρ^- decay (which is most easily used in the helicity formalism) and the Z^o/γ^*rest frame's variables for ρ^- decay which are most easily measured experimentally. In the future, with silicon-vertex-detectors the τ^- rest frame may be known experimentally for a large sub-sample of $\tau^-\tau^+$pair events in which case a simpler 4 variable function $I\left(E_\rho, E_{\bar{\rho}}, \tilde{\phi}_a, \tilde{\phi}_b\right)$could be used, see ref. [9].

Notice that measurement of β_a by the simple 4-variable SCSC function is possbile only because of the occurence of the Wigner rotation. In ρ_{hh}, $\cos\beta_a$ appears multiplied by $\cos\omega_1$. Also notice the necessity of the τ^- being polarized for measurement of β_a. Here this is achieved by using the spin-correlation technique. At γ^* energies, β_a cannot be measured by only analyzing the decay of an unpolarized τ^- via $\tau^- \to \rho^-\nu \to (\pi^-\pi^o)$. At the Z^o, the dependence goes as $P_\tau \cos\beta_a$ where $P_\tau \simeq -0.14$ is the degree of tau polarization. Lastly, notice that unlike in K_{l3} decays there are no large final-state interactions in the "standard lepton model", such as an electromagnetic rescattering between the ρ (or its decay products) and the ν.

Two useful generalizations of the above $I\left(E_\rho, E_{\bar{\rho}}, \tilde{\theta}_1, \tilde{\theta}_2\right)$ for the $\{\rho^-, \rho^+\}$ are [10]

- to the $\tau \to a_1\nu$ decay mode in which the $a_1$has the opposite CP quantum number to that of the ρ [13]

Table 4
2 Tests for "Non-CKM-Type Leptonic CP Violation" in $\tau^- \to a_1^- \nu$ decay, both $(2\pi^-\pi^+)$ and $(2\pi^o\pi^-)$ (ideal statistical errors):

E_{cm}	$\sigma(r_a)$ $CP\tilde{T}_{FS}, CP$	$\sigma(\tilde{\beta}) \simeq \sigma(\beta_a)$ CP $\tilde{T}_{FS}$	$\sigma(\beta')$ $CP\tilde{T}_{FS}, CP$
M_Z	0.3%	$\sim 10^o$	$\sim 15^o$
$10GeV$	0.05%	$\sim 3^o$	$\sim 3^o$
$4GeV$	0.05%	$\sim 4^o$	$\sim 5^o$

- to include ν_R couplings [see next Section].

For the $\tau^- \to a_1^- \nu \to (\pi^-\pi^-\pi^+)\nu, (\pi^o\pi^o\pi^-)\nu$ modes, the composite-decay-density matrix is given by

$$\begin{aligned}
&\rho_{hh} = \\
&(1 + h\cos\theta_1^\tau)\left[\sin^2\omega_1 \cos^2\tilde{\theta}_1\right. \\
&+(1 - \tfrac{1}{2}\sin^2\omega_1)\sin^2\tilde{\theta}_1 \\
&+\tfrac{r_a^2}{2}(1 - h\cos\theta_1^\tau)\left[(1+\cos^2\omega_1)\cos^2\tilde{\theta}_1\right. \\
&+\left(1 + \tfrac{1}{2}\sin^2\omega_1\right)\sin^2\tilde{\theta}_1] \\
&-h\tfrac{r_a}{\sqrt{2}}\cos\beta_a \sin\theta_1^\tau \sin 2\omega_1\left[\cos^2\tilde{\theta}_1 - \tfrac{1}{2}\sin^2\tilde{\theta}_1\right]
\end{aligned}$$

Here $\tilde{\theta}_1$ specifies the normal to the $(\pi^-\pi^-\pi^+)$ decay triangle, instead of the π^- momentum direction used for $\tau^- \to \rho^-\nu$. Also the Dalitz plot for $(\pi^-\pi^-\pi^+)$ has been integrated over so that, following Berman & Jacob, it is not necessary to separate the form-factors for $a_1^- \to (\pi^-\pi^-\pi^+)$. The $(\pi^o\pi^o\pi^-)$ mode is similarly treated.

For the two CP tests Table 4 shows that the sensitivity of the $a_1$mode, versus the ρ mode, is about 2 times better for the r_a measurement and is about 5 times worse for the β measurements.

2.3. Tests for a (V-A) + "something" Structure

It is straightforward to include ν_R and $\bar{\nu}_L$ couplings in spin-correlation functions. The helicity amplitudes for $\tau^- \to \rho^-\nu_{L,R}$ for both $(V \mp A)$ couplings and m_ν arbitrary are:

For ν_L so $\lambda_\nu = -\frac{1}{2}$,

$$\begin{aligned}
&A\left(0,-\tfrac{1}{2}\right) = g_L\left(\tfrac{E_\rho+q_\rho}{m_\rho}\right)\sqrt{m_\tau\left(E_\nu+q_\rho\right)} \\
&-g_R\left(\tfrac{E_\rho-q_\rho}{m_\rho}\right)\sqrt{m_\tau\left(E_\nu-q_\rho\right)} \\
&A\left(-1,-\tfrac{1}{2}\right) = g_L\sqrt{2m_\tau\left(E_\nu+q_\rho\right)} \\
&-g_R\sqrt{2m_\tau\left(E_\nu-q_\rho\right)}.
\end{aligned}$$

For ν_R so $\lambda_\nu = \frac{1}{2}$,

$$\begin{aligned}
&A\left(0,\tfrac{1}{2}\right) = -g_L\left(\tfrac{E_\rho-q_\rho}{m_\rho}\right)\sqrt{m_\tau\left(E_\nu-q_\rho\right)} \\
&+g_R\left(\tfrac{E_\rho+q_\rho}{m_\rho}\right)\sqrt{m_\tau\left(E_\nu+q_\rho\right)} \\
&A\left(1,\tfrac{1}{2}\right) = -g_L\sqrt{2m_\tau\left(E_\nu-q_\rho\right)} \\
&+g_R\sqrt{2m_\tau\left(E_\nu+q_\rho\right)}
\end{aligned}$$

and $A\left(-1,\frac{1}{2}\right) = 0$. Note that g_L, g_R denote the 'chirality' of the coupling and $\lambda_\nu = \mp\frac{1}{2}$ denote the handedness of $\nu_{L,R}$.

The resulting S2SC formulas are also very simple for including ν_R and $\bar{\nu}_L$ couplings,

$$\begin{aligned}
&I\left(E_\rho, E_{\bar{\rho}}, \tilde{\theta}_1, \tilde{\theta}_2\right)|_{\nu_R,\bar{\nu}_L} = I_4 \\
&+\left(\lambda_R\right)^2 I_4\left(\rho \to \rho^R\right) + \left(\bar{\lambda}_L\right)^2 I_4\left(\bar{\rho} \to \bar{\rho}^L\right) \\
&+\left(\lambda_R\bar{\lambda}_L\right)^2 I_4\left(\rho \to \rho^R, \bar{\rho} \to \bar{\rho}^L\right)
\end{aligned}$$

where $\lambda_R \equiv \frac{|A(0,\frac{1}{2})|}{|A(0,-\frac{1}{2})|}$, $\bar{\lambda}_L \equiv \frac{|B(0,-\frac{1}{2})|}{|B(0,\frac{1}{2})|}$ give the moduli's of the ν_R and $\bar{\nu}_L$ amplitudes versus the standard amplitudes. The corresponding composite density matrices for $\tau \to \rho\nu$ with ν_R and $\bar{\nu}_L$ final state particles are given by the substitution rules:

$$\begin{aligned}
\rho_{hh}^R &= \rho_{-h,-h}\left(r_a \to r_a^R, \beta_a \to \beta_a^R\right) \\
\bar{\rho}_{hh}^L &= \bar{\rho}_{-h,-h}\left(r_b \to r_b^L, \beta_b \to \beta_b^L\right)
\end{aligned}$$

where the ν_R and $\bar{\nu}_L$ moduli ratios and phase differences are defined by $r_a^R \equiv \frac{|A(1,\frac{1}{2})|}{|A(0,\frac{1}{2})|}$, $r_b^L \equiv \frac{|B(-1,-\frac{1}{2})|}{|B(0,-\frac{1}{2})|}$, $\beta_a^R \equiv \phi_1^a - \phi_0^{aR}$, $\beta_b^L \equiv \phi_{-1}^b - \phi_0^{bL}$.

The presence of $(V+A)$ couplings in $\tau^- \to A^-\nu_\tau$ is characterized by the value of the "chirality parameter" $\xi_A \equiv \frac{|g_L|^2-|g_R|^2}{|g_L|^2+|g_R|^2} = \frac{2Re(v_A a_A^*)}{|v_A|^2+|a_A|^2}$. Note that $\xi_A = -\langle h_{\nu_\tau}\rangle$, twice the negative of the ν_τ helicity, in the special case of only V and A couplings and $m_\nu = 0$. Using spin-correlations, the ALEPH [14] and the ARGUS [15] collaborations have measured ξ_A. The current world average is $\xi_A = 1.002 \pm 0.032$, see M. Davier's talk [16].

Using both the above 4 variable and the analogous 7 variable S2CS functions, we have obtained their associated ideal statistical errors for measurements of ξ_ρ, and ξ_{a_1} so as to see what are the "ideal" limits and to see whether it is useful to include additional variables: Using I_4, for respectively $\{\rho^-,\rho^+\}$, $\{a_1^-,a_1^+\}$ we find $\sigma = 0.006, 0.010$ at M_Z, $\sigma = 0.0012, 0.0018$ at $10GeV$, and $\sigma = 0.0013, 0.0018$ at $4GeV$. This shows for $\{\rho^-,\rho^+\}$ that by using I_4, instead of the simpler 2 variable $I(E_\rho, E_{\bar{\rho}})$ spin- correlation function, there is about a factor of 8 improvement. However, using the 3 additional variables $(\phi, \tilde{\phi}_1, \tilde{\phi}_2)$ in I_7 gives less than a 1% improvement. Even if in addition the τ^-momentum direction is known via a SVX detector, there is only an $\sim$ 11% improvement. Hence, for $(V+A)$ versus $(V-A)$ more than 4 variables is not statistically helpful.

2.4. Tests for Additional Tensorial, Scalar, and Pseudo- Scalar Couplings

Historically in the study of the weak charged-current in muonic and in hadronic processes, it has been an important issue to determine the "complete Lorentz structure" directly from experiment in a model independent manner. Here the I_4 and I_7 functions can be used for this purpose to study the τ charged-current [7] since these functions depend directly on the 4 helicity amplitudes for $\tau^- \to \rho^-\nu$ and on the 4 amplitudes for the CP- conjugate process. We also obtain the associated "ideal" sensitivities.

We first consider the "traditional" couplings for $\tau^- \to \rho^-\nu$ which characterize the most general Lorentz coupling

$$\rho_\mu^* \bar{u}_{\nu_\tau}(p)\,\Gamma^\mu u_\tau(k)$$

where $k_\tau = q_\rho + p_\nu$. It is convenient to treat the vector and axial vector matrix elements separately. We introduce a parameter Λ = "the scale of New Physics". In effective field theory this is the scale at which new particle thresholds are expected to occur. In old-fashioned renormalization theory it is the scale at which the calculational methods and/or the principles of "renormalization" breakdown, see for example [17]. While some terms of the following types do occur as higher-order perturbative-corrections in the standard model, such SM contributions are "small" versus the sensitivities of present tests in τ physics, c.f. [18,19,11,20–22].

In terms of the "traditional" tensorial and spin-zero couplings

$$V_{\nu\tau}^\mu \equiv \langle\nu|v^\mu(0)|\tau\rangle = \bar{u}_{\nu_\tau}(p)\,[g_V\gamma^\mu + \frac{f_M}{2\Lambda}\iota\sigma^{\mu\nu}(k-p)_\nu + \frac{g_{S^-}}{2\Lambda}(k-p)^\mu]u_\tau(k)$$

$$A_{\nu\tau}^\mu \equiv \langle\nu|a^\mu(0)|\tau\rangle = \bar{u}_{\nu_\tau}(p)\,[g_A\gamma^\mu\gamma_5 + \frac{f_E}{2\Lambda}\iota\sigma^{\mu\nu}(k-p)_\nu\gamma_5 + \frac{g_{P^-}}{2\Lambda}(k-p)^\mu\gamma_5]u_\tau(k)$$

Notice that $\frac{f_M}{2\Lambda}$ = a "tau weak magnetism" type coupling, and $\frac{f_E}{2\Lambda}$ = a "tau weak electricity" type coupling. Both the scalar g_{S^-} and pseudo-scalar g_{P^-}couplings do not contribute for $\tau^- \to \rho^-\nu$ since $\rho_\mu^* q^\mu = 0$, nor for $\tau^- \to a_1^-\nu$.

By Lorentz invariance, there is the equivalence theorem that for the vector current

$$S \approx V + f_M, \quad T^+ \approx V + S^-$$

and for the axial-vector current

$$P \approx A + f_E, \quad T_5^+ \approx A + P^-$$

where

$$\Gamma_V^\mu = g_V\gamma^\mu + \frac{f_M}{2\Lambda}\iota\sigma^{\mu\nu}(k-p)_\nu + \frac{g_{S^-}}{2\Lambda}(k-p)^\mu + \frac{g_S}{2\Lambda}(k+p)^\mu + \frac{g_{T^+}}{2\Lambda}\iota\sigma^{\mu\nu}(k+p)_\nu$$

$$\Gamma_A^\mu = g_A\gamma^\mu\gamma_5 + \frac{f_E}{2\Lambda}\iota\sigma^{\mu\nu}(k-p)_\nu\gamma_5 + \frac{g_{P^-}}{2\Lambda}(k-p)^\mu\gamma_5 + \frac{g_P}{2\Lambda}(k+p)^\mu\gamma_5 + \frac{g_{T_5^+}}{2\Lambda}\iota\sigma^{\mu\nu}(k+p)_\nu\gamma_5$$

Table 5
Limits on Λ in GeV for Real Coupling Constants

	$\{\rho^-, \rho^+\}$ mode		$\{a_1^-, a_1^+\}$ mode	
	At M_Z	10, or 4 GeV	At M_Z	10, or 4 GeV
1st Class Currents				
$V+A$, for ξ_A	0.006	0.0012	0.010	0.0018
f_M, for Λ	$214GeV$	1,200	282	1,500
S	$306GeV$	1,700	64	345
T_5^+	$506GeV$	2,800	371	2,000
2nd Class Currents				
f_E, for Λ	$214GeV$	1,200	282	1,500
P	$306GeV$	1,700	64	345
T^+	$506GeV$	2,800	371	2,000

For $V+A$ only, the entry is for ξ_A.

Table 6
"Reality structure" of J^μ_{Lepton} current's form factors:

Form Factor:	Class I Current	Class II Current	**T invariance**
V, A, f_M, P^-	Real parts	Imaginary parts	$Re \neq 0, Im = 0$
f_E, S^-	Imaginary parts	Real parts	$Re \neq 0, Im = 0$

The matrix elements of the divergences of these charged-currents are

$$(k-p)_\mu V^\mu = [g_V(m_\nu - m_\tau) + \frac{g_{S^-}}{2\Lambda} q^2 + \frac{g_S}{2\Lambda}(m_\nu^2 - m_\tau^2) + \frac{g_{T^+}}{2\Lambda}(q^2 - [m_\tau - m_\nu]^2)]\bar{u}_\nu u_\tau$$

$$(k-p)_\mu A^\mu = [g_A(m_\nu + m_\tau) + \frac{g_{P^-}}{2\Lambda} q^2 + \frac{g_P}{2\Lambda}(m_\nu^2 - m_\tau^2) + \frac{g_{T_5^+}}{2\Lambda}(q^2 - m_\tau^2 + m_\nu^2)]\bar{u}_\nu \gamma_5 u_\tau$$

Both the weak magnetism $\frac{f_M}{2\Lambda}$ and the weak electricty $\frac{f_E}{2\Lambda}$ terms are divergenceless. On the other hand, since $q^2 = m_\rho^2$, when $m_\nu = m_\tau$ there are non-vanishing terms due to the couplings S^-, T^+, A, P^-, T_5^+.

Table 5 gives the limits on these additional couplings assuming a $(V-A)+$"something" structure for the tau charged-current. Real coupling constants are assumed. Notice that at M_Z the scale of $\Lambda \approx$few $100GeV$ can be probed; and at $10GeV$ or at $4GeV$ the scale of $1-2TeV$ can be probed.

In compiling the entries in Table 5, we have used the idea of 1st and 2nd class currents [23]. This is suggested by a 3rd-family perspective of a possible "$\tau \leftrightarrow \nu_\tau$ symmetry" in the structure of the tau lepton currents. At the level of the masses, this is a badly broken symmetry but it might still be relevant to 3rd- family currents. Recall that $\frac{m_b}{m_t} \sim \frac{5}{174} \sim 3\%$, and $\frac{m_\nu}{m_\tau} < \frac{24}{1777} \sim 1.7\%$ so this symmetry is badly broken for the 3rd family. However, for the other leptons this symmetry is empirically (and also for Harari's theoretical values) more strongly broken since $\frac{m_{\nu_e}}{m_e} < 10^{-5}$, and $\frac{m_{\nu_\mu}}{m_\mu} < 0.15\%$.

We assume that the effective charged-current $J_{Lepton}{}^{Charged}$ is Hermitian and has such an SU(2) symmetry, so that we can identify the ν_τ and the τ^-spinors. Thereby, we obtain for the "traditional couplings" and real form factors that the "Class I" couplings are V, A, f_M, P^-, and that the "Class II" couplings are f_E, S^- if we define $J^\mu_{Lepton} = J^\mu_I + J^\mu_{II}$ where for $U = \exp(\iota \pi I_2)$

$$\left.\begin{matrix} (J_I^\mu)^\dagger = -U J_I^\mu U^{-1} & First \\ (J_{II}^\mu)^\dagger = U J_{II}^\mu U^{-1} & Second \end{matrix}\right. \; Class$$

This classification is particularly useful in considering the reality structure of the charged-current [24]. As show in Table 6 there is a "clash" between the "Class I and Class II" structures and the consequences of time-reversal invariance. In

Table 7
Limits on Λ in GeV for Pure Imaginary Coupling Constants

	$\{\rho^-, \rho^+\}$ mode		$\{a_1^-, a_1^+\}$ mode	
	At M_Z	10, or 4 GeV	At M_Z	10, or 4 GeV
1st Class Currents:				
$V+A$, for ξ_A	0.006	0.0012	0.010	0.0018
f_M, for $(\Lambda)^2$	$(12GeV)^2$	$(28)^2$	$(15)^2$	$(34)^2$
S	$(14GeV)^2$	$(33)^2$	$(6)^2$	$(13)^2$
T_5^+	$(22GeV)^2$	$(50)^2$	$(18)^2$	$(42)^2$
2nd Class Currents:				
f_E, for $(\Lambda)^2$	$(12GeV)^2$	$(28)^2$	$(15)^2$	$(34)^2$
P	$(14GeV)^2$	$(33)^2$	$(6)^2$	$(13)^2$
T^+	$(22GeV)^2$	$(50)^2$	$(18)^2$	$(42)^2$

For $V+A$ only, the entry is for ξ_A.

Table 8
Contributions to $A(\lambda_\rho, \lambda_\nu)$ from the general $\rho_\alpha^*(\overline{u}_\nu \Gamma^\alpha u_\tau)$ coupling versus $m_\nu \to 0$ limit:

	$V-A$	$V+A$	$S+P$	$S-P$
ν_L Helicity Amplitudes:				
$A(0,-1/2)$	•	∘	•	∘
$A(-1,-1/2)$	•	∘		
ν_R Helicity Amplitudes:				
$A(0,1/2)$	∘	•	∘	•
$A(1,1/2)$	∘	•		

Here • = " occurs even if $m_\nu = 0$" and ∘ = " requires $m_\nu \neq 0$."
The $T^+ \pm T_5^+$ couplings are respectively mixtures of the $V \mp A$ and the $S \pm P$.
So, the $T^+ \pm T_5^+$ have the same columnar entries as that shown for the $V \mp A$.

particular, there are the useful theorems that (a) ($\tau \leftrightarrow \nu_\tau$ symmetry) + (T invariance) $\Longrightarrow$ Class II currents are absent, (b) ($\tau \leftrightarrow \nu_\tau$ symmetry) + (existence of J_I^μ and J_{II}^μ) $\Longrightarrow$ violation of T invariance, and (c) (existence of J_{II}^μ) + (T invariance) $\Longrightarrow$($\tau \leftrightarrow \nu_\tau$ symmetry) in J_{Lepton}^μ is broken.

Table 7 shows the limits on such couplings assuming a pure- imaginary coupling constant. In the case of $(V-A)$ the limits on the β's in Sec. 2.2 cover this situation. Notice that the limits here are in $(\Lambda)^2$and are $\Lambda \sim$ few $10GeV$'s because, unlike for Table 6, this is not due to an interference effect in the S2SC functions.

Besides the 3rd-family perspective of a possible $\tau \leftrightarrow \nu_\tau$ symmetry, it is also instructive to consider the situation from the perspective of "Chiral Combinations" of the various Lorentz couplings, see Table 8. Note that the $S \pm P$ couplings do not contribute to the transverse ρ or $a_1$transitions. Tables 9 and 10 give the limits on Λ in the case of purely real and imaginary coupling constants for these "Chiral Couplings".

The results in the tables in this section easily follow from the dependence of the helicity amplitudes for $\tau^- \to \rho^- \nu$ on the presence of $(S \pm P)$ couplings with m_ν arbitrary:

$$A(0,-\tfrac{1}{2}) = g_{S+P}(\tfrac{m_\tau}{2\Lambda})\tfrac{2q_\rho}{m_\rho}\sqrt{m_\tau(E_\rho + q_\rho)} + g_{S-P}(\tfrac{m_\tau}{2\Lambda})\tfrac{2q_\rho}{m_\rho}\sqrt{m_\tau(E_\rho - q_\rho)}$$

$$A(-1,-\frac{1}{2}) = 0$$

$$A(0,\tfrac{1}{2}) = g_{S+P}(\tfrac{m_\tau}{2\Lambda})\tfrac{2q_\rho}{m_\rho}\sqrt{m_\tau(E_\rho - q_\rho)} + g_{S-P}(\tfrac{m_\tau}{2\Lambda})\tfrac{2q_\rho}{m_\rho}\sqrt{m_\tau(E_\rho + q_\rho)}$$

Table 9
"Chiral Couplings": Limits on Λ in GeV for Real Coupling Constants

	$\{\rho^-,\rho^+\}$ mode		$\{a_1^-,a_1^+\}$ mode	
	At M_Z	10, or 4 GeV	At M_Z	10, or 4 GeV
$V+A$, for ξ_A	0.006	0.0012	0.010	0.0018
$S+P$, for Λ	$310GeV$	1,700	64	350
$S-P$, for $(\Lambda)^2$	$(11GeV)^2$	$(25)^2$	$(4)^2$	$(7)^2, (10)^2$
f_M+f_E, for Λ	$210GeV$	1,200	280	1,500
f_M-f_E, for $(\Lambda)^2$	$(9GeV)^2$	$(20)^2$	$(10)^2$	$(24)^2$

For the ρ and a_1 modes, the $T^+ + T_5^+$ coupling is equivalent to the $V-A$ coupling; and $T^+ - T_5^+$ is equivalent to $V+A$.

Table 10
"Chiral Couplings": Limits on Λ in GeV for Pure Imaginary Coupling Constants

	$\{\rho^-,\rho^+\}$ mode		$\{a_1^-,a_1^+\}$ mode	
	At M_Z	10, or 4 GeV	At M_Z	10, or 4 GeV
$V+A$, for ξ_A	0.006	0.0012	0.010	0.0018
$S+P$, for$(\Lambda)^2$	$(11GeV)^2$	$(25)^2$	$(4)^2$	$(10)^2$
$S-P$, for $(\Lambda)^2$	$(11GeV)^2$	$(25)^2$	$(4)^2$	$(7)^2, (10)^2$
f_M+f_E, for$(\Lambda)^2$	$(9GeV)^2$	$(20)^2$	$(10)^2$	$(24)^2$
f_M-f_E, for $(\Lambda)^2$	$(9GeV)^2$	$(20)^2$	$(10)^2$	$(24)^2$

For the ρ and a_1 modes, the $T^+ + T_5^+$ coupling is equivalent to the $V-A$ coupling; and $T^+ - T_5^+$ is equivalent to the $V+A$.

$$A(1,\frac{1}{2}) = 0$$

Finally, as shown in Tables 11 and 12, the helicity amplitudes themselves provide a simple framework for characterizing a "complete measurement" of the $\tau^- \to \rho^-\nu$: In the case that only ν_L coupling's exist (compare the discussion in Sec.2.2), there are only 2 amplitudes. So 3 measurements (of r_a, β_a,and $|A(0,-\frac{1}{2})|$ via $\{\rho^-, B^+\}\ |_{B\neq\rho}$) will provide a "complete measurement". In the case that ν_R coupling's also exist, then there are 2 more amplitudes, $A(0,\frac{1}{2})$ and $A(1,\frac{1}{2})$. Then there are 2 additional CP tests: $r_a^R = r_b^L$ and $\beta_a^R = \beta_b^L$. Then to achieve an "almost" complete measurement, 3 additional quantities must be determined, e.g. by the I_4 S2SC function: r_a^R, β_a^R and $\lambda_R \equiv \frac{|A(0,\frac{1}{2})|}{|A(0,-\frac{1}{2})|}$. However, to also measure the relative phase of the ν_L and ν_R amplitudes, $\beta_a^o \equiv \phi_o^{aR} - \phi_o^a$ or $\beta_a^1 \equiv \phi_1^a - \phi_{-1}^a$, requires, e.g., the occurrence of a common final state which arises from both ν_L and ν_R.

3. "ADDITIONAL STRUCTURE" IN NEUTRAL-CURRENT COUPLINGS?

Here, our discussion is brief and we refer the reader to other talks at this conference [11,12] on the Lorentz structure of neutral-current couplings. Again it is important to perform a complete systematic search for possible additional S, P, and T couplings, with either real or imaginary coupling constants, and to determine the experimental bounds on the associated scales Λ which probe for possible new physics. The helicity formalism in the context of "beam-referenced spin-correlation" functions provides a simple framework for such an investigation since simple BRSC functions exist [22] and the relevant helicity amplitudes are simply expressed in terms of the most general Lorentz couplings (see Sec. 3 of Ref. [25]).

To obtain BRSC functions, the simpler $I(E_1, E_2)$ spin-correlation function is generalized by including the polar and azimuthal angles of the incident e^- beam versus the final $\pi^-$and π^+ mo-

Table 11
Limits on ν_R and $\bar{\nu}_L$ couplings in terms of the ratios of the moduli of the helicity amplitudes:

	$\{\rho^-, \rho^+\}$ mode		$\{a_1^-, a_1^+\}$ mode	
	At M_Z	10, or 4 GeV	At M_Z	10, or 4 GeV
λ_R	$(7\%)^2$	$(3\%)^2$	$(13\%)^2$	$(6\%)^2$
$\lambda_R r_a^R$	$(7\%)^2$	$(3\%)^2$	$(10\%)^2$	$(4\%)^2$

Elements of "error matrix" are given in the next table.

Table 12
Elements of error matrix for limits on ν_R and $\bar{\nu}_L$ couplings in terms of the helicity amplitudes for respectively $\tau \to \rho\nu$, and $\tau \to a_1\nu$:

	$\{\rho^-, \rho^+\}$ mode		$\{a_1^-, a_1^+\}$ mode	
	At M_Z	10, or 4 GeV	At M_Z	10, or 4 GeV
Diagonal elements:				
$a = \lambda_R$	$(8\%)^2$	$(4\%)^2$	$(18\%)^2$	$(8\%)^2, (9\%)^2$
$b = \lambda_R r_a^R$	$(8\%)^2$	$(4\%)^2$	$(18\%)^2$	$(8\%)^2, (9\%)^2$
$c =$				
$(\lambda_R)^2 r_a^R \cos\beta^R$	$(13\%)^2$	$(6\%)^2, (10\%)^2$	$(41\%)^2$	$(20\%)^2, (24$ **Correlations :**
ρ_{ab}	-0.75	-0.77	-0.95	$-0.96, -0.97$
ρ_{ac}	-0.27	$-0.17, 0.06$	-0.56	$0.029, 0.019$
ρ_{bc}	0.085	$.017, 0.003$	0.04	$-0.041, -0.026$

menta [e.g. $\hat{z}$ along the final π^- with the orthogonal $\hat{x}$ in $(\hat{z}, \pi^+)$ half-plane. Then $\theta_{beam}, \phi_{beam}$ describes the incident e^- in this $\hat{x}$, $\hat{y}$, $\hat{z}$ right-handed coordinate system].

By now, there is a considerable literature on tests for such anomalous couplings, in particular for electromagnetic and/or weak dipole moments in $\gamma^* \to \tau^-\tau^+$ [18,19] and in $Z^o \to \tau^-\tau^+$ [26,20,22,21,27–30]. There are experimental bounds on the real and imaginary parts of a weak dipole moment $\tilde{d}(q^2)$ from the ALEPH and OPAL collaborations [31,12].

For Z^o, or $\gamma^* \to \tau^-\tau^+$, there are 4 independent (complex) helicity amplitudes $T(\pm\mp)$ and $T(\pm\pm)$ where the matrix element

$$\langle \Theta_\tau, \Phi_\tau | J = 1, M \rangle = D^{1*}_{M\lambda}(\Phi_\tau, \Theta_\tau, -\Phi_\tau) T(\lambda_1, \lambda_2)$$

Table 13 lists the discrete symmetry relations for these production amplitudes. In principle, these amplitudes can be completely determined by measurement of the BRSC function for the process $e^-e^+ \to Z^o$, or $\gamma^* \to \tau^-\tau^+ \to (\pi^-\nu)(\pi^+\bar{\nu})$.

Table 14 lists the 4 distinct tests for $CP/\tilde{T}_{FS}$ violation in these production amplitudes. Analogous to the Michel parameters in μ decay, here there are 9 vertex intensity parameters which characterize possible $CP/\tilde{T}_{FS}$ violation; so there are 5 vertex intensity relations which can be used as consistency checks if such a violation is observed [22].

Table 13
Discrete symmetry relations for production amplitudes for $Z^o, \gamma^* \to \tau^-\tau^+$:

If Decay	**Then**
CP invar.	$T(++) = T(--)$
P invar.	$T(+-) = T(-+)$, $T(++) = T(--)$
C invar.	$T(+-) = T(-+)$

P and C invariances can be tested for in $\gamma^* \to \tau^-\tau^+$.

Table 15 lists the ideal sensitivity for a complete measurement of these production amplitudes.

As noted below Table 15, unless experimental

Table 14
4 distinct tests for CP / $\tilde{T}_{FS}$ violation in the production process $Z^o, \gamma^* \rightarrow \tau^- \tau^+$:

If symmetry	Then
$\tilde{T}_{FS}$	
	$\beta_{+-} \equiv \phi_{+-} - \phi_{-+} = 0$
	$\beta_{++} \equiv \phi_{++} - \phi_{--} = 0$
	$2\beta_o \equiv \phi_{++} + \phi_{--} - \phi_{+-} - \phi_{-+} = 0$
CP	
	$\beta_{++} = 0$
	$\lambda = 0$

$\lambda \equiv (|T(++)| - |T(--)|)/(|T(++)| + |T(--)|)$

Table 15
Errors for a complete measurement of the Z^o, or $\gamma^* \rightarrow \tau^- \tau^+$ helicity amplitudes:

$(\tau - \gamma^* - \tau)$ vertex at $10GeV$	at $4GeV$
$\lvert t(-+)\rvert = \lvert t(+-)\rvert$ to 0.4%	0.4%
$\lvert t(++)\rvert = \lvert t(--)\rvert$ to 0.8%	0.2%
β_{+-} to 0.5^o	0.7^o
β_o to 1^o	0.7^o
β_{++} to 4^o	1.5^o
$\sigma(\lambda) = 0.007$	0.024
$(\tau - Z^o - \tau)$ vertex at M_Z	
$\lvert T(-+)\rvert$ to 1%	
$\lvert T(+-)\rvert$ to 1%	
β_{+-} to 3^o	

Without experimental surprises, the Z^o helicity-changing $T(++), T(--)$ will be unmeasured since $\sigma(|T(--)|)/|T(--)| = 6$.

surprises are discovered, at LEP the "helicity-changing neutral-current" amplitudes $T(\pm\pm)$ will remain unmeasured. But experimentally "Is there something where the Standard Model says there is nothing?" Here again for the τ lepton the situation is less unfavorable than for the other charged leptons since

$$\frac{|T(--)|}{|T(-+)|} \simeq \sqrt{2}\frac{v_f m_{lepton}}{a_f M_Z} \simeq 10^{-3}$$

for τ, but is 10^{-4} for μ, 10^{-6} for e.

Also at $10GeV$, and at $4GeV$, there are 3 tests for C and P invariances as shown with their associated sensitivities in Table 16. The vertex intensity parameters are defined in Ref. [22]:

$$\omega, \eta \equiv (Re[\{T(++) + T(--)\}T^*(\mp\pm)])/N$$

$$\zeta \equiv 2Re[T(++)T^*(--)]/(|T(++)|^2 + |T(--)|^2)$$

are mainly functions of the helicity-changing amplitudes. But,

$$\alpha_H \equiv P_\tau \equiv [|T(+-)|^2 - |T(-+)|^2]/N$$

depends mainly on the helicity-conserving amplitudes. Here $N \equiv \sum |T(\lambda_1, \lambda_2)|^2$.

4. CONCLUSIONS

(1) In searching for new 3rd-family phenomena in the context of large and growing $(\tau^- \tau^+)$ data samples, we have two powerful tools

Table 16
Ideal statistical errors with respect to C and P tests for the production process $\gamma^* \rightarrow \tau^- \tau^+$:

$(\tau - \gamma^* - \tau)$ vertex		$10GeV$	$4GeV$
"C or P good"	$\Longrightarrow \alpha_H = 0$	0.002	0.002
	$\Longrightarrow \omega = \eta$	3%	2%
"P or CP good"	$\Longrightarrow \zeta = 1$	6%	1%

Note that $\alpha_H = P_\tau$, the tau polarization.

- tau polarimetry
- $(\tau^- \tau^+)$ spin correlations.

In addition, in the near future there is the very exciting possiblity that longitudinally polarized beams will be available in a "Tau Charm Factory" [32,33].

While the analogous tools [6] can also someday be used in $(t\,\bar{t})$ physics, there is at present a crucial difference. Until, at least, near the end of this century, there will only be large data samples for the other than "t" members of the 3rd-family.

(2) There are observables for tests for "Additional Structure" in

- tau charged-current couplings
- tau neutral-current couplings.

For instance, in charged-current couplings

$f_M(q^2)$ tau weak magnetism
$f_E(q^2)$ tau weak electricity
can be probed to new physics scales of
$\Lambda_{Real Coupling}$ $1.2-1.5TeV$ at 10, or $4GeV$
$\Lambda_{Imag. Coupling}$ $28-34GeV$ at 10, or $4GeV$.
For neutral-current couplings, there are
(i) tests for "anomalous couplings" in $Z^o \to \tau^- \tau^+$
(ii) C and P tests in $\gamma^* \to \tau^- \tau^+$

In principle, by spin-correlation techniques the Lorentz stucture of the neutral- current couplings can be completely determined and that of the charge-current couplings can almost be completely determined from, for instance, the $\{\rho^-, \rho^+\}$and $\{a_1^-, a_1^+\}$ modes.

(3) There are observables for testing for T and/or CP violation in tau decays, and in the Z^o and $\gamma^* \to \tau^- \tau^+$ production processes.

ACKNOWLEDGEMENTS

For helpful discussions, we thank participants at this conference as well as theorists and experimentalists at Cornell, DESY, and Valencia. This work was partially supported by U.S. Dept. of Energy Contract No. DE-FG 02- 96ER40291.

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