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Using the Inverse Chirp-Z Transform for Time-Domain Analysis of Simulated Radar Signals

by

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Abstract

There exists a need to develop a method to locate underground voids, or caches. In the past, ground penetrating radar (GPR) operating in the time domain mode has been used. In this paper, we turn our attention to stepped frequency radar, capable of making frequency domain reflection coefficient measurements. We then apply the inverse Chirp-Z transform (ICZT) to this data, generating a time domain response. The scenario under consideration is that of an airborne radar passing over the surface of the earth. The radar is directed toward the surface and is capable of measuring the reflection coefficient, seen looking toward the earth, as a function of frequency. The frequency domain data in this work is simulated and is generated from a transmission line model of the problem. Using the ICZT we convert this frequency domain data to the time domain. Once in the time domain, reflections due to discontinuities appear at times indicating their relative distance from the source. The discontinuities occurring beyond the surface of the earth could be indicative of underground structures. The ICZT allows a person to zoom in on the time span of interest by specifying the starting time, the time resolution, and the number of time steps.

I. Introduction

Ground penetrating radar has been in use for many years. At the Idaho National Engineering Laboratory, GPR has been used in locating underground pipe, and in trying to locate buried waste. Recently, there has been increased interest in trying to locate underground voids or caches.

Traditional GPR operates in an impulse mode. It transmits a series of pulses and after each pulse, it watches for the returned signal. The timing of this returned signal gives an indication of the depth of the objects causing the reflections.

For this work, we began by studying impulse GPR. The thought of making measurements in the frequency domain, rather than the time domain as in the case of the traditional GPR, led us to decide to investigate the use of a stepped frequency radar rather than an impulse radar. Other researchers have also studied stepped frequency radar.[1,2]

The problem we considered was that of an aircraft flying over the earth with a radar looking down toward the surface. The

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operating frequency range of the radar is such that the electromagnetic waves will penetrate the earth sufficiently deep to reflect from underground discontinuities. In addition, the radar is capable of measuring the ratio of the reflected signal to that of the transmitted signal (i.e., the reflection coefficient).

This problem is modeled as a series of transmission lines with impedances, propagation characteristics, and lengths dependent upon the medium. A transmission line model is equivalent to normal incidence of the electromagnetic energy. Using this transmission line model, we calculate the reflection coefficient as a function of frequency. This frequency domain data is then converted to the time domain.

The problem described, and the model, are illustrated in Figure 1, in which the transmission line L_1 represents the transmission path from the aircraft to the surface of the earth. The characteristic impedance and propagation constant will be that of free space. Transmission line L_2 represents the soil from the surface of the earth to the ceiling of the underground cache. Its characteristic impedance Z is

$$Z = \sqrt{\frac{\mu}{\epsilon}}$$

and its propagation constant γ is

$$\gamma = j\omega\sqrt{\mu\epsilon}$$

where $j = \sqrt{-1}$, ω is the radian frequency, μ is the permeability, and ϵ is permittivity

$$\mu = \mu' - j\mu''$$

and

$$\epsilon = \epsilon' - j\epsilon''$$

For transmission line L_3 we considered that the void was filled with air so this transmission line had the same characteristics as transmission line L_1. And finally, the soil is lossy enough that the earth beneath the void is considered infinite in extent. This allows us to set the load impedance, Z_L, equal to the impedance of the soil.

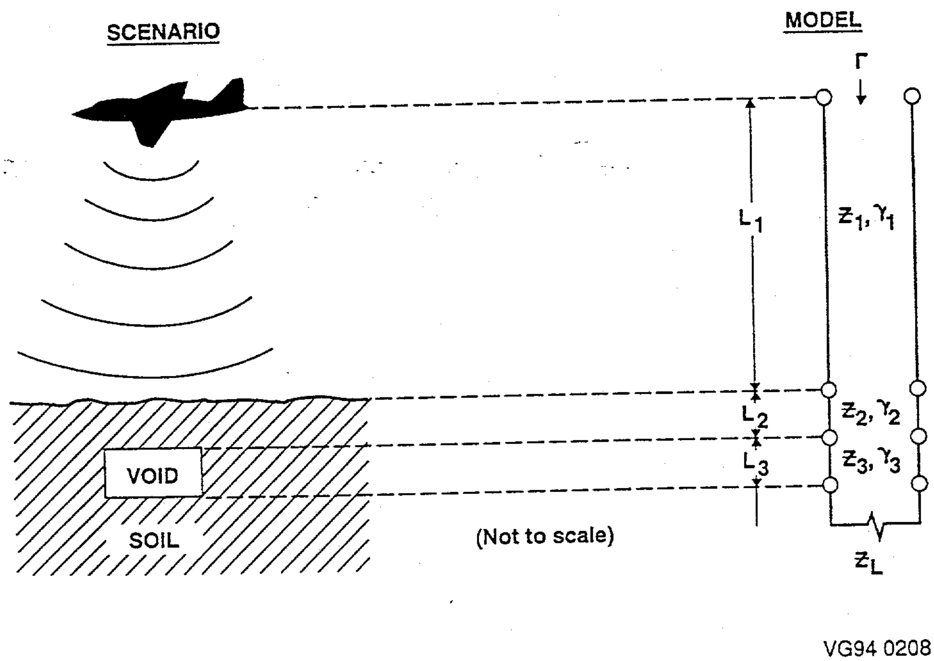


Figure 1. Illustration showing the scenario studied and the transmission line model used for analysis.

To calculate the input impedance of the network, we make multiple use of the transmission line impedance transformation equation shown below.

$$Z_{in} = Z_1 \frac{Z_L + Z_1 \tanh(\gamma_1 L_1)}{Z_1 + Z_L \tanh(\gamma_1 L_1)}$$

The reflection coefficient of this network is then calculated by

$$\Gamma = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} \quad , \quad Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$$

For an example, we chose a medium dry soil with a relative dielectric constant of 15 and a conductivity of

$$\sigma = 4.624(10^{-7}) * f^{1.636} \quad ,$$

$$f \ in \ MHz$$

$$125 \ MHz < f < 10 \ GHz$$

We placed the aircraft at an altitude of 4000 feet. A 10-foot deep void was placed 20 feet below the surface of the earth. Using the model described, the reflection coefficient was calculated and is shown in Figure 2.

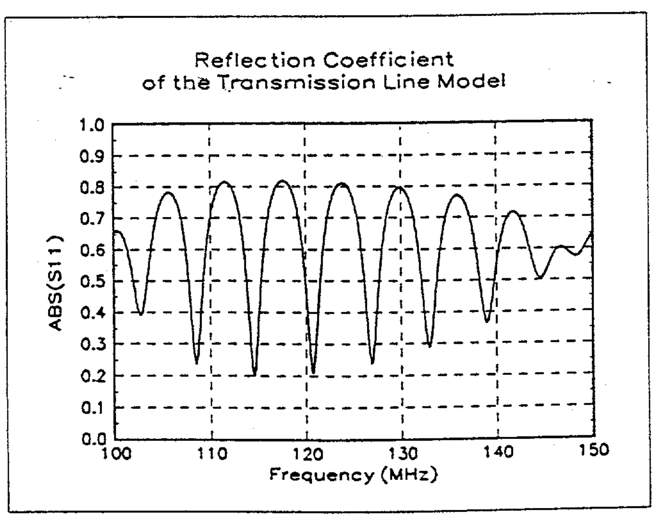


Figure 2. Reflection coefficient calculated from the transmission line model in Figure 1.

II. Frequency to Time Transformation

A time domain plot of the reflection coefficient data indicates when in time reflections occurred. This can be seen by first looking at the definition for the inverse Discrete Fourier transform written as

$$h(n) = \frac{1}{N} \sum_{n=0}^{N-1} H(k) e^{j\frac{2\pi}{N}nk}$$

where n is the time index, k is the frequency index, N is the number of data, H(k) is the frequency sequence, and h(n) is the time sequence.

From linear systems theory we know that Y(k) = H(k)*X(k) where X(k) is the Discrete Fourier transform (DFT) of the input sequence, Y(k) is the DFT of the output sequence, and H(k) is the transfer function, the DFT of the impulse response of the system. For an impulse input signal, X(k) = 1 so Y(k) = H(k). Substituting Y(k) for H(k) in (8) then shows that the results of taking IDFT of the output frequency sequence will be the impulse response of the system. Because we are converting reflection coefficient data, the impulse response will show when in time reflections occurred. Even though we use the IDFT above, the meaning of the time domain response remains the same no matter what technique we use to obtain it.

In order to achieve time domain plots with the desired resolution and time span, it is necessary to first develop relationships between them and the frequencies used in the initial calculations. From Fourier transform theory for a periodic sequence, the fundamental period is given by N*T where N is the number of samples and T is the sample interval. The frequency resolution or step size, f_{step}, is 1/(N*T).

When working in the frequency domain with a stepped-frequency system, we can immediately write $f_{step} = f_{span}/N$ where f_{span} is the frequency span, and N is the number of intervals. But from above the frequency resolution is 1/(N*T) so equating this with the expression for f_{step} shows that $T = 1/f_{span}$. This means that the sample interval in the time domain is equal to the inverse of the frequency span in the frequency domain. The result is that the total unaliased time available in the time domain is given by $1/f_{step}$. The time resolution in the time domain is given by $1/f_{span}$.

To determine the parameters of the stepped-frequency radar, one must first choose the time, or spatial resolution, desired. Of course time and distance are related by the velocity. Ignoring for the moment that the velocity of propagation of the electromagnetic wave will vary depending upon the medium through which the wave travels and just using c_0, we can choose a desired distance resolution and calculate the required f_{step} and f_{span}, based on N, the number of frequency steps. The graph in Figure 3 represents this relationship. In addition, this figure shows the maximum unaliased range.

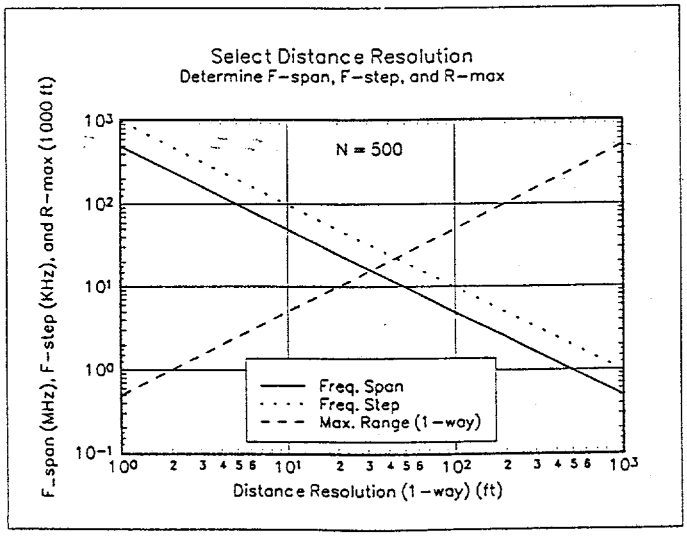


Figure 3. Graph showing the relationship between f_{span}, f_{step}, maximum range, and the one-way distance resolution.

Notice that absolute frequency is not specified. For the problem under consideration, it is necessary to select a frequency range that is suitable. Soil is known to be lossy to high frequencies. In fact, most traditional GPRs operate below 1 GHz. Based on a desired resolution of 10 feet, which is the height of the underground cache, Figure 3 provides the following approximate specifications:

$$f_{step} = 100 \text{ KHz}$$

$$f_{span} = 50 \text{ MHz}$$

After several trials, we chose the frequency range of 100 MHz to 150 MHz with N = 500.

Using the reflection coefficient data in Figure 3, we calculate the IDFT producing the plot shown in Figure 4. The total time shown on the axis is the unaliased time. Notice that most of the response in the time domain occurs close together. This is a result of the discontinuities in the transmission line model occurring first at 4000 feet, then 20 feet beyond that, and then 10 feet beyond that. If one were to try to expand this region in the time domain by simply zooming in on the graph, one would

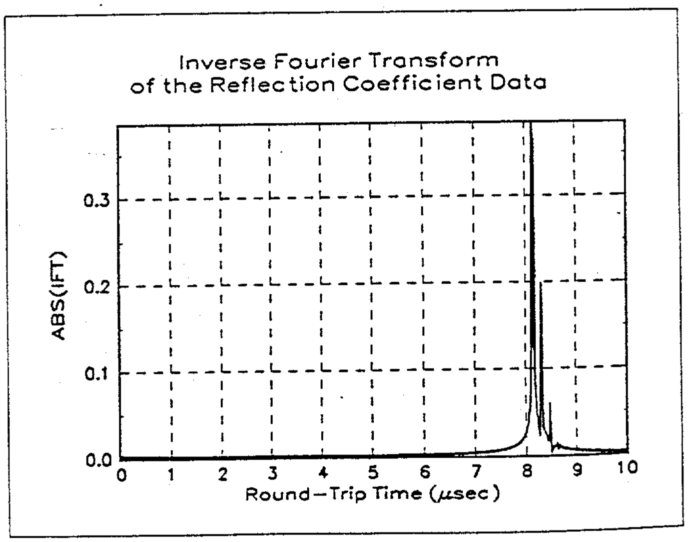


Figure 4. The time domain response calculated using the inverse Fourier transform.

find that the resolution is poor because there would only be a few points in that region. One could perform the IDFT again but this time zero padding the frequency data. This would result in a time domain plot with finer resolution however, the additional computation time could be significant.

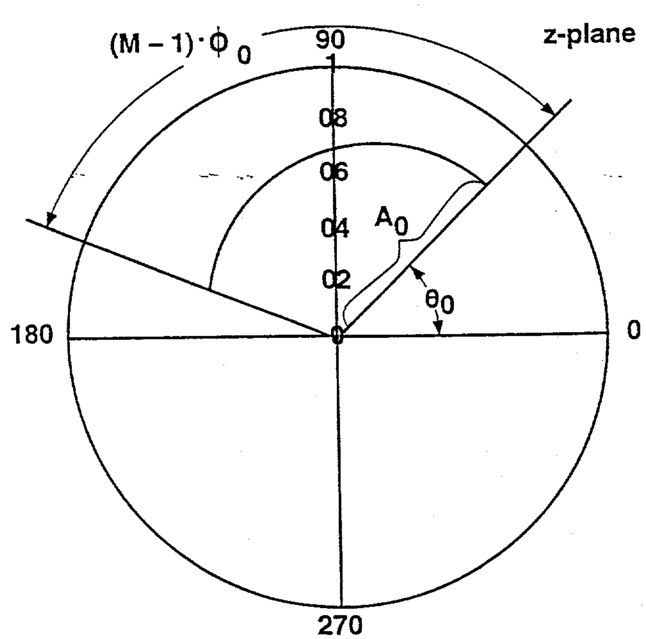
An alternative to this approach is to use the inverse Chirp-Z transform (ICZT). The Chirp-Z transform is a generalization of the Z-transform. Whereas the Z-transform is restricted to the unit circle in the z-plane, the Chirp-Z has no such restriction [3]. The Chirp-Z transform is defined by

$$X(z) = \sum_{n=0}^{N=1} x(n) z^{-n}$$

$$z = AW^{-k}, \quad K = 0,1,\ldots M-1$$

$$W = W_0 \cdot e^{j2\pi\phi_0}$$

$$A = A_0 \cdot e^{j2\pi\theta_0}$$



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where A_0 is the starting radius, θ_0 is the starting angle, and ϕ_0 is the angle step size. W_0 is the parameter that determines if the contour spirals in or out or if the radius stays constant as frequency changes. If W_0 is greater than 1, the contour spirals in. For W_0 less than 1 the contour spirals out. If $W_0 = 1$ then the radius is constant. In the case of $A_0 = W_0 = 1$, $\theta_0 = 0$, and $\phi_0 = 2\pi/N$, the Chirp-Z transform is equal to the Z transform.

Any transform that converts data from the time domain to the frequency domain has an inverse transform to convert data from the frequency domain to the time domain. This inverse transform can be obtained (to within a scaling factor) by taking the complex conjugate of the transform of the complex conjugate of the frequency domain data. For the case of the Chirp-Z, the inverse Chirp-Z transform (ICZT) can be written, $\text{ICZT}(X(k)) = [\text{CZT}(X(k)^*)]^*$.

It is not clear if this will work for the Chirp-Z transform in general; however, it will work for the Chirp-Z that operates on the unit circle. This requires $A_0 = W_0 = 1$. It is by this procedure that we calculated the ICZT in this paper.

For the problem under consideration, we can make use of the ICZT to transform the frequency domain data to the time domain. The goal is to be able to zoom in on the region of interest. To do that we make use of the fact that we know the distance to the earth and this will correspond to the time to the first peak in the time domain. Keeping in mind that the reflection coefficient data represents the round-trip travel of the electromagnetic wave, we can calculate the time to the first peak by $T_1 = 2L_1/c_0$.

By converting this to a fraction of 2π, the total unaliased time which is represented by a complete path around the unit circle, we can select the starting angle for the ICZT transform. This is given by $t_{start} = 2L_1/T_{max}$ where T_{max} is the unaliased time and is equal to N*T with N and T defined previously. The angle step size, ϕ_0, and the number of points, M, are chosen to provide the view we're interested in and were selected as ϕ_0 = -1/10000 and M = 501.

The ICZT is performed on the reflection coefficient data in Figure 2. The resulting time domain plot is shown in Figure 5, which shows the two peaks indicating the discontinuities caused by the ceiling and floor in the void.

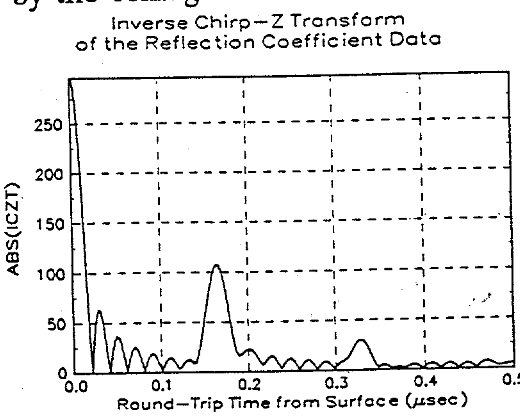

Figure 5. The time domain response calculated using the inverse Chirp-Z transform.

Also notice that in the time domain there are ripples. These result from the use of the rectangular window used in the frequency domain. If we apply a different window to the frequency data we can reduce these ripples. Figure 6 shows the time domain results after performing the ICZT on the reflection coefficient data weighted with a Hamming window.

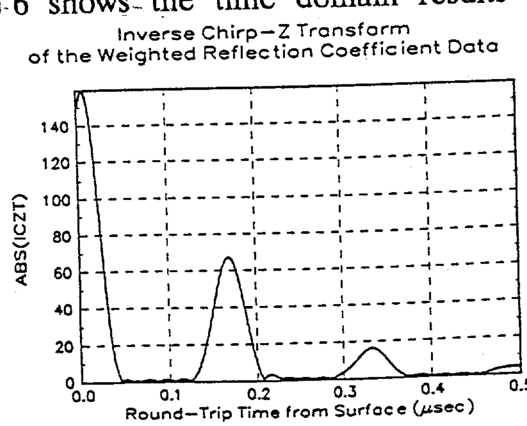

Figure 6. The time domain response of the weighted reflection coefficient data.

III. Conclusions

This work demonstrates the use of the inverse Chirp-Z transform in processing frequency domain reflection coefficient data to aid in the detection of underground voids. The analysis is performed on simulated data generated by modeling the problem with transmission lines. While no claims are being made about the idea of the ICZT, or its use to convert frequency domain reflection data to the time domain, the application of this technique to the problem of locating underground voids appears to be new.

IV. Acknowledgments

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V. References

1. K. Iizuka and A. P. Freundorfer, "Detection of Nonmetallic Buried Objects by a Step Frequency Radar," Proc. IEEE, Vol. 71, No. 2, Feb. 1983, pp. 276-279.

2. K. Iizuka, A. P. Freundorfer, K. H. Wu, H. Mori, H. Ogura, and V. Nguyen, "Step-frequency Radar," J. Appl. Phys, 56 (9), 1 Nov 1984, pp. 2572-2583.

3. L. R. Rabiner, R. W. Schafer, and C. M. Rader, "The Chirp z-Transform Algorithm and Its Application," The Bell System Technical Journal, May-June 1969, pp. 1249-1292.