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Boundary Conditions for Granular Flows at Randomly Fluctuating Bumpy Boundaries

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Boundary Conditions for Granular Flows at Randomly Fluctuating Bumpy Boundaries

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Abstract

In this paper we focus attention on the interactions between rapid flows of identical, smooth spheres that interact with bumpy boundaries through inelastic collisions. The boundaries translate with specified mean velocities, and deviate about the mean with specified fluctuation velocities. Based upon Maxwellian velocity distribution functions that describe the velocities of both the flow particles and boundaries, we calculate the rates at which linear momentum and kinetic energy are exchanged between the two. Using these exchange rates, we write down conditions that ensure that both momentum and energy are balanced at bumpy boundaries. Finally, we employ a constitutive theory that is consistent with these conditions to calculate the granular temperature and solid fraction profiles within a granular material confined between two parallel, bumpy surfaces that randomly fluctuate about zero mean velocity.

1. INTRODUCTION

In the past several years, considerable effort has been devoted to quantifying the influence that boundaries exert on the granular flows with which they interact. That the effects may be profound has been demonstrated experimentally by Craig et. al. [1987], who found that when the internal surfaces of their shear cell were relatively rough the stresses induced were considerably higher than those induced when the surfaces were relatively smooth; and through numerical simulation by Campbell and Gong [1987], who found that shear flows of disks between parallel walls were critically influenced by the geometry of the walls.

In order to calculate the effects of a containing boundary on a granular flow, it is neccessary to satisfy conditions that at least express the balance of momentum and energy at such a boundary. Phenomenological conditions of this type were first proposed by Hui et. al. [1984] and were later improved upon by Johnson and Jackson [1987]. Jenkins and Richman [1986] have employed the formal methods of averaging to obtain conditions that apply to either two-dimensional systems of identical disks or three-dimensional systems of identical spheres that interact with smooth, bumpy boundaries. The distribution function upon which the these conditions are based has since been improved upon by Richman and Chou [1988] for systems of disks, and by Richman [1988] for systems of spheres. Averaging techniques based upon simpler distribution functions have been employed by Pasquarell and Ackermann [1989] and Pasquarell [1991] to obtain conditions that apply at smooth, bumpy boundaries, and by Jenkins [1991] to obtain conditions at flat, frictional surfaces.

In this paper, we are concerned with the effects of vibrating boundaries on the granular flows that they contain. An experimental study of these effects has been conducted by Savage [1988], who vibrated the bottom panel of a rectangular box that contained round polystyrene beads at specified frequencies and at amplitudes that diminished with distance from the center of the box. As a result of the non-uniformity in vibrational amplitude, the induced mean velocities throughout the assemblies were upwards near the centerline of the box and downward near its sides. Recently Jackson [1991] proposed a phenomenological energy flux condition to account for the energy supplied to the flow by the boundary's vibrational motion, and predicted that inclined flows would be strongly influenced by small changes in this motion.

Here we focus on smooth, bumpy boundaries that translate with known mean velocities and deviate about the means with known fluctuation velocities. We write down conditions that ensure that both momentum and energy are balanced at these boundaries and employ formal methods of averaging to calculate the rates at which momentum and energy are transferred from the boundaries to the flows. Finally, we employ the resulting boundary conditions and a corresponding constitutive theory to determine the influence of the boundaries on the granular temperature and solid fraction profiles within a granular material that is excited by and confined between two parallel surfaces that randomly fluctuate about zero mean velocity.

2. PRELIMINARIES

We are concerned here with flows of identical, smooth, nearly elastic spheres of diameter σ and mass m that interact through nearly elastic collisions with smooth but bumpy boundaries that translate with mean velocities U and fluctuate about the mean with mean square velocities V^2.

The bumpy boundaries are flat walls to which identical, smooth, hemispherical particles of diameter d are randomly attached at an average distance s apart. On average, the nearest neighbors of any hemisphere form a half torus with inner diameter d+2s, outer diameter 3d+2s, and height d/2. The complete range of s/d is from -1, which corresponds to a perfectly flat boundary at a distance d/2 from the flat wall to which the bumps are attached, to $-1+\sqrt{(1+2\sigma/d)}$, which is the maximum value that prevents any flow particle from colliding with the flat wall. The fraction of surface area of each wall particle that is accesible to any flow particle is $(1-\cos\theta)$, where θ is defined by the relation $\sin\theta=(d+s)/(d+\sigma)$. In the range of s/d described, the boundary may be made effectively rougher by increasing θ from zero to a maximum value that depends on σ/d.

Each wall-flow particle collision is described by the velocities $\mathbf{c}_1$ of the wall particle and $\mathbf{c}$ of the flow particle just prior to impact, and by the unit vector $\mathbf{k}$ directed from the center of the wall particle to that of the flow particle at impact. If the coefficient of restitution that accounts for the energy dissipated during impact is e_w, and the velocity of the wall is unchanged by the impact, then in terms of the relative velocity $\mathbf{g}\equiv\mathbf{c}_1-\mathbf{c}$ the changes in linear momentum and kinetic energy experienced by a flow particle due to the impact are given respectively by,

$$m(\mathbf{c}'-\mathbf{c}) = m(1+e_w)(\mathbf{g}\cdot\mathbf{k})\mathbf{k} \quad , \tag{1}$$

in which $\mathbf{c}'$ is the velocity of the flow particle immediately after the collision, and

$$\frac{m}{2}(c'^2-c^2) = m(1+e_w)[(\mathbf{g}\cdot\mathbf{k})(\mathbf{U}\cdot\mathbf{k})+(\mathbf{g}\cdot\mathbf{k})(\mathbf{C}_1\cdot\mathbf{k})-\frac{1}{2}(1-e_w)(\mathbf{g}\cdot\mathbf{k})^2], \tag{2}$$

in which $\mathbf{C}_1$ is the fluctuation velocity $\mathbf{c}_1-\mathbf{U}$ of the wall particle.

The statistics associated with collisions between wall particles and flow particles are described by two distribution functions: a single particle distribution function f defined such that $f(\mathbf{c},\mathbf{r})d\mathbf{c}$ gives the number of flow particles per unit volume centered at position $\mathbf{r}$ with velocities $\mathbf{c}$ within the range $d\mathbf{c}$; and a probability distribution function p defined such that $p(\mathbf{c})d\mathbf{c}$ gives the probability that a specified wall particle has velocity $\mathbf{c}$ within $d\mathbf{c}$. At impact, the center position of the wall particle is $\mathbf{x}$, and the distance between centers of the colliding particles is $\bar{\sigma}\equiv(\sigma+d)/2$. The frequency of collisions per unit area of flat wall that involve flow particles with velocities $\mathbf{c}$ in the range $d\mathbf{c}$, wall particles with velocities $\mathbf{c}_1$ in the range $d\mathbf{c}_1$, and occur within an area of contact centered about $\mathbf{k}$ within an element $d\mathbf{k}$ of solid angle on the surface of the wall particle, is

$$(\chi/\pi\sin^2\theta)f(\mathbf{c},\mathbf{x}+\bar{\sigma}\mathbf{k})p(\mathbf{c}_1)(\mathbf{g}\cdot\mathbf{k})d\mathbf{k}d\mathbf{c}d\mathbf{c}_1 \quad . \tag{3}$$

Here the factor χ accounts for the effects of excluded volume and the shielding of flow particles from wall particles by other flow particles, and the product $\mathbf{g}\cdot\mathbf{k}$ must be positive for a collision to occur.

3. BOUNDARY CONDITIONS

Due to repeated collisions between the grains and the bumps, both momentum and energy are exchanged between a flow and its containing boundary. In particular, a unit area of the boundary supplies momentum to the flow at a rate **M**, supplies energy to the flow at rate **M·U** due to its mean motion and at a rate F due to its fluctuating motion, and absorbs energy from the flow at a rate D due to the inelasticity of the boundary-flow collisions. For the bumpy boundaries of interest here, the supply rate **M** is a statistical average of the change in momentum given by expression (1). The rates **M·U**, F, and D are the averages of the first, second, and third terms in the change in energy given by expression (2).

In order to write down the forms of the required boundary conditions that apply to any boundary, we focus on a parallelepiped fixed within the flow that has two opposite sides of unit area, one of which remains coincident with a unit area of the boundary whose unit inward normal is **N**, while the other four sides shrink to zero. In this limit, the balance of momentum at the boundary requires that,

$$\mathbf{M} = \boldsymbol{P}\cdot\mathbf{N} \quad , \tag{4}$$

where $\boldsymbol{P}$ is the pressure tensor, and the balance of energy requires that,

$$\mathbf{M}\cdot\mathbf{v} + F - D = \mathbf{Q}\cdot\mathbf{N} \quad , \tag{5}$$

where **Q** is the energy flux, and **v** is the slip velocity equal to the difference between the mean boundary velocity **U** and the mean flow velocity adjacent to the boundary. The flux of fluctuation energy normal to the boundary is determined by contributions from the slip work rate **M·v**, which is due to equal tractions **M** acting through velocities that differ by an amount **v**, the supply rate F, and the dissipation rate D.

The transfer rates **M**, F, and D vary with the geometry of the boundary, and therefore depend on **N**, s/d, and and σ/d. Each is a statistical average of its corresponding transfer in a single wall-flow particle collision, and therefore depends on e_w. The rate **M**, for example, is the average over all possible collisions of the change in momentum $m(1+e_w)(\mathbf{g}\cdot\mathbf{k})\mathbf{k}$ given in equation (1) weighted by the collision frequency (3). The rates F and D are similarly weighted averages of the second term $m(1+e_w)(\mathbf{g}\cdot\mathbf{k})(\mathbf{C}_2\cdot\mathbf{k})$ and third term $m(1-e_w^2)(\mathbf{g}\cdot\mathbf{k})^2/2$ of the change in energy given in equation (2).

In order to illustrate the averaging procedure, we assume that the distribution functions $f(\mathbf{c},\mathbf{r})$ and $p(\mathbf{c})$ are Maxwellians:

$$f(\mathbf{c},\mathbf{r}) = \frac{n}{(2\pi w^2)^{3/2}}\exp\left[\frac{-(\mathbf{c}-\mathbf{u})\cdot(\mathbf{c}-\mathbf{u})}{2w^2}\right] \quad , \tag{6}$$

in which the particle number density is $n(\mathbf{r})$, the mean flow velocity is $\mathbf{u}(\mathbf{r})$, and the granular temperature is $w^2(\mathbf{r})$; and

$$p(\mathbf{c}) = \frac{1}{(2\pi V^2)^{3/2}} \exp\left[\frac{-(\mathbf{c}-\mathbf{U})\cdot(\mathbf{c}-\mathbf{U})}{2V^2}\right] , \tag{7}$$

in which the boundary's mean velocity is $\mathbf{U}$, and its mean square fluctuation velocity is V^2. If the velocity integrations are carried out first, then the intermediate expressions for $\mathbf{M}$, F, and D may be written compactly in terms of the quantity,

$$\Phi = \frac{(\mathbf{U}-\mathcal{U})\cdot\mathbf{k}}{[2\mathcal{W}(1+V^2/\mathcal{W}^2)]^{1/2}} , \tag{8}$$

where $\mathcal{U}$ and $\mathcal{W}$ are $\mathbf{u}$ and w evaluated at $\mathbf{x}+\sigma\mathbf{k}$. The resulting integral expression for the rate at which momentum is supplied by the boundary to the flow is,

$$\mathbf{M} = \frac{\rho\chi(1+e_w)}{\pi^{3/2}\sin^2\theta}\int \mathbf{k}(\mathcal{W}^2+V^2)[\sqrt{\pi}(\frac{1}{2}+\Phi^2)\mathrm{erfc}(-\Phi)+\Phi\exp(-\Phi^2)]d\mathbf{k} , \tag{9}$$

where the flow density ρ is equal to the product mn. Similarly, the energy supply rate F is given by the integral,

$$F = \frac{2^{1/2}\rho\chi(1+e_w)V^2}{\pi^{3/2}\sin^2\theta}\int (\mathcal{W}^2+V^2)^{1/2}[\exp(-\Phi^2)+\sqrt{\pi}\Phi\mathrm{erfc}(-\Phi)]d\mathbf{k} , \tag{10}$$

and the energy dissipation rate D is given by,

$$D = \frac{\rho\chi(1-e_w^2)}{2^{1/2}\pi^{3/2}\sin^2\theta}\int (\mathcal{W}^2+V^2)^{3/2}[(1+\Phi^2)\exp(-\Phi^2) + \sqrt{\pi}\Phi(\frac{3}{2}+\Phi^2)\mathrm{erfc}(-\Phi)]d\mathbf{k} . \tag{11}$$

In principle, it remains only to carry out the $\mathbf{k}$-integrations (9), (10), and (11) over that portion of a wall particle's surface area that is accessible to the flow particles.

Although it is not possible to express the results of the exact $\mathbf{k}$-integrations for $\mathbf{M}$, F, and D in closed form, there are circumstances under which approximate closed form expressions may be obtained. If, for example, the dimensionless gradients $\sigma\nabla w/w$ and $\sigma\nabla\rho/\rho$ are of order σ/L, where L is a characteristic length L over which the mean flow fields vary, while the gradient $\sigma\nabla u/w$ is of order $(\sigma/L)^{1/2}$, then to within an error of order (σ/L), the quantity Φ is approximated by,

$$\Phi = \frac{[\mathbf{v}-\bar{\sigma}(\mathbf{N}-\mathbf{k})\cdot\nabla\mathbf{u}]\cdot\mathbf{k}}{[2w(1+V^2/w^2)]^{1/2}} \quad , \tag{12}$$

in which **v** is equal to **U-u** and all mean fields are evaluated at $\mathbf{r}=\mathbf{x}+\bar{\sigma}\mathbf{N}$. If, in addition, the dimensionless slip velocity v/w is of order $(\sigma/L)^{1/2}$, and the quantity $(1-e_w)$ is of order (σ/L), then to within an error of order (σ/L) the Cartesian components of **M** are given by,

$$M_i = \rho\chi(w^2+V^2)\left\{N_i + \frac{2^{1/2}}{[\pi(1+V^2/w^2)]^{1/2}}\left[\frac{v_j}{w}I_{ij} + \frac{\bar{\sigma}}{w}\frac{\partial u_j}{\partial r_k}(I_{ijk}+I_{ij}N_k)\right]\right\} \quad , \tag{13}$$

in which all mean fields are evaluated at **r**. The tensor components I_{ij} and I_{ijk}, which depend on the measure θ of bumpiness and the orthogonal triad **N**, t, and τ at the boundary, are defined by,

$$I_{ij} \equiv \frac{2}{3}\{2[\csc^2\theta(1-\cos\theta)+\cos\theta]N_iN_j + [2\csc^2\theta(1-\cos\theta)-\cos\theta](t_it_j+\tau_i\tau_j)\}, \tag{14}$$

and

$$I_{ijk} \equiv (\sin^2\theta-2)N_iN_jN_k - \frac{\sin^2\theta}{2}[N_i(t_jt_k+\tau_j\tau_k)+N_j(t_kt_i+\tau_k\tau_i)+N_k(t_it_j+\tau_i\tau_j)]. \tag{15}$$

The corresponding lowest order approximation to integral (10) for F is given by,

$$F = (2/\pi)^{1/2}4\rho\chi V^2(w^2+V^2)^{1/2}(1-\cos\theta)\csc^2\theta \quad , \tag{16}$$

and, to within an error of order $(\sigma/L)^{3/2}$, the approximation to integral (11) for D is,

$$D = (2/\pi)^{1/2}2\rho\chi(1-e_w)(w^2+V^2)^{3/2}(1-\cos\theta)\csc^2\theta \quad . \tag{17}$$

Just as the mean fields in expression (13) for **M** are evaluated at **r**, so too are those that appear in approximate expressions (16) and (17) for F and D.

For boundaries that do not fluctuate about their mean velocities, Richman [1988] based all averaging at the boundary on a corrected Maxwellian flow-particle velocity distribution. For such boundaries, V^2 and F vanish, expression (17) for D reduces to Richman's result, and expression (13) for **M**, differs from his by a term not obtained here because the correction to the Maxwellian (6) has been ignored.

4. A BOUNDARY VALUE PROBLEM

Of interest here are the steady, gravity-free motions of granular materials that are confined between two parallel bumpy boundaries that randomly vibrate about zero mean velocities. The boundaries have mean square fluctuation velocities V^2, and are separated by a fixed distance 2L. The grains are identical spheres of mass density α. Under these circumstances, the profiles of granular temperature w^2 and solid fraction ν are induced entirely by the fluctuations of the boundary; the resulting normal pressure P_{22} is constant throughout; and the velocity field **u**, the slip velocity **v**, and the shear stresses all vanish.

We establish an x_1-x_2-x_3 Cartesian coordinate system such that the x_2-direction is normal to the boundaries, which are located symmetrically at $x_2=-L$ and $x_2=+L$ and are infinite in the x_1- and x_3-directions. The solid fraction ν and the dimensionless measure $W \equiv w/V$ of granular temperature then depend only on the dimensionless distance $y \equiv x_2/\sigma$ from the midplane between the boundaries, and the balances of mass and momentum are identically satisfied. Furthermore, if Q_2 is the x_2-component of the energy flux, and γ is the collisional rate per unit volume of energy dissipation, then in terms of their dimensionless counterparts $q \equiv Q_2/\alpha V^3$ and $\Gamma \equiv \sigma\gamma/\alpha V^3$, the balance of energy reduces to,

$$q' + \Gamma = 0 \quad , \tag{18}$$

where a prime denotes differentiation with respect to y. In these flows, there is a net loss of energy due to inelastic collisions between particles.

For q, Γ, and the dimensionless normal pressure $P \equiv P_{22}/\alpha V^2$, we employ a kinetic constitutive theory that is based on the Maxwellian distribution (6) and, for simplicity, applies only to dense flows. In this theory, which is obtained from that derived by Jenkins and Richman [1985] by neglecting the contributions to the constitutive relations from the corrections to the Maxwellian and from particle transport, the normal pressure P is given by,

$$P = 4\nu G W^2 \quad , \tag{19}$$

where $G(\nu)=\nu(2-\nu)/2(1-\nu)^3$; the component q of the energy flux is,

$$q = \frac{-2P}{\pi^{1/2}} W' \quad ; \tag{20}$$

and the rate Γ of energy dissipation is,

$$\Gamma = \frac{6(1-e)PW}{\pi^{1/2}} \quad , \tag{21}$$

where e is the coefficient of restitution between flow particles. The assumptions regarding the magnitudes of the gradients of the mean fields and the difference (1-e) made in deriving relations (19), (20), and (21) are consistent with those made in deriving expressions (13), (16), and (17) for **M**, F, and D.

When equations (20) and (21) are employed to eliminate q and Γ from equation (18), the energy equation becomes simply,

$$W'' - \lambda^2 W = 0 \quad , \tag{22}$$

in which $\lambda^2 \equiv 3(1-e)$. The profile that is symmetric about y=0 is therefore,

$$W = \frac{\Omega \cosh\lambda y}{\cosh\lambda\beta} \quad , \tag{23}$$

where β is the value L/σ of y at the upper boundary. The constants Ω and P remain to be determined. When they are known, constitutive relation (19) determines the solid fraction profile.

The x_1- and x_3-components of the momentum condition (4) at the upper boundary y=β are identically satisfied because the shear stresses, slip velocity, and velocity gradients all vanish. In order that the solid fraction at the boundary be a free parameter, the x_2-component requires that $\chi=4G\Omega^2/(1+\Omega^2)$. The constant Ω is determined by the energy flux boundary condition (5) at y=β. If expressions (16), (17), and (20) are employed to eliminate F, D, and Q_2, and expression (23) is employed to eliminate W' from the intermediate result, then balance between the supply, dissipation, and flux of energy at the boundary is given by,

$$1 - \epsilon(1+\Omega^2) = \Lambda\Omega(1+\Omega^2)^{1/2} \quad , \tag{24}$$

where ε is equal to $(1-e_w)/2$, and the parameter Λ depends on the measure λ of flow particle inelasticity, the measure θ of boundary roughness, and the dimensionless half-width β, according to

$$\Lambda = \frac{\lambda \tanh \lambda\beta}{2^{3/2}(1-\cos\theta)\csc^2\theta} \quad . \tag{25}$$

The parameter Λ increases monotonically as either the flow particles become more inelastic, the distance between the plates increases, or the boundary becomes smoother. The ratio Ω^2 of the temperature at the boundary to the mean square fluctuation velocity of the boundary is then fixed by,

$$1+\Omega^2 = \frac{(\Lambda^2-2\epsilon)+[(\Lambda^2-2\epsilon)^2+4(\Lambda^2-\epsilon^2)]^{1/2}}{2(\Lambda^2-\epsilon^2)} \quad . \tag{26}$$

Solution (26) of energy balance (24) ensures that the supply rate F exceeds the dissipation rate D to precisely compensate for the net loss within the flow.

For fixed values of ϵ and Λ, equation (26) determines Ω. If, in addition, the values of λ, β and therefore θ are fixed, then equation (23) determines the profile W(y). For prescribed values of P, the profile $\nu(y)$ is then obtained by inverting constitutive relation (19). Alternatively, for prescribed values of the depth-averaged solid fraction,

$$\bar{\nu} \equiv \frac{1}{\beta}\int_0^\beta \nu(y)\,dy \quad , \tag{27}$$

we vary the value of P until the profile $\nu(y)$ obtained by inverting relation (19) has a depth-averaged value that agrees with its prescribed value.

5. RESULTS AND DISCUSSION

Of primary interest here are the effects of the boundaries' fluctuation velocity, geometry, and dissipative character on the mean profiles and normal pressure induced throughout the flow. In the boundary value problem described above, the fluctuations of the boundaries are entirely responsible for the agitation of the grains. Consequently, both the granular temperature and the normal pressure scale with the square V^2 of the boundaries' fluctuation speed. It remains to describe the effects of bumpiness θ and inelasticity $(1-e_w)$ on the solutions.

We focus attention on values of Λ between zero (for perfectly elastic flow particles) and .55 (for e=.8, large values of β, and flat boundaries). In Figure 1 we show the variations of the ratio Ω with the parameter Λ for e_w=.95, .9, and .8. The The dimensionless difference on the left-hand-side of equation (24) corresponds to the dimensional difference (F-D), and must compensate for the net loss of energy within the flow. As the boundary becomes bumpier, the factor $(1-\cos\theta)\csc^2\theta$ that magnifies this difference increases.

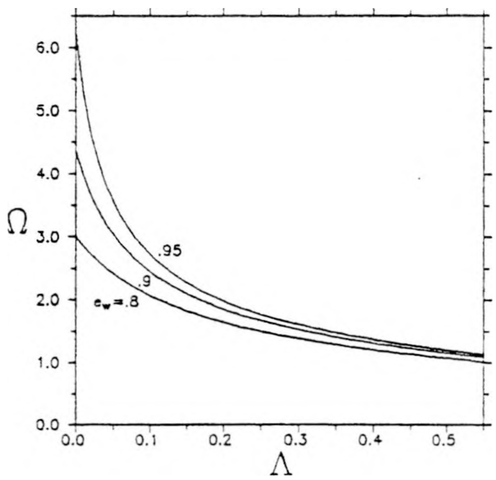

Figure 1. The variations of Ω with Λ for e_w=.95, .9, and .8.

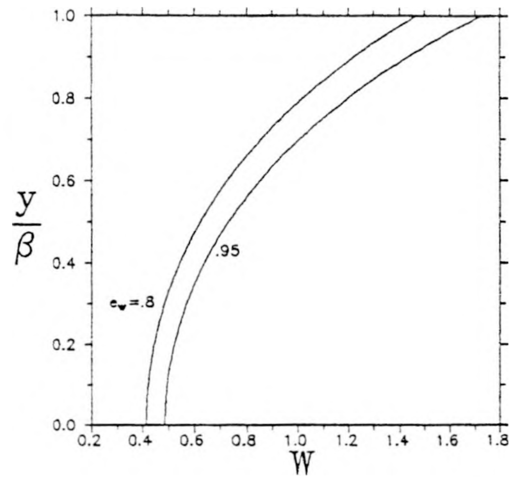

Figure 2. The variations of W with y/β for e_w=.95 and .8 when e=.95, β=5, and θ=0.

Consequently, as Λ decreases, the ratio Ω increases. However, the increase is moderated by an accompanying increase in net energy loss in the flow. If, for example, $e=e_w$=.95 and β=5, then as the boundary evolves from perfectly flat (θ=0) relatively bumpy (θ=π/3) the ratio Ω increases by twenty one percent from 1.65 to 1.99.

The same energy balance described by condition (24) dictates that as e_w increases, the ratio Ω must also increase. Again, the increase in Ω is moderated by a corresponding increase in the net energy loss in the flow. If, for example, e=.95, β=5, and θ=0, then as e_w varies from .8 to .95, the ratio Ω increases by seventeen percent from 1.47 to 1.72. The extremes in the profiles of W(y) for this example are shown in Figure 2. Because the solid fraction does not appear in the energy equation (22), the profiles of granular temperature are independent of $\bar{\nu}$.

For a fixed value of β, the normal pressure throughout the flow depends on θ, e_w, e, and $\bar{\nu}$. However, constitutive relation (19) and solution (23) demonstrate that for the same value of β, the ratio P/Ω^2 and the profile ν(y) depend only on e and $\bar{\nu}$. In Figure 3, the variations of P/Ω^2 with $\bar{\nu}$ are shown for e=.95 and .9 when β=5. Not suprisingly, the normal pressure increases with $\bar{\nu}$ and e. In fact, the increase in P with e is more pronounced than it might appear in Figure 3 because as e increases so too does Ω. In Figure 4, solid fraction profiles are shown for $\bar{\nu}$=.4 and .45 when e=.95 and β=5. For these values of $\bar{\nu}$, e, and β, these profiles prevail regardless of the boundary parameters e_w and θ.

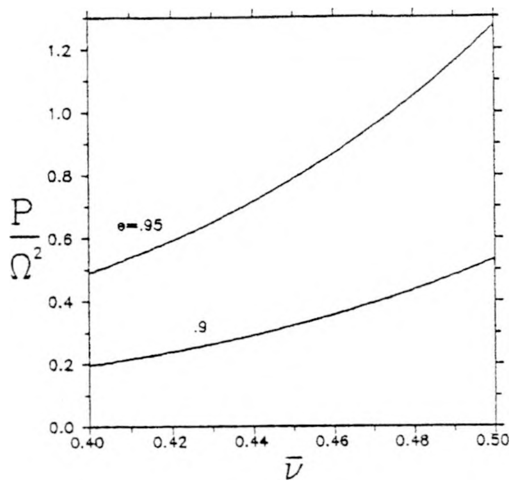

Figure 3. The variations of P/Ω^2 with $\bar{\nu}$ for e=.95 and .9 when β=5.

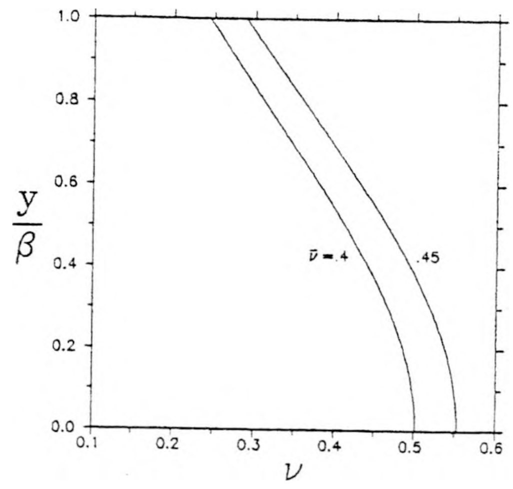

Figure 4. The variations of ν with y/β for $\bar{\nu}$=.4 and .45 when e=.95 and β=5.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Craig, K., Buckholz, R.H., Domoto, G., 1987. The Effect of Shear Surface Boundaries on Normal and Shear Stresses for Rapid Shearing Flow of Dry Cohesionless Metal Powder - An Experimental Study, J. Tribology, Vol. 109, pp. 232-237.

Campbell, C.S., Gong, A., 1987. Boundary Conditions for Two-Dimensional Granular Flows, Proc. Sino-US Int. Symp. Multiphase Flows, Zhejiang Univ. Press, Hangzhou, China, Vol. I, pp. 278-283.

Hui, K., Haff, P.K., Ungar, J.E., Jackson, R., 1984. Boundary Conditions for High-Shear Grain Flows, J. Fluid Mech., Vol. 145, pp. 223-233.

Jackson, R., 1991. Aerated and Vibrated Chute Feeders For Particulate Materials, Final Project Report, US Dept. of Energy, DOE Contract DE-FG22-86PC90518.

Jenkins, J.T., 1991. Boundary Conditions for Rapid Granular Flows: Flat Frictional Walls, J. Appl. Mech., in press.

Jenkins, J.T., Richman, M.W., 1985. Grad's 13-Moment System for a Dense Gas of Inelastic Spheres, Arch. Rat. Mech. Anal., Vol. 87, pp. 355-377,

Jenkins, J.T., Richman, M.W., 1986. Boundary Conditions for Plane Flows of Smooth, Nearly Elastic, Circular Disks, J. Fluid Mech., Vol. 171, pp. 53-69.

Johnson, P.C., Jackson, R., 1987. Frictional-Collisional Constitutive Relations for Granular Material, with Application to Plane Shearing, J. Fluid Mech., Vol. 176, pp. 67-94.

Pasquarell, G.C., Ackermann, N.L., 1989. Boundary Conditions for Planar Granular Flows, ASCE J. Eng. Mech., Vol. 115, 1283-1302.

Pasquarell, G.C., 1991. Granular Flows: Boundary Conditions for Slightly Bumpy Walls, ASCE J. Eng. Mech., Vol. 117, pp. 312-328.

Richman, M.W., 1988. Boundary Conditions Based upon a Modified Maxwellian Velocity Distribution for Flows of Identical, Smooth, Nearly Elastic Spheres, Acta Mech., Vol. 75, pp. 227-240.

Richman, M.W., Chou, C.S., 1988. Boundary Effects on Granular on Granular Shear Flows of Smooth Disks, ZAMP, Vol. 39, pp. 885-901.

Savage, S.B., 1988. Streaming Motions in a Bed of Vibrationally Fluidized Dry Granular Material, J. Fluid Mech., Vol. 194, pp. 457-478.