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The Off-Shell Axial Anomaly via the $\gamma^* \pi^0 \to \gamma$ Transition

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The $\gamma^*\pi^0 \to \gamma$ form factor, including the extension off the pion mass-shell, is obtained from a generalized impulse approximation within a QCD-based model field theory known to provide an excellent description of the pion charge form factor. This approach implements dressing of the vertex functions and propagators consistent with dynamical chiral symmetry breaking, gauge invariance, quark confinement and perturbative QCD. Soft nonperturbative behavior, dictated by the axial anomaly, is found to evolve to the perturbative QCD limit only for $Q^2 \geq 20$ GeV^2.

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1. Introduction. The pion charge form factor for space-like momenta has long been used as one of the simplest but non-trivial testing grounds for applications of QCD to hadronic properties. A closely related quantity that has received less attention is the $\gamma^*\pi^0 \to \gamma$ transition form factor. Here the photon momentum dependence maps out a particular off-shell extension of the axial anomaly. [1] Presently available data for this transition form factor in the space-like region $Q^2 < 2.5$ GeV^2 is from the CELLO [2] collaboration at the PETRA storage ring where the process $e^+e^- \to e^+e^-\pi^0$ was measured with geometry requiring one of the two intermediate photons to be almost real. There is currently renewed interest in this transition form factor of the pion due to the prospect of obtaining higher precision data over a broader momentum range via virtual Compton scattering from a proton target at CEBAF. [3] In this case a (virtual) pion is supplied by the target and a final real photon selected through the excellent missing mass spectrometry available at CEBAF. It is anticipated that suitable electron scattering geometry can minimize the t-channel momentum to the extent that the pion mechanism will dominate other contributions such as s-channel resonances. [3] An extrapolation to the pion mass shell will be needed to deduce the physical transition form factor.

In this work we present results of a calculation of the $\gamma^*\pi^0 \to \gamma$ vertex as a function of both the virtual photon and pion momenta. The mechanism employed is the generalized impulse approximation or quark triangle diagram in which the confining quark propagator and the photon-quark and pion-quark vertices are dynamically dressed quantities consistent with nonperturbative Schwinger-Dyson equation (SDE) studies [4], and hence, also with asymptotic QCD. The constraints of electromagnetic gauge invariance and dynamical chiral symmetry breaking are obeyed and lead to the calculation being completely determined by the amplitudes describing the dynamical quark propagator. For the latter we take a recently developed model which has been shown to provide an excellent description of the space-like pion charge form factor, as well as a variety of soft chiral-physics quantities. [5] No adjustment of parameters is made in the present application. The approach naturally identifies the momentum scale where soft nonperturbative behavior evolves to the hard perturbative QCD limit.

2. The $\pi^\circ\gamma\gamma$ Vertex Function. We use a Euclidean-space formulation with metric $\delta_{\mu\nu}$. The action for the three-point interaction can be written as

$$S[\pi^0\gamma\gamma] = \int \frac{d^4P d^4Q}{(2\pi)^8} A_\mu(-P-Q)A_\nu(Q)\pi^0(P)\Lambda_{\mu\nu}(P,Q), \tag{1}$$

where A_ν is the electromagnetic field and the momentum assignments are shown in Fig. 1. The general form of the vertex allowed by CPT is

$$\Lambda_{\mu\nu}(P,Q) = -i\frac{\alpha}{\pi f_\pi}\epsilon_{\mu\nu\alpha\beta}P_\alpha Q_\beta \; g(Q^2,P^2,P\cdot Q) \tag{2}$$

where $\epsilon_{4123}=1$, α is the fine-structure constant, f_π is the pion decay constant, and g is the off-mass-shell invariant amplitude. With the one photon mass-shell condition $(P+Q)^2=0$, the invariant amplitude, denoted by $g(Q^2,P^2)$, is the object of the present work. For a physical pion the shape of the $\gamma^*\pi^0 \to \gamma$ transition form factor is given by $g(Q^2, -m_\pi^2)$. At the fully on-mass-shell point $(Q^2 = 0, P^2 = -m_\pi^2)$, the amplitude for the $\pi^0 \to \gamma\gamma$ decay is provided by $2\Lambda_{\mu\nu}\epsilon_\mu(1)\epsilon_\nu(2)$ where ϵ_μ is a photon polarization vector. The chiral limit for the strength of this decay amplitude is fixed at $\frac{\alpha}{\pi f_\pi}$ by the axial anomaly [6] which gives an excellent account of the $\pi^0 \to \gamma\gamma$ decay width of 7.7 eV. Thus $g(0,0) = 1/2$ follows only from gauge invariance and chiral symmetry in quantum field theory and provides a stringent check upon model calculations.

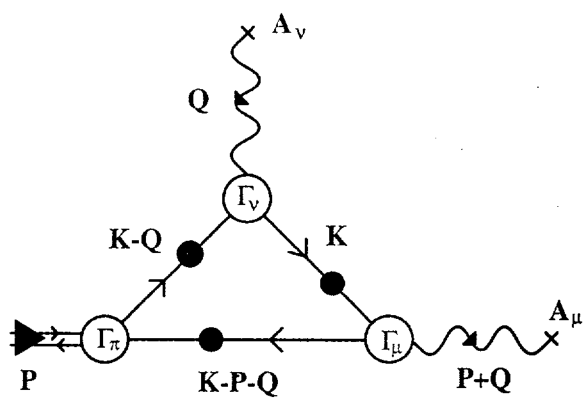

FIG. 1. The quark triangle diagram for the generalized impulse approximation to the $\gamma^*\pi^0\gamma$ vertex.

Within the impulse approximation, and with momentum assignments corresponding to the quark triangle diagram of Fig. 1, the vertex function is given by the integral

$$\Lambda_{\mu\nu}(P,Q) = -\mathrm{tr}\int \frac{d^4k}{(2\pi)^4} S(k-P-Q)\Gamma_\mu(k-\frac{P}{2}-\frac{Q}{2}; -P-Q)S(k) \times\Gamma_\nu(k-\frac{Q}{2}; Q)S(k-Q)i\gamma_5\tau_3\Gamma_\pi(k-\frac{P}{2}-Q; P). \quad (3)$$

Here $S(k)$ is the dressed quark propagator, and $\Gamma_\pi(p;q)$ and $\Gamma_\mu(p;q)$ represent respectively the pion-quark Bethe-Salpeter amplitude and the dressed quark-photon vertex corresponding to incoming boson momentum q and relative $\bar{q}q$ momentum p. The trace is over spin, flavor (u and d only) and color. We require that the dynamical quantities S, Γ_μ and Γ_π

be mutually consistent with electromagnetic gauge invariance, dynamical chiral symmetry breaking, confinement and the known behavior of perturbative QCD.

This generalized impulse approximation, with consistently dressed elements, can be derived as the lowest-order term in the meson loop expansion of the electromagnetically gauged version [7] of the model-QCD field theory known as the Global Color-symmetry Model (GCM). [8,9] This replaces the gluon sector by a momentum dependent finite range effective gluon propagator, thus formalizing the Abelian approximation to QCD and properly embedding the chiral anomalies [9]. For the quark propagator, written as $S(p) = -i\gamma \cdot p\ \sigma_V(p^2) + \sigma_S(p^2)$, we employ the parameterized amplitudes [5]

$$\bar{\sigma}_S(x) = c\ e^{-2x} + \frac{1-e^{-b_1 x}}{b_1 x}\frac{1-e^{-b_3 x}}{b_3 x}\left(b_0 + b_2 \frac{1-e^{-b_4 x}}{b_4 x}\right) + \frac{\bar{m}}{x+\bar{m}^2}(1-e^{-2(x+\bar{m}^2)}) \tag{4}$$

$$\bar{\sigma}_V(x) = \frac{2(x+\bar{m}^2) - 1 + e^{-2(x+\bar{m}^2)}}{2(x+\bar{m}^2)^2} - c\ \bar{m}\ e^{-2x}, \tag{5}$$

with $x = p^2/\lambda^2$, $\bar{\sigma}_S = \lambda\sigma_S$, $\bar{\sigma}_V = \lambda^2\sigma_V$, $\bar{m} = m/\lambda$, where m is the bare quark mass and λ is the momentum scale. The resulting $S(p)$ is an entire function in the complex momentum plane, a sufficient condition for confinement, as it ensures the absence of quark production thresholds in S-matrix amplitudes. [10] When $b_0 = b_2 = 0$, this propagator accurately represents the entire function solution of the simple, infra-red dominant, confining model SDE developed in Ref. [11]. The more general parameterization with $b_0,\ b_2 \neq 0$ allows a good representation, at moderate momenta, of the behavior found in realistic SDE studies [4] while remaining consistent with the requirements of perturbative QCD in the deep Euclidean region up to $\ln(p^2)$ corrections. The parameters are $\lambda = 0.516$ GeV, $c = 0.0406$, $m =$ 6.1 MeV, and $(b_0, b_1, b_2, b_3, b_4) = (0.118, 2.51, 0.525, 0.169, 1 \times 10^{-4})$. The fitted soft chiral physics quantities produced by this parameterization are [5] $f_\pi = 83.9$ MeV, $<\bar{q}q>_{1\mathrm{GeV}^2} =$ (211 MeV)^3, $m_\pi = 127$ MeV, $r_\pi^{em} = 0.596$ fm together with reproduction of the experimental $\pi\pi$ scattering lengths to within 20%. In this approach, choice of quark propagator parameters is equivalent to an implicit choice of effective gluon propagator underlying the model field theory. [4]

With $S(p)^{-1} = i\gamma \cdot p\ A(p^2) + B(p^2) + m$, representations for Γ_μ and Γ_π may be obtained in terms of A and B while satisfying the constraints of gauge invariance and dynamical chiral symmetry breaking. In particular, the chiral limit ($m = 0$) result for the mass-shell Γ_π is $\Gamma_\pi(p; P^2 = 0) = B(p^2, m = 0)/f_\pi$ since the SDE for $B(p^2)$ and the Bethe-Salpeter equation for Γ_π become identical. [12] For finite m we use $\Gamma_\pi(p; P) \approx B(p^2, m)/f_\pi$ since this produces the PCAC determination of m_π. We use the Ball-Chiu [13] ansatz for the dressed quark photon vertex which is $\Gamma_\mu(k; Q) = \hat{Q}\bar{\Gamma}_\mu(k; Q)$, where $\hat{Q} = \frac{1}{2}(\tau_3 + \frac{1}{3})$ is the quark charge operator, and

$$\bar{\Gamma}_\mu(k; Q) = -i\gamma_\mu \frac{1}{2}\Big(A(k_+) + A(k_-)\Big) + \frac{k_\mu}{k \cdot Q}\Big[i\gamma \cdot k\Big(A(k_-) - A(k_+)\Big) + \Big(B(k_-) - B(k_+)\Big)\Big] \tag{6}$$

with $k_\pm = k \pm \frac{Q}{2}$. This vertex satisfies the Ward-Takahashi identity, transforms correctly and has the correct perturbative limit. The above mutually consistent model for S, Γ_π and Γ_μ has been shown to provide an excellent description of the data for the spacelike pion charge form factor at the level of the corresponding impulse approximation. [5]

The $\pi^0\gamma\gamma$ vertex function in (3) is now completely specified in terms of the quark propagator. The integral is devoid of spurious quark threshold singularities, is naturally convergent and is evaluated by numerical quadrature. At $Q^2 = 0$, our numerical evaluation of the coupling constant yields $g_{\pi^0\gamma\gamma} = g(0, -m_\pi^2) = 0.497$ in agreement with the previous application of this model [5], and in good agreement with the experimental value 0.504 ± 0.019. The chiral limit of this approach has been shown [5] to correctly incorporate the exact result $g(0,0) = 1/2$ produced by the axial anomaly independent of the form of quark propagator amplitudes. Our numerical evaluation of the chiral limit reproduces this result.

3. The Transition Form Factor. The obtained shape of the form factor $F(Q^2) = g(Q^2, -m_\pi^2)/g(0, -m_\pi^2)$ at the pion mass-shell is displayed in Fig. 2 by the solid line along with the CELLO collaboration data. [2] Also shown there is a recent result from a QCD sum rule approach [14], and a monopole form [15] that interpolates from the leading asymptotic behavior $8\pi^2 f_\pi^2/Q^2$ argued from the perturbative QCD factorization approach. [16] In the latter two approaches there is considerable ambiguity due to: A) the unknown momentum scale at which perturbative behavior should set in; and B) assumptions for the pion wavefunction and how it should evolve to non-perturbative momentum transfer. [14] Within the present approach, both the photon coupling and the produced pion wavefunction evolve with Q^2 in a way determined by the evolution of the dressed quark propagators. This produces, in a single expression, both the ultra-violet behavior required by perturbative QCD and the infra-red limit dictated by the axial anomaly.

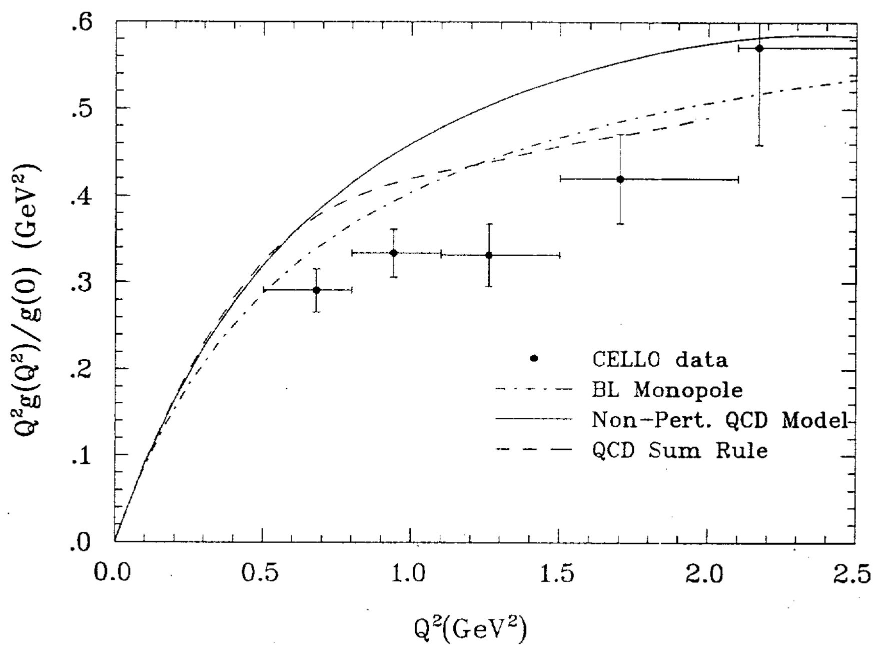

FIG. 2. The $\gamma^*\pi^0\gamma$ transition form factor at the pion mass-shell. The data is from Ref. [2], the solid line is the present work, the dashed line is the monopole shape which interpolates in from the leading asymptotic behavior produced by perturbative QCD factorization [15], and the dot-dashed line is a recent QCD sum rule calculation [14].

The predicted scale for transition from perturbative to non-perturbative regimes can be unfolded as follows. The asymptotic behavior produced by the quark triangle diagram can be obtained in closed form without factorization by application of Feynman integral techniques with bare photon coupling and free quark propagators thereby preserving gauge invariance. We note that the employed pion Bethe-Salpeter amplitude $B(p^2,m)/f_\pi$ has the correct leading power law behavior $m\lambda^2/p^2 f_\pi$ which implements the hard gluon contribution that dominates perturbative QCD. The asymptotic form $F(Q^2) \sim A\,[Q^2 + B + C\ \ln(Q^2/Q_0^2)/Q^2]^{-1}$, where $Q_0 = 1$ GeV^2, is found to incorporate the first three leading terms obtained this way and also provides an excellent fit to the numerical results for $Q^2 \geq 10$ GeV^2 with $A = 0.53$ GeV^2, $B = 3.1$ GeV^2 and $C = -3.8$ GeV^4. Our direct calculation of this anomalous process; i.e. without employing a factorization prescription, yields $A/f_\pi^2 = 75$, in agreement with the normalization ($8\pi^2 = 79$) and leading asymptotic behavior (Q^{-2}) obtained from factorization. [16] It must be emphasized, however, that the interplay of soft and hard mechanisms prevents the A/Q^2 term from dominating to better that 85% until $Q^2 \geq 20$ GeV^2. The recent QCD sum rule approach [14] assumed that at $Q^2 = 3$ GeV^2 the leading term provides an accurate foundation for generating the soft physics behavior. This is not borne out by the present investigation; the non-leading contribution is still providing 50% of the strength at that scale.

Off the pion-mass shell ($P^2 > -m_\pi^2$), we use for the pion vertex $\Gamma_\pi(p;P) \approx B(p^2,m)/\sqrt{Z(P^2)}$, where $Z(P^2)$ is obtained from the calculated tree-level inverse propagator $(P^2 + m_\pi^2)Z(P^2)$ for the $\bar{q}q$ mode in the pion channel produced by the model. [7] With this convention, the departure of Z from its mass-shell value f_π^2 defines a vertex that communicates with pions via the conventional point propagator to facilitate subsequent applications. The calculated $\gamma^*\pi^0 \to \gamma$ transition form factor $F(Q^2, P^2) = g(Q^2, P^2)/g(0, -m_\pi^2)$, for a significant range of virtual pion momenta ($P^2 \geq -m_\pi^2$), is well represented as a suppression of the mass-shell form factor $F(Q^2)$ by the expression $F(Q^2, P^2) \simeq F(Q^2)\ f(P^2)$. The calculated suppression factor is shown in Fig. 3. The form

$$f(P^2) = \frac{1}{1 + (P^2 + m_\pi^2)/a^2 - (P^2 + m_\pi^2)^2/b^4}, \tag{7}$$

with $a = 1.38$ GeV, and $b = 1.55$ GeV provides a fit to within 7% for $-m_\pi^2 \leq P^2 \leq 1$ GeV^2 and $Q^2 \leq 2.5$GeV^2. This is more than sufficient to cover the range of minimum P^2 allowed by typical electron scattering angles considered to favor the $\gamma^*\pi^0 \to \gamma$ mechanism in a feasibility study for an experiment on virtual Compton scattering on a proton target at CEBAF. [3] This prediction of the dependence upon virtual pion momentum is, of course, subject to uncertainities associated with the definition of an off-mass-shell $\bar{q}q$ correlation in the pion channel. An illustration of the uncertainties can be obtained by comparing these results with those obtained in the point-pion limit (Γ_π = constant). We find that this leads to at most a 12% decrease of the suppression factor for the momenta considered in Fig. 3.

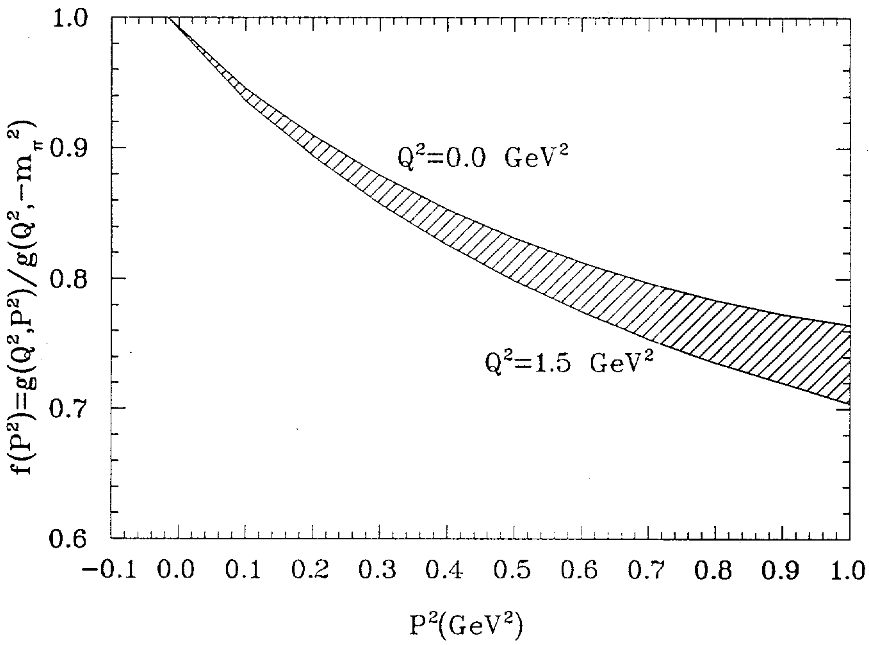

FIG. 3. The reduction factor $g(Q^2,P^2)/g(Q^2,-m_\pi^2)$ for the $\gamma^*\pi^0\gamma$ transition produced by extrapolating the pion off its mass-shell in the space-like direction. For any γ^* momenta between those shown, the results lie between the two curves.

4. Summary. We present here for the first time a calculation of the $\gamma^*\pi^0 \to \gamma$ form factor that maps out the off-shell behavior with virtual pion momentum. We find that the dependence on the virtual-pion momentum is smooth and well described by a simple suppression factor, which is qualitatively independent of the details of the pion interpolating field.

A previous quark triangle diagram study [17] was limited to the pion mass shell. Further distinguishing features of the present work are that nonperturbative dressing of the photon-quark vertex is included, the quark propagators have confining dynamical self-energy amplitudes rather than a constant constituent mass, and the pion Bethe-Salpeter amplitude is completely determined by the quark propagator as required by dynamical chiral symmetry breaking. The correct mass-shell axial anomaly is naturally generated by our approach, the Q^2 dependence is in reasonable accord with the available data, and no parameters are adjusted to achieve this. A significant result of our study is that for this anomalous process, soft nonperturbative effects remain significant for $Q^2 < 20$ GeV^2. For the charge form factor $F_\pi(Q^2)$, a similar scale is produced from the same model. [5]

We conclude that the generalized impulse approximation captures the dominant spacelike physics of both the elastic and transition electromagnetic form factors of the pion. With more reliable data for the transition form factor, adjustment of the present quark propagator model parameters should allow a high quality fit to both the transition and elastic pion form factors as well as the other pion data mentioned. Since this approach is encoded in a QCD-based model field theory, applications to other hadronic processes not dominated by chiral symmetry (e.g. vector meson masses and mixing [18]) can proceed with a minimum of new parameters.

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