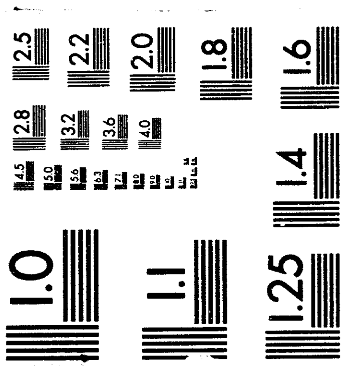
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UCRL-ID-115701

Filament to Mandrel Gap Analysis

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October 1993

Lawrence Livermore National Laboratory

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

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Filament to Mandrel Gap Analysis

Resulting Gap from Filament Winding Over a Cone-Cylinder Transition Mandrel (Reference NMTP No. 93838)

Paul Geraghty

October 27, 1993

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Appendix 16

Problem Description

Problem Statement

The composites industry employs a method of high speed continuous reinforcement lay-down called filament winding. This is a process where resin impregnated tows, bundles of filament, are wound over a rotating mandrel.[1] The tows, hereafter referred to as filament, are laid down over the rotating mandrel at a prescribed wind angle.

Consider a cylindrical filament winding mandrel with conical features such that the filament is tangent to both the cylinder and the cone simultaneously. A gap is formed between the points of tangency. The gap distance is measured along a line normal to the filament and intersecting the mandrel's axis of rotation. The maximum distance occurs between the filament and a point on the intersection of the cylinder and the cone.

The problems this paper addresses are: given a cylindrical filament winding mandrel with conical features, what is the maximum gap size for a given wind angle; conversely what is the wind angle for a given maximum gap size; and what does the geometry need to be in the mandrel transition area between the cylinder and the cone such that the filament remains in contact with the mandrel at all times?

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[1] ASM International, Engineered Materials Handbook, Volume 1, Composits: ASM International, Metals Park, Ohio 44073, 1987-1989

Problem Geometry

The mandrel geometry is shown in figure 1. The constants for the calculations that follow are the radiuses of the cylinder and the cone R' and R" as well as the cone height H. The variables are the angles theta (θ), xi (ξ) and the gap ε. The relationship between theta and xi is: $\xi = 90° - \theta$. Note, for all calculations, the angle units will be radians not degrees.

A right hand coordinate system is used. Its origin is located at the point of intersection between the mandrel centerline and the cone-cylinder interface. The y-axis coincides with the mandrel centerline. The angles theta and xi are rotations about the z-axis and represent the wind angle and the lead angle respectively.

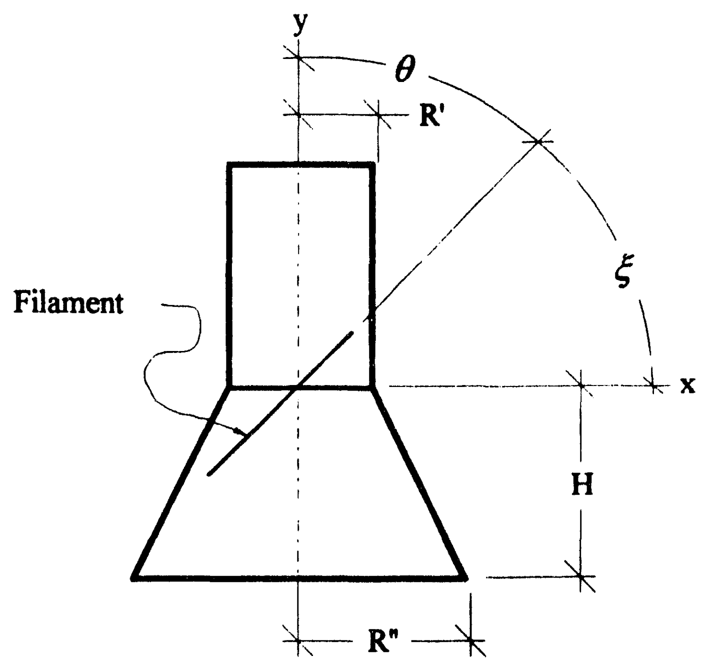


Figure 1 Mandrel Geometry

If we cut the mandrel with a plane that makes the same angle as the filament we get the cross section shown in figure 2.

The cross section will be made up of two curves, one ellipse and one general conic. The general conic may be an ellipse, hyperbola, or parabola depending on the angle of the cutting plane. The filament will be tangent to these two curves. The gap, ε, between the filament and the mandrel can also be seen in this cross section.

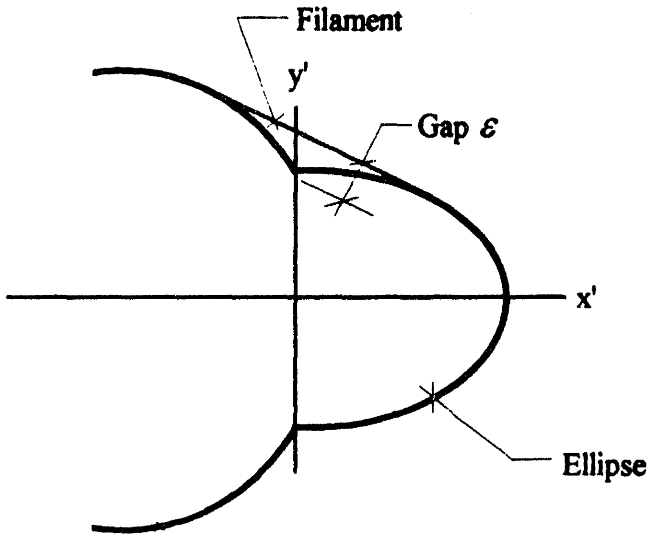


Figure 2 Cross Section of Mandrel

The three dimensional right hand coordinate system (x,y,z) shown in figure 1 can be transformed into the two dimensional right hand coordinate system (x',y') of the cross section by the following equations.

$$y' = -z$$

$$x' = \frac{y}{Sin(\xi)} = \frac{x}{Cos(\xi)}$$

This cross section is the key to solving the problem of finding the gap for the given angle. It transforms the problem from a three dimensional problem into a two dimensional problem.

Calculations

Calculation Outline

As you can see from figure 2, the filament is tangent to an ellipse and a general conic. The shape of the ellipse and the general conic will determine the location of the tangent points and this in turn will determine the distance from the filament to the ellipse-general conic intersection. In short, the filament to mandrel gap distance.

In order to be able to calculate the gap distance for a given angle or an angle for a given gap distance we must first fully characterize the relationship between the angle ξ and the shapes of the ellipse and general conic as well as the tangency relationship of the filament to these two curves. This is the goal of this calculation section. I will derive the needed equation along the following steps:

- Derive the equation of the ellipse as a function of the angle ξ, x', and y'.
- Find an expression for the ellipse so that y' is a dependent variable in terms of the independent variables ξ, x'.
- Derive the slope equation for a line tangent to the ellipse.
- Derive an equation for the y'-axis intercept of a line tangent to the ellipse.
- Derive the equation of the general conic as a function of the angle ξ, x', and y'.
- Find an expression for the general conic so that y' is a dependent variable in terms of the independent variables ξ, x'.

- Derive the slope equation for a line tangent to the general conic.
- Derive an equation for the y'-axis intercept of a line tangent to the general conic.
- Derive an expression that relates the gap size to the y'-axis intercept of a line tangent to the ellipse and its slope.
- Derive an expression that relates the gap size to the y'-axis intercept of a line tangent to the general conic and its slope.
- Combine the gap equations to find a solution for the gap given an angle problem as well as a solution for the angle given a gap problem.

After these expressions are derived I will derive the equation for the fillet curve. This curve, if machined into the mandrel, will eliminate any gap between the mandrel and the filament in the cone-cylinder interface area.

Ellipse Equations

The general equation of an ellipse centered on the origin is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

where

a is the semi-major of the ellipse
b is the semi-minor of the ellipse

For our case the semi-minor is just the radius of the mandrel cylinder, R'. The semi-major is a function of the angle ξ, or

$$a = \frac{R'}{Cos(\xi)}$$

This leaves us with our equation for the ellipse

$$\left(\frac{Cos(\xi)x'}{R'}\right)^2 + \left(\frac{y'}{R'}\right)^2 - 1 = 0 \qquad \text{Eq 1}$$

Solving this equation for y' gives

$$y' = \left[(R')^2 - (Cos(\xi)x')^2\right]^{0.5} \qquad \text{Eq 2}$$

To find the equation for the slope of a line tangent to the ellipse we must take the derivative of Eq 2 with respect to x'. This gives:

$$\frac{dy'}{dx'} = -\frac{Cos^2(\xi)x'}{y'}$$

Substituting Eq 2 into this equation gives

$$\frac{dy'}{dx'} = -\frac{Cos^2(\xi)x'}{\left[(R')^2 - (Cos(\xi)x')^2\right]^{0.5}} \qquad \text{Eq 3}$$

The equation for the y'-axis intercept is:

$$b = y' - \frac{dy'}{dx'}x'$$

Substituting Eq 2 and Eq 3 into this equation gives

$$b = \left[(R')^2 - (Cos(\xi)x')^2\right]^{0.5} + \frac{Cos^2(\xi)(x')^2}{\left[(R')^2 - (Cos(\xi)x')^2\right]^{0.5}} \qquad \text{Eq 4}$$

General Conic Equations

Now consider the cone. If we cut the cone with a plane normal to the y-axis, the x-z plane, we get a circle. The equation of this circle is:

$$x^2 + z^2 = r^2$$

where r is the radius of the circle. The radius of the circle is different depending on where we cut the cone. In other words, the radius is dependent on the y dimension. This dependence can be expressed mathematically by the equation:

$$r = R' + \frac{R'-R''}{H}y$$

And we know from the Problem Description section that the relationship between the x ,y ,z coordinate system and the x', y' is

$$x = x'\,Cos(\xi)$$
$$y = x'\,Sin(\xi)$$
$$z = -y'$$

Now substituting these last four equations into the circle equation gives

$$(x' \, Cos(\xi))^2 + (-y')^2 = \left(R' + \frac{(R' - R'')x' \, Sin(\xi)}{H} \right)^2 \qquad \text{Eq 5}$$

This is the equation that describes the general conic curve. Solving this equation for y' gives

$$y' = \left[\left(R' + \frac{(R' - R'')x' \, Sin(\xi)}{H} \right)^2 - (x' \, Cos(\xi))^2 \right]^{0.5} \qquad \text{Eq 6}$$

To find the equation for the slope of a line tangent to the general conic we must take the derivative of Eq 6 with respect to x'. This gives:

$$\frac{dy'}{dx'} = \frac{((R')^2 - R'R'')Sin(\xi)}{Hy'} + \frac{(R' - R'')^2 Sin^2(\xi)x'}{H^2 y'} - \frac{Cos^2(\xi)x'}{y'} \qquad \text{Eq 7}$$

And of course we can make the substitution of Eq 6 for y' to make the slope only a function of x' and ξ. As you can see, this would be a big hairy equation so I'm not going write it out here.

The equation for the y'-axis intercept of the tangent line is:

$$b = y' - \frac{dy'}{dx'} x' \qquad \text{Eq 8}$$

Substituting Eq 6 and Eq 7 into this equation gives us y'-axis intercept as a function of x' and ξ. This is another big equation so I'm not going write it out.

Gap Expressions

Now that we have the equations that describe the ellipse and general conic as well as the equations for the tangent line and y'-axis intercept, we can develop the equation for the gap, ε.

Consider the geometry shown in figure 3. As you can see, to find an expression for the gap distance ε, we must only solve a simple right triangle.

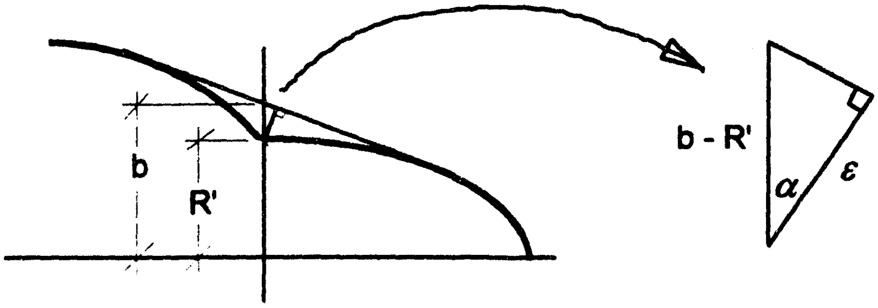


Figure 3 Gap Geometry

Using basic trigonometry we can see that

$$\varepsilon = (b - R')Cos(\alpha)$$

The angle α is related to the slope of the tangent line (the filament) by the equation

$$\alpha = Tan^{-1}\left(\frac{dy'}{dx'}\right)$$

We have two equations for the gap size, one in terms of the ellipse

$$\varepsilon_{ellipse} = (b_{ellipse} - R')Cos\left(Tan^{-1}\left(\frac{dy'}{dx'}\right)_{ellipse}\right) \qquad \text{Eq 9}$$

and one in terms of the general conic

$$\varepsilon_{conic} = (b_{conic} - R')Cos\left(Tan^{-1}\left(\frac{dy'}{dx'}\right)_{conic}\right) \qquad \text{Eq 10}$$

Solution Equations

All of this analysis leaves us with the equations

$$\varepsilon_{ellipse} - \varepsilon_{conic} = 0 \qquad \text{Eq 11}$$

$$b_{ellipse} - b_{conic} = 0 \qquad \text{Eq 12}$$

$$\left(\frac{dy'}{dx'}\right)_{ellipse} - \left(\frac{dy'}{dx'}\right)_{conic} = 0 \qquad \text{Eq 13}$$

These equations hold true because the gap, y'-intercept, and slope all describe the same line therefore they must be equal regardless if we calculate them using the ellipse or the general conic.

These equations contain four variables $x'_{ellipse}$, x'_{conic}, ε, and ξ. With three equations we can solve for three unknowns. So if we specify a gap, ε, we can solve for the angle ξ and vice versa. All we need to do is solve these three equations simultaneously. Unfortunately these equations are complex and non-linear and I can't solve them algebraically. The next section of this paper will explain a method for solving these equations numerically.

Fillet Definition

One of the byproducts of the analysis is the we get the location of the filament tangent points. With these two points we derive an equation of a curve that, if machined into the mandrel, will allow the filament to make the transition between the cone and the cylinder without loosing contact with the mandrel.

The equation of this curve is:

$$x = \left[\left(\frac{y}{Tan(\xi)} \right)^2 + \left(z_1 + \frac{(z_1 - z_2)(y - y_1)}{y_1 - y_2} \right)^2 \right]^{0.5}$$

where the point tangent to the cylinder is

$$P_1 = (x_1, y_1, z_1)$$

and the point tangent to the cone is

$$P_2 = (x_2, y_2, z_2)$$

Solution

Numerical Solution Technique

Numerical solution techniques typically involve an equation with two variables arranged so that the equation equals zero. Lets say:

$$f(x, y) = 0$$

We solve this type of equation by specifying one variable and guessing at the other. We set the equation up so that if, after we guess, the right side is grater then zero then we must guess a smaller number. If the right side is less then zero then we must guess a larger number. We continually make better guesses, making the right side closer and closer to zero. Once the right side is within a zero tolerance, say ± 0.00001, then the problem is considered solved.

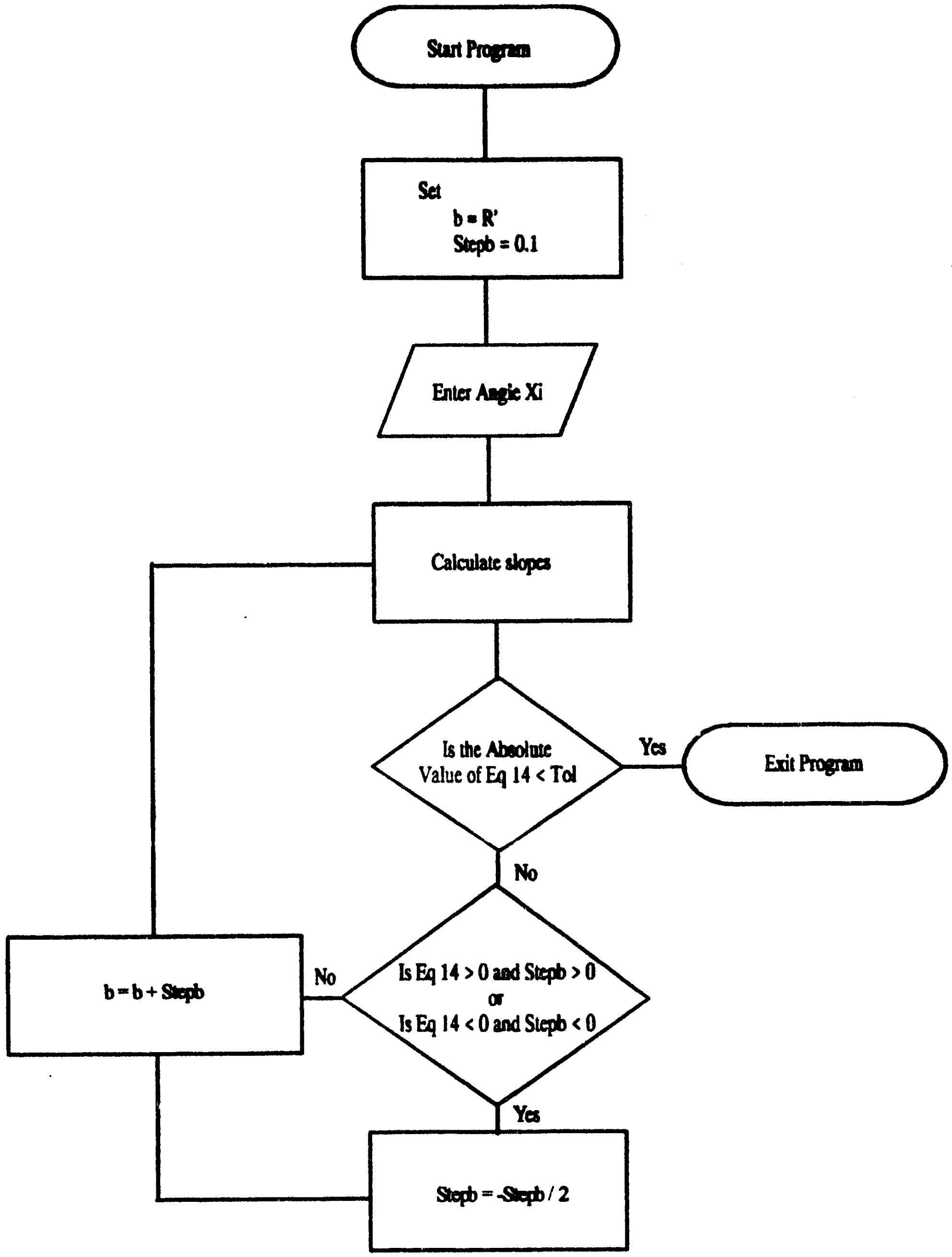
This the the technique I'll use the solve the equations Eq11, Eq 12, and Eq 13.

Solving for the Gap

The gap size ε, given the angle ξ, can be solved by finding the y'-axis intercept that will give

$$\left(\frac{dy'}{dx'}\right)_{conic} - \left(\frac{dy'}{dx'}\right)_{ellipse} = 0 \qquad \text{Eq 14}$$

The procedure can best be described by a flow chart. The following flow chart is the method I employed to solve for the gap ε.



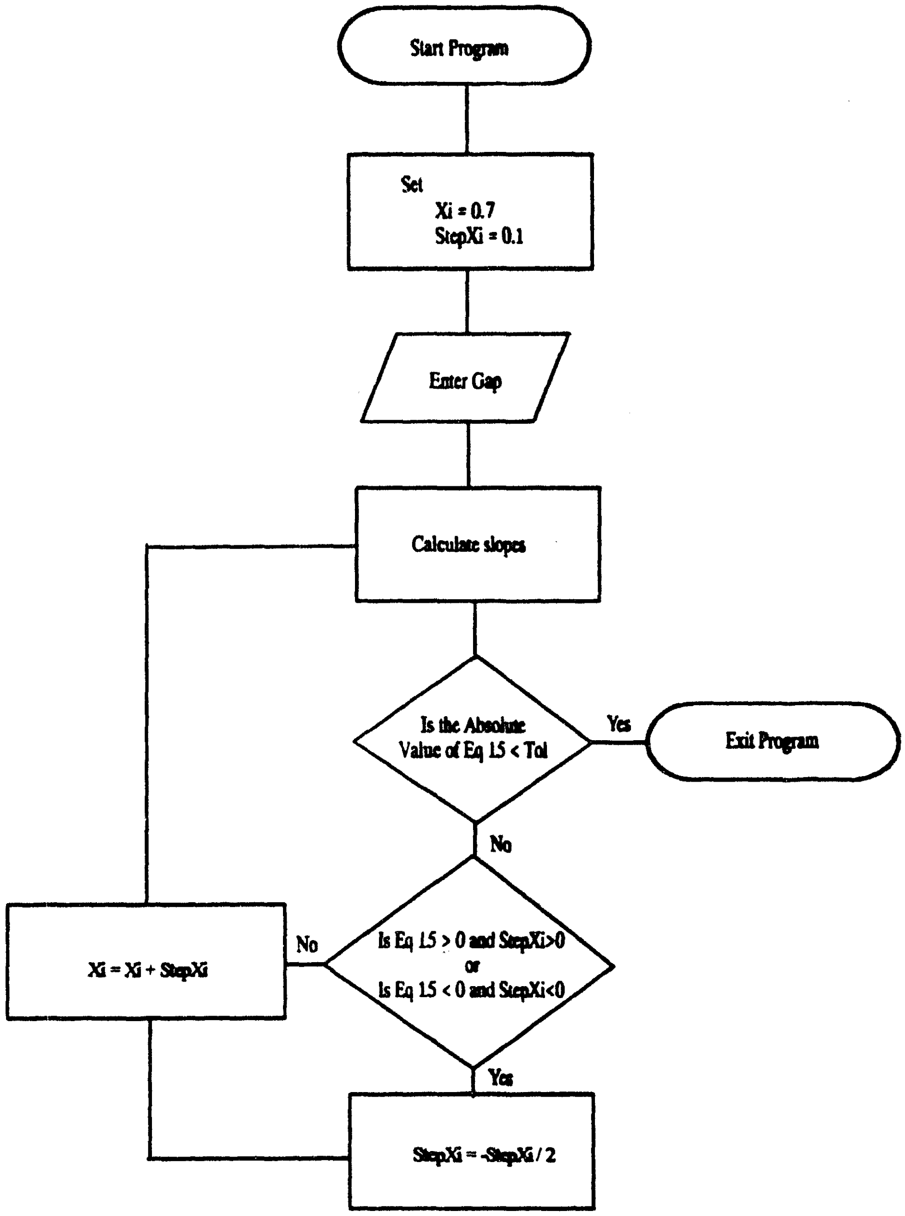
Note that to calculate the slopes, the numerical technique must be employed again.

Solving for the Angle

The angle ξ, given the gap size ε, can be solved by finding the angle ξ that will give

$$\left(\frac{dy'}{dx'}\right)_{ellipse} - \left(\frac{dy'}{dx'}\right)_{conic} = 0 \qquad \text{Eq 15}$$

The procedure can best be described by a flow chart. The following flow chart is the method I employed to solve for the angle ξ.



The appendix of this paper describes the program that I had written to solve this problem. Its available if you contact me.

Appendix

Wire Program

WIRE.EXE is a program that I developed that will numerically solve the filament winding gap and wind angle equations derived in this paper. With it, you enter the mandrel geometry, the known gap or angle, then the program will calculate the unknown gap or angle. It will also derive the fillet equation for the transition fillet as well as an arc approximation and error value.

System Requirements

To use WIRE.EXE (labeled "Filament Winding Gap" in the program manager window) you need the following:

- IBM-compatible personal computer with an 80286 processor or higher.
- A hard disk.
- A 3 1/2" floppy drive.
- An EGA, VGA, 8514, Hercules, or compatible display.
- One megabyte of memory.
- A mouse.
- Microsoft MS-DOS version 3.1 or later.
- Windows version 3.0 or later in standard or enhanced mode.

Setting Up and Starting Wire

Use the setup program (SETUP.EXE) to install Wire on your computer.

To set up Wire:

1. Insert Disk in drive A.
2. Start Windows.
3. In the Windows Program Manager, choose Run from the File menu. Windows displays the Run dialog box.
4. Type **a:setup** in the Command Line box.
5. Choose the OK button.
6. Follow the setup instructions on the screen.

After installing the files you need, the Setup program creates a Lawrence Livermore Nat'l Lab program group and puts the Wire icon in the group.

——— Wire.exe icon

Filament
WindingGap

To Start Wire:

1. In the Windows Program Manager, open the program group that contains the Wire icon.
2. Double-click the Wire icon.

   Wire starts and displays the data form.

How the Program Works

When the program starts it will display the data form. As you'll see, this form is separated into four sections: Mandrel Geometry, Dependent and Independent Variables, Fillet Definition, and a picture similar to figure 1 of this paper. The data form also contains two command buttons: Find Theta and Xi, and Find Gap.

To use this program you first enter all of the mandrel geometry data. This data is required. Then enter your known angle or gap. Once this is done, select the Find Gap button if you entered an angle, or select the Find Theta and Xi button if you entered a gap distance. The program will then calculate the unknown gap or angles. Note, when entering data you can clear the currant field by pressing the space bar.

Once the program calculates for the unknown variable it will also display a curve equation in the Fillet Definition area of the data form. Along with the equation there is a y-axis range for which this equation is valid. If this curve feature is machined into the mandrel, then the filament will remain in contact with the mandrel throughout the transition from cone to cylinder.

The equation in the fillet area is complex and is not easily machined, therefore it may be advantageous to machine a simple arc instead. If you move the mouse pointer into the Fillet Definition area and click the left mouse button a dialog box will pop up. This dialog box will contain the fillet radius and a error value. The error value is the maximum deviation between the curve defined by the equation and the arc approximation. The error value is given so that you can judge whether or not the arc approximation is close enough to meet your needs. This arc is not, in general, tangent to the cone or cylinder. This arc must pass through the starting and ending points of the fillet curve defined in the Fillet Definition area.

Once the calculations are complete you may obtain a hard copy by way of the print menu. You have the choice of ether printing the data or a bit map copy of the data form. I suggest that you print the data instead of the form because it contains the most information and prints much faster than the form. The form has the advantage in that it prints a copy of the mandrel picture.

Contacting Me If You Need More Information

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