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ROCKING RESPONSE OF TANKS CONTAINING TWO LIQUIDS

Yu Tang
Reactor Engineering Division
Argonne National Laboratory
Argonne, IL 60439

ABSTRACT

A study on the dynamic response of upright circular cylindrical liquid-storage tanks containing two different liquids under a rocking base motion with an arbitrary temporal variation is presented. Only rigid tanks were studied. The response quantities examined include the hydrodynamic pressure, sloshing wave height and the associated frequencies, base shear and moments. Each of these response quantities is expressed as the sum of the so-called impulsive component and convective component. Unlike the case of tanks containing one liquid, in which the response is controlled by one parameter, height-to-radius ratio, the response of tanks containing two different liquids are controlled by three parameters: height-to-radius ratio, and mass density ratio and height ratio of the two liquids. The interrelationship of the responses of the tank-liquid system to rocking and lateral base excitations is established by examining numerical results extensively. The study shows that some of the response quantities for tank-liquid system under a rocking base motion can be determined from the available data for the response of an identical tank under a horizontal base motion.

INTRODUCTION

Liquid storage tanks are important components of industrial facilities and, when located in earthquake prone regions, should be designed to withstand the earthquakes to which they may be subjected. The dynamic response of liquid-storage tanks subjected to earthquakes has been a subject of numerous studies in the past 30 years. For reviews of the previous studies on this topic, the reader is referred to Haroun (1980), Haroun and Ellaithy (1985), Veletsos (1984), Veletsos and Tang (1986) and Veletsos and Tang (1987) for responses of laterally, vertically and rockingly excited tank-liquid systems. Most of the previous studies were focused on the tank containing only one liquid. However, there are cases as stated in Tang (1993a) in which the

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density of the tank content is not uniform. For such cases, the dynamic responses of tanks containing liquids with different densities must be studied. To respond to this need, Tang (1993a) presented a solution for dynamic response of rigid tanks containing two liquids under a horizontal base motion. In that study, however, the effect of the gravitation on the interface motion of two liquids was neglected. This gravitational effect was later introduced in Tang (1993b), and the dynamic characteristics of the flexible tanks containing two liquid was also studied by Tang (1992). All these studies are aimed at the understanding of the dynamic behavior of tanks containing two liquids under lateral base motions. Adding to these efforts presented in this paper is the dynamic response of tanks containing two liquids under rocking base motions. It should be noted that base rocking motion can occur in a ground-supported tank or in an elevated tank under earthquake motions. Due to the flexibility of either the supporting soil or the supporting tower, the tank base will experience a rocking component of motion, even for a purely translational free field motion.

The objectives of this paper are: (1) to present the exact solution for the dynamic response of tanks containing two liquids under base rocking motions; (2) to elucidate the interrelationship of the responses of a tank to rocking and lateral base motions; and (3) to provide a foundation for the study of the soil-structure interaction analysis of tanks containing two liquids. The response functions examined are the hydrodynamic pressure, the sloshing motion and the associated natural frequencies, the base shear and moments. How to utilize these response functions in the tank design can be found in Veletsos (1984). In this paper, each of these response functions is expressed as the sum of the so-called impulsive and convective components of the response (Housner, 1957). This division is necessary because it is essential to the approach used by Veletsos and Yang (1977), Veletsos and Tang (1989) and Tang (1992) in the analysis of flexible tanks. The impulsive component of the response represents the effects of the part of the liquid that moves in unison with the tank, whereas the convective component represents the effects of the part of the liquid that associates with the sloshing motion.

In this paper, the interrelationship of the responses of a tank containing two liquids under base rocking motion and under base horizontal motion is established by examining numerical data of the response quantities under the two base motions extensively. It is found, similar to the case of the responses of tanks containing one liquid, that for tanks containing two liquids some of the response quantities may be evaluated from the existing data for the same tank-liquid system under lateral excitations. Similar to the response of tanks containing two liquids under horizontal base motions presented in Tang (1993b), a tank containing two liquids under a base rocking motion has two natural frequencies associated with each sloshing mode of vibration. These natural frequencies are the same as those for tanks in lateral excitation.

SYSTEM DESCRIPTION

The tank-liquids system investigated is shown in Fig. 1. It is a ground-supported upright circular cylindrical tank of radius R that is filled with two liquids to a total height of H. The lower portion liquid, identified as Liquid I, has heavier mass density, ρ_1, and the upper portion liquid, identified as Liquid II, has lighter mass density, ρ_2. The heights of Liquid I and II are H_1 and H_2, respectively. The tank wall is assumed to be of uniform thickness and clamped to a rigid base. Both liquids are considered to be incompressible and inviscid. The response of the liquids is assumed to be linear.

Let r, θ, z_1 denote the radial, circumferential, and vertical axial coordinates of a point in the Liquid I, and let r, θ, and z_2 be the corresponding coordinates for a point in Liquid II as shown in Fig. 1. The origins of the two coordinate systems are at the central axis of the cylindrical tank.

The base motion experienced by the tank is an angular acceleration, denoted by $\ddot{\theta}_b(t)$, acting in the direction along the θ=90° coordinate axis. The temporal variation of $\ddot{\theta}_b(t)$ can be arbitrary.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

Given the conditions that the liquids are incompressible and inviscid, the hydrodynamic pressures induced at Liquid I and Liquid II, denoted by p_1 and p_2 respectively, must satisfy the Laplace equations

$$\nabla^2 p_1 = 0 \tag{1a}$$

in the region $0 \le r \le R$, $0 \le \theta \le 2\pi$, and $0 \le z_1 \le H_1$, and

$$\nabla^2 p_2 = 0 \tag{1b}$$

in the region $0 \le r \le R$, $0 \le \theta \le 2\pi$, and $0 \le z_2 \le H_2$.

The liquid acceleration at an arbitrary point along n-direction is given by

$$a_n = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial n} \tag{2}$$

for points in Liquid I, and

$$a_n = -\frac{1}{\rho_2}\frac{\partial p_2}{\partial n} \tag{3}$$

for points in Liquid II.

The boundary conditions for Liquid I are:

(a) The vertical acceleration of Liquid I at the tank base must equal the acceleration of the base plate, i.e.,

$$-\frac{1}{\rho_1}\frac{\partial p_1}{\partial z_1}\bigg|_{z_1=0} = r\cos\theta\ \ddot{\theta}_b(t) \tag{4a}$$

(b) The radial acceleration of Liquid I adjacent to the tank wall must equal the acceleration of the tank wall, i.e.,

$$-\frac{1}{\rho_1}\frac{\partial p_1}{\partial r}\bigg|_{r=R} = -z_1\cos\theta\ \ddot{\theta}_b(t)\ \text{, and} \tag{4b}$$

The boundary conditions for Liquid II are:

(a) The radial acceleration along the tank wall is given by

$$-\frac{1}{\rho_2}\frac{\partial p_2}{\partial r}\bigg|_{r=R} = -(z_2 + H_1)\cos\theta\ \ddot{\theta}_b(t)\ , \tag{4c}$$

(b) At free surface, the linearized boundary condition is

$$\left(\frac{\partial^2 p_2}{\partial t^2} + g\frac{\partial p_2}{\partial z_2}\right)_{z_2=H_2} = 0 \tag{4d}$$

where g is the gravitational acceleration, and

The boundary conditions at the interface of two liquids are as follows.

(a) Continuity of vertical accelerations, i.e.,

$$-\frac{1}{\rho_1}\frac{\partial p_1}{\partial z_1}\bigg|_{z_1=H_1} = -\frac{1}{\rho_2}\frac{\partial p_2}{\partial z_2}\bigg|_{z_2=0} \tag{4e}$$

and

(b) Kinematic and pressure conditions:

b.1 Kinematic condition. If $\eta(r,\theta,t)$ represents the height of the small disturbance at the interface above the still interface level, the $\eta(r,\theta,t)$ is related to p_1 by

$$\frac{\partial^2 \eta}{\partial t^2} = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial z_1}\bigg|_{z_1=H_1} \tag{5}$$

b.2 Pressure condition. If the gravitational effect is considered for the interface motion, there is a discontinuity of the hydrodynamic pressure with the amount of $(\rho_1 - \rho_2)g\eta$ at the interface, i.e.,

$$p_1\big|_{z_1=H_1} - p_2\big|_{z_2=0} = (\rho_1 - \rho_2)g\eta \tag{6}$$

Eliminating η between Eqs. (5) and (6) and making use of the Eq. (4e), one obtains the following equation for the interface boundary conditions in addition to Eq. (4e).

$$\left(\frac{\partial^2 p_1}{\partial t^2} + g\frac{\partial p_1}{\partial z_1}\right)\bigg|_{z_1=H_1} = \left(\frac{\partial^2 p_2}{\partial t^2} + g\frac{\partial p_2}{\partial z_2}\right)\bigg|_{z_2=0} \tag{4f}$$

Also,

p_1 and p_2 are finite at r=0 (4g)

The solutions for p_1 and p_2 are expressed as the sum of the impulsive component and convective component, i.e.,

$$p_1 = p_1^{ir} + p_2^{cr} \tag{7}$$

and

$$p_2 = p_2^{ir} + p_2^{cr} \tag{8}$$

where the superscript i = impulsive component; the superscript c = convective component; and the superscript r is used as a reminder for rocking.

The detailed derivation of these solutions are given in Appendix I. The solutions for these functions are summarized as follows.

$$p_1^{ir}(r,\theta,z_1,t) = C_o^{Ir}(r,z_1)\ \ddot{x}_T(t)\ \rho_1 R\cos\theta \tag{9}$$

$$p_2^{ir}(r,\theta,z_2,t) = C_o^{IIr}(r,z_2)\ \ddot{x}_T(t)\ \rho_2 R\cos\theta \tag{10}$$

$$p_1^{cr}(r,\theta,z_1,t) = \left[\sum_{n=1}^{\infty}\sum_{k=1}^{2} C_{nk}^{Ir}(r,z_1)\ A_{nk}^{r}(t)\right]\rho_1 R\cos\theta \tag{11}$$

and

$$p_2^{cr}(r,\theta,z_2,t) = \left[\sum_{n=1}^{\infty}\sum_{k=1}^{2} C_{nk}^{IIr}(r,z_2)\ A_{nk}^{r}(t)\right]\rho_2 R\cos\theta \tag{12}$$

where superscripts I and II denote Liquids I and II, respectively. The expressions for dimensionless functions $C_o^{Ir}(r,z_1)$, $C_o^{IIr}(r,z_2)$, $C_{nk}^{Ir}(r,z_1)$ and $C_{nk}^{IIr}(r,z_2)$ are given in Appendix I; $\ddot{x}_T(t)$ = $H\ddot{\theta}_b(t)$ = the horizontal acceleration of the tank wall at the level of the still liquid surface, and the functions $A_{nk}^{r}(t)$, k= 1 and 2 are the pseudoacceleration functions for the nth sloshing mode of vibration, $J_1\left(\lambda_n \frac{r}{R}\right)$, and are defined by

$$A_{nk}^{r}(t) = \omega_{nk}\int_o^t \ddot{x}_T(t)\sin\left(\omega_{nk}(t-\tau)\right)d\tau \tag{13}$$

in which ω_{nk}, k=1,2, = the natural frequencies associated with the nth sloshing mode of vibration. Note that for the nth sloshing mode of vibration there are two natural frequencies, ω_{n1} and ω_{n2}. This phenomena has been discussed in Tang (1993b). Introducing the notation, Λ_{nk}, for a nondimensional coefficient that is related to ω_{nk} by the equation

$$\Lambda_{nk} = \frac{\omega_{nk}^2 R}{\lambda_n g} \tag{14}$$

It is shown in Appendix I that Λ_{nk}, k=1, 2, are the roots of the characteristic equation given by

$$a\,\Lambda_n^2 - b\,\Lambda_n + c = 0 \tag{15}$$

where

$$a = 1 + \alpha\,\tanh\beta_{1n}\,\tanh\beta_{2n} \tag{16a}$$

$$b = \tanh\beta_{1n} + \tanh\beta_{2n} \tag{16b}$$

and

$$c = (1-\alpha)\,\tanh\beta_{1n}\,\tanh\beta_{2n} \tag{16c}$$

in which $\beta_{1n} = \lambda_n \frac{H_1}{R}$, $\beta_{2n} = \lambda_n \frac{H_2}{R}$ and $\alpha = \rho_2/\rho_1$.

It can be shown that the discriminant of Eq. (15) $D = b^2 - 4ac > 0$ for $\alpha > 0$; therefore, Eq. (15) has two real and unequal roots. Explicitly, these two roots are given by

$$\Lambda_{n1} = \frac{b + \sqrt{D}}{2a} \tag{17}$$

and

$$\Lambda_{n2} = \frac{b - \sqrt{D}}{2a} \tag{18}$$

Obviously, $\Lambda_{n1} > \Lambda_{n2}$; hence $\omega_{n1} > \omega_{n2}$. The reasons for this reverse order numbering for the natural frequencies were given in Tang (1993b). Note that, the Eq. (15) is identical to the characteristic equation presented in Tang (1993b) for tanks subjected to a lateral base motion, i.e., the natural frequencies for the sloshing motion in a tank undergoing rocking motion are the same as those of the sloshing motion in an identical tank subjected to a lateral excitation. The numerical results for values of ω_{nk} are available in Tang (1993b).

PRESENTATION OF RESULTS

The dynamic response of a tank containing two liquids under rocking base excitations is controlled by three parameters, H/R, H_2/H_1 and α, which are the same control parameters as those for the dynamic response of a tank containing two liquids under lateral base excitations.

Hydrodynamic Pressure: Impulsive Component

The impulsive pressure exerted on the tank wall is conveniently expressed in the form

$$p^{ir}(\theta,z,t) = C_0^r(z)\ \ddot{x}_T(t)\ \rho_1 R\cos\theta \tag{19}$$

where $C_0^r(z)$ is given by

$$C_0^r(z) = C_0^{Ir}(r,z_1)\Big|_{r=R} \qquad \text{for } 0 \le z \le H_1 \tag{20a}$$

$$C_0^r(z) = \alpha C_0^{IIr}(r,z_2)\Big|_{z=R} \qquad \text{for } H_1 \le z \le H \tag{20b}$$

The coordinate z used in Eq. (19) is related to z_1 and z_2 by the equations

$$z = z_1 \qquad \text{for } 0 \le z \le H_1 \tag{21a}$$

and

$$z = z_2 + H_1 \qquad \text{for } H_1 \le z \le H \tag{21b}$$

It is clearly shown in Eq. (19) that the timewise variation of the impulsive pressure is the same as that of the base excitation. This indicates that the impulsive pressure is produced by a portion of liquid that moves in unison with the tank wall.

The distributions of $C_0^r(z)$ for α = 0.25, 0.5, 0.75 and 1 for a broad tank, H/R=0.5, and a tall tank, H/R=3 are shown in Fig. 2 for H_2/H_1 = 0.5 and in Fig. 3 for H_2/H_1 = 2. Note that for a broad tank, the impulsive pressure decreases monotonically from bottom to top, whereas for a tall tank the shape of impulsive pressure distribution has a double curvature. The maximum value occurs at a point away from the bottom. These trends are in agreement with those presented in Veletsos and Tang (1987) for tanks containing one liquid undergoing rocking motion. The impulsive pressure decreases as the value of α decreases. This reduction is more pronounced in tall tanks and for larger values of H_2/H_1.

Base Shear: Impulsive Component

The impulsive component of base shear, $Q^{ir}(t)$, is given by

$$Q^{ir}(t) = \int_0^{2\pi} \int_0^{H_1} p_1^i \Big|_{r=R} R\cos\theta \, dz_1 \, d\theta$$

$$+ \int_0^{2\pi} \int_0^{H_2} p_2^i \Big|_{r=R} R\cos\theta \, dz_2 \, d\theta \tag{22}$$

Substituting Eqs. (9) and (10) into Eq. (22) and performing the integration, one obtains the expression for $Q^{ir}(t)$ which is given as

$$Q^{ir}(t) = S_0^{Ir} M_{\ell 1} \ddot{x}_T(t) + S_0^{IIr} M_{\ell 2} \ddot{x}_T(t) \tag{23}$$

in which $M_{\ell 1} = \rho_1 \pi R^2 H_1$ = total liquid mass of liquid I; $M_{\ell 2} = \rho_2 \pi R^2 H_2$ = total mass of liquid II; and S_0^{Ir} and S_0^{IIr} = dimensionless coefficients dependent on the values of H/R, H_2/H_1 and α.

In Eq. (23), the first term on the right-hand side is the base shear contributed from Liquid I, and the second term is from Liquid II. Since Eq. (22) involves only simple integrations of hyperbolic functions, the expressions for S_0^{Ir} and S_0^{IIr} are not given herein. The expression given by Eq. (23) has a physical meaning; it gives the volume ratio of each liquid that may be considered as added mass to the tank wall. However, to study the effect of two liquids on the total base shear, it is also desirous to have an expression that can be used for comparison with an identical tank that contains one liquid. Therefore, $Q^{ir}(t)$ is also expressed as

$$Q^{ir}(t) = r_{0s}^{ir} M_\ell^1 \ddot{x}_T(t) \tag{24}$$

in which $M_\ell^1 = \pi \rho_1 R^2 H$ = the total liquid mass if the tank is filled with Liquid I; and r_{0s}^{ir} = dimensionless coefficient related to S_0^{Ir} and S_0^{IIr} by the equation

$$r_{0s}^{ir} = \frac{S_0^{Ir} H_1 + \alpha S_0^{IIr} H_2}{H} \tag{25}$$

The values of S_0^{Ir} and S_0^{IIr} for different control parameters, H/R, H_2/H_1 and α, are available in Tang and Chang (1993). It is found that for the same values of H/R and H_2/H_1, the value of S_0^{Ir} increases as the value of α increases, whereas the value of S_0^{IIr} decreases with increasing value of α.

Examination of the numerical value of r_{0s}^{ir} reveals that this quantity can be computed from the quantities in the expressions for the base moment at a section immediately below the tank base of an identical tank-liquid system excited laterally. Specifically, presented in Tang (1993b), for an identical system excited laterally by an acceleration of $\ddot{x}(t)$, the base moment $M^i(t)$ at a section immediately above the tank base can be expressed as

$$M^i(t) = r_{0M}^i M_\ell^1 H \ddot{x}(t) \tag{26}$$

and the base moment induced by the pressure exerted on the tank base, denoted by $\Delta M^i(t)$, is given by

$$\Delta M^i(t) = \Delta r_{0M}^i M_\ell^1 H \ddot{x}(t) \tag{27}$$

Then, it is found that the value of r_{0s}^{ir} is related to the values of r_{0M}^i and Δr_{0M}^i by the equation

$$r_{0s}^{ir} = r_{0M}^i + \Delta r_{0M}^i \tag{28}$$

The relation defined by Eq. (28) may be explained by a generalization of Betti's principle stated in Veletsos and Tang (1987). In that paper, the same relation as that defined by Eq. (28) is found for tank containing one liquid under rocking motion. Numerical values for r_{0M}^i and Δr_{0M}^i have been presented in Tang (1993a) and Tang and Chang (1992); therefore, the value of r_{0s}^r may be computed by direct application of Eq. (28).

Base Moments: Impulsive Components

The base moment $M^{ir}(t)$ at a section immediately above the tank base is computed from

$$M^{ir}(t) = \int_0^{2\pi} \int_0^{H_1} p_1^i \Big|_{r=R} R z_1 \cos\theta \, dz_1 \, d\theta$$

$$+ \int_0^{2\pi} \int_0^{H_2} p_2^i \Big|_{r=R} R(z_2 + H_1) \cos\theta \, dz_2 \, d\theta \qquad (29)$$

and the result is expressed as

$$M^{ir}(t) = C_{0M}^{Ir} \ddot{x}_T(t) M_{\ell 1} H_1 + C_{0M}^{IIr} \ddot{x}_T(t) M_{\ell 2} H_2 \qquad (30)$$

Again, since Eq. (29) involves only the simple integrations of hyperbolic functions, the expressions for C_{0M}^{Ir} and C_{0M}^{IIr} are not given herein.

The numerical values of C_{0M}^{Ir} and C_{0M}^{IIr} are available in Tang and Chang (1993). It shows that for the same values of H/R and H_2/H_1, the value of C_{0M}^{Ir} increases as the value of α increases, but the value of C_{0M}^{IIr} has an opposite trend.

Equation (30) is also rewritten as

$$M^{ir}(t) = r_{0M}^{ir} M_\ell^1 H \ddot{x}_T(t) \qquad (31)$$

in which r_{0M}^{ir} = a dimensionless coefficient defined by

$$r_{0M}^{ir} = C_{0M}^{Ir} \left(\frac{H_1}{H}\right)^2 + \alpha \, C_{0M}^{IIr} \left(\frac{H_2}{H}\right)^2 \qquad (32)$$

The value of r_{0M}^{ir} is presented graphically in Fig. 4 for different values of H_2/H_1 as a function of α. Four different values of H/R are considered; they are 0.5, 1, 2 and 3. From Fig. 4 it is noted that the value of r_{0M}^{ir} decreases with decreasing value of α, and the decrease is more rapid for large values than for small values of H_2/H_1.

The base moment induced by the impulsive pressure exerted on the tank base is denoted by $\Delta M^{ir}(t)$ which is given by the following equation

$$\Delta M^{ir}(t) = \int_0^{2\pi} \int_0^R p_1^r \Big|_{z_1=0} r^2 \cos\theta \, dr \, d\theta \tag{33}$$

and may be expressed in the form as

$$\Delta M^{ir}(t) = \Delta C_{0M}^r \ddot{x}_T(t) M_{\ell 1} H_1 \tag{34}$$

in which ΔC_{0M}^r = a dimensionless coefficient. The value of ΔC_{0M}^r is available in Tang and Chang (1993). It shows that the value of ΔC_{0M}^r decreases rapidly as the value of H/R increases.

For the reason identified earlier, Eq. (34) is also expressed as

$$\Delta M^{ir}(t) = \Delta r_{0M}^{ir} \ddot{x}_T(t) M_\ell^1 H \tag{35}$$

to compare the result with that corresponding to $\alpha = 1$. The coefficient Δr_{0M}^r is related to ΔC_{0M}^r by the equation

$$\Delta r_{0M}^{ir} = \Delta C_{0M}^r \left(\frac{H_1}{H} \right)^2 \tag{36}$$

The values of Δr_{0M}^{ir} are shown in Fig. 5. It can be seen that for a tall tank this moment is much smaller than that for a broad tank. Note that in Fig. 5, the vertical scale for H/R=1 is one-fifth of that for H/R = 0.5.

Hydrodynamic Pressure: Convective Component

The convective pressure exerted on the tank wall may be obtained by evaluating the pressures defined by Eqs. (11) and (12) at r=R. This approach is similar to that used in obtaining the impulsive pressure exerted on the tank wall presented above. However, in this paper a physically motivated approach is used to derive the convective pressure instead, and the results obtained by the two approaches will be checked against each other.

Since the natural modes of vibration and the natural frequencies of the sloshing motion in a tank-liquid system are independent of the excitation that the system is experienced, the heightwise distribution of the convective hydrodynamic wall pressure induced by the rocking motion will be identical to that induced by the lateral motion. If the heightwise distribution of the convective component of the hydrodynamic pressure under lateral excitation is given by the function $C_{nk}(z)$,(see Tang 1993a), then, following from the above argument, the convective pressure, $p^{cr}(z,\theta,t)$, under rocking can be expressed as

$$p^{cr}(z,\theta,t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk}\, C_{nk}(z)\, A_{nk}^{r}(t)\right] \rho_l R \cos\theta \tag{37}$$

in which γ_{nk} = a dimensionless proportionality factor that remains to be determined.

Base Shear: Convective Component

It is shown in Tang (1993b) that the convective base shear, $Q^c(t)$, for tank-liquid excited laterally is given by

$$Q^c(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} S_{nk}^{I}\, A_{nk}(t)\right] M_{\ell 1} + \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} S_{nk}^{II}\, A_{nk}(t)\right] \tag{38}$$

or, it may be expressed differently as

$$Q^c(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} r_{nk}^{s}\, A_{nk}(t)\right] M_{\ell}^{1} \tag{39}$$

in which $A_{nk}(t)$ = the instantaneous pseudoacceleration induced by a base acceleration $\ddot{x}(t)$ in a SDF system of circular natural frequency ω_{nk}. $A_{nk}(t)$ is obtained from Eq. (13) by replacing $\ddot{x}_T(t)$ by $\ddot{x}(t)$. By application of the same analogy used in obtaining convective pressure, Eq. (37), for system in rocking motion from that for system in lateral motion, one obtains the convective base shear for system in rocking motion from Eqs. (38) which is given as

$$Q^{cr}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} S_{nk}^{I} A_{nk}^{r}(t)\right] M_{\ell 1}$$

$$+ \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} S_{nk}^{II} A_{nk}^{r}(t)\right] M_{\ell 2} \tag{40}$$

or from Eq. (39), one obtains

$$Q^{cr}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} r_{nk}^{s} A_{nk}^{r}(t)\right] M_{\ell}^{1} \tag{41}$$

Note that Eqs. (40) and (41) can also be obtained from Eq. (22) by replacing function $p^{ir}(t)$ by the function $p^{cr}(t)$ of Eq. (37) and performing the integrations.

Base Moments: Convective Components

It is shown in Tang (1993b) that the convective base moment, $M^{c}(t)$, at a section immediately above the tank base for tank-liquid system excited laterally can be expressed as

$$M^{c}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} C_{nk}^{MI} A_{nk}(t)\right] M_{\ell 1} H_{1}$$

$$+ \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} C_{nk}^{MII} A_{nk}(t)\right] M_{\ell 2} H_{2} \tag{42}$$

and the base moment induced by the pressure exerted on the tank base, denoted by $\Delta M^{c}(t)$, is given by

$$\Delta M^{c}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \Delta C_{nk}^{M} A_{nk}(t)\right] M_{\ell 1} H_{1} \tag{43}$$

Eqs. (42) and (43) can be expressed differently as

$$M^{c}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} r_{nk}^{M} A_{nk}(t)\right] M_{\ell}^{1} H \tag{44}$$

and

$$\Delta M^{c}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \Delta r_{nk}^{M} A_{nk}(t)\right] M_{\ell}^{1} H \tag{45}$$

Equations (42) and (43) are useful for providing the information about the contribution of each liquid, and Eqs. (44) and (45) are useful for comparing with the results of an identical tank filled with one liquid to assess the effect of two-liquid interaction.

Applying the same analogy that used in obtaining the convective components of the hydrodynamic pressure and base shear for system in rocking from the corresponding results for system excited laterally, one obtains, from Eqs. (42) and (43), the expressions of the convective components of the base moments for system in rocking

$$\begin{aligned} M^{cr}(t) \quad &= \quad \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} C_{nk}^{MI} A_{nk}^{r}(t)\right] M_{\ell 1} H_{1} \\ &+ \quad \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} C_{nk}^{MII} A_{nk}^{r}(t)\right] M_{\ell 2} H_{2} \end{aligned} \tag{46}$$

and

$$\Delta M^{cr}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} \Delta C_{nk}^{M} A_{nk}^{r}(t)\right] M_{\ell 1} H_{1} \tag{47}$$

or from Eqs. (44) and (45), one obtains

$$M^{cr}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} r_{nk}^{M} A_{nk}(t)\right] M_{\ell}^{1} H \tag{48}$$

and

$$\Delta M^{cr}(t) = \left[\sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} \, \Delta r_{nk}^{M} \, A_{nk}^{r}(t)\right] M_{\ell}^{1} \, H \tag{49}$$

Surface and Interface Sloshing Displacements

The surface sloshing wave height, $d^r(r,\theta,t)$, of an arbitrary point at the liquid surface may be determined from

$$p_2^{cr}\Big|_{z_2=H_2} = \rho_2 g \, d(r,\theta,t) \tag{50}$$

and the sloshing wave height at the interface of the two liquids, $\eta^r(r,\theta,t)$, may be determined from

$$\eta(r,\theta,t) = \frac{1}{(\rho_1-\rho_2)g}\left(p_1^{cr}\Big|_{z_1=H_1} - p_2^{cr}\Big|_{z_2=0}\right) \tag{51}$$

Substituting p_1^{cr} and p_2^{cr} in Eqs. (50) and (51) by Eqs. (11) and (12), one may obtain the expressions for $d(r,\theta,t)$ and $\eta(r,\theta,t)$, and the expressions for the maximum values of these two functions are obtained by evaluating $d(r,\theta,t)$ and $\eta(r,\theta,t)$ at $r = R$ and $\theta = 0$. Alternatively, the expressions for these maximum values may be obtained as follows. If the expressions for computing these maximum values for two liquids tank system excited laterally are given by (Tang 1993b)

$$d(R,0,t) = \sum_{n=1}^{\infty} \sum_{k=1}^{2} d_{nk} \frac{A_{nk}(t)}{g} R \tag{52}$$

and

$$\eta(R,0,t) = \sum_{n=1}^{\infty} \sum_{k=1}^{2} \eta_{nk} \frac{A_{nk}(t)}{g} R \tag{53}$$

their counterparts for the same system in rocking may be expressed as

$$d^{r}(R,0,t) = \sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk} \, d_{nk} \frac{A_{nk}^{r}(t)}{g} R \tag{54}$$

and

$$\eta^r(R,0,t) = \sum_{n=1}^{\infty} \sum_{k=1}^{2} \gamma_{nk}\, \eta_{nk} \frac{A^r_{nk}(t)}{g} R \tag{55}$$

Once the proportionality factor, γ_{nk}, is known, the maximum values of the surface and interface sloshing wave heights may be computed from the corresponding solutions for liquid-tank system excited laterally by making use of Eqs. (54) and (55).

Now, the generalized Betti's principle stated in Veletsos and Tang (1987) is invoked herein to determine the proportionality factor, γ_{nk}. In noting that for a tank subjected to lateral and rocking base motions of the same timewise variations and for each mode of vibration of the sloshing motion in the tank, the work done by the base shear for the tank in rocking through the displacement of the laterally excited tank is equal to the work done by the foundation moment for laterally excited tank acting through the rotation of the tank in rocking, from Eqs. (41), (44) and (45) one obtains

$$\gamma_{nk}\, r^s_{nk} = r^M_{nk} + \Delta r^M_{nk} \tag{56}$$

therefore,

$$\gamma_{nk} = \frac{r^M_{nk} + \Delta r^M_{nk}}{r^s_{nk}} \tag{57}$$

With γ_{nk} determined, all the convective components of the response quantities for tank in rocking can be calculated. Note that the determination of γ_{nk} from Eq. (57) requires only the results for the laterally excited tank; therefore, there is no need to solve the governing equations and the boundary conditions to obtain the convective components of the response for tank in rocking. However, in this paper, for the purpose of confirming the accuracy, the convective components of the response quantities are also computed from the convective pressure defined by Eqs. (11) and (12) which are obtained by direct solving the governing equations and the boundary conditions. The identical numerical results obtained by two approaches confirm the accuracy.

Extensive numerical data for laterally excited tank containing two liquids are available in Tang (1993a and 1993b), and also because all the convective component of the response quantities for tank in rocking can be computed from their counterparts for the laterally excited tank, no numerical results for the convective components of the response are presented herein.

CONCLUSIONS

The complete solutions for tanks containing two liquids under rocking excitation have been presented. Each dynamic response quantity is expressed as the sum of the impulsive and convective components so that the solutions presented may provide a rational basis for evaluating the effects of tank flexibility and soil-structure interaction. It is found that many response quantities for a tank in rocking may be evaluated from the corresponding response quantities for an identical tank excited laterally; especially, all the convective components of the response quantities can be evaluated by such way.

APPENDIX I. SOLUTION FOR RIGID TANKS

Impulsive Component of Solution

The impulsive component of the hydrodynamic pressures p_1^{ir} and p_2^{ir} must satisfy

$$\nabla p_1^{ir} = 0 \tag{58a}$$

and

$$\nabla p_2^{ir} = 0 \tag{58b}$$

and the following boundary conditions:

$$-\frac{1}{\rho_1}\left.\frac{\partial p_1^{ir}}{\partial z_1}\right|_{z_1=0} = r\ddot{\theta}_b(t)\cos\theta \tag{59a}$$

$$-\frac{1}{\rho_1}\left.\frac{\partial p_1^{ir}}{\partial r}\right|_{r=R} = -z_1\ddot{\theta}_b(t)\cos\theta \tag{59b}$$

$$-\frac{1}{\rho_2}\left.\frac{\partial p_2^{ir}}{\partial r}\right|_{r=R} = -\left(z_2 + H_1\right)\ddot{\theta}_b(t)\cos\theta \tag{59c}$$

$$\left.p_2^{ir}\right|_{z_2=H_2} = 0 \tag{59d}$$

$$-\frac{1}{\rho_1}\left.\frac{\partial p_1^{ir}}{\partial z_1}\right|_{z_1=H_1} = -\frac{1}{\rho_2}\left.\frac{\partial p_2^{ir}}{\partial z_2}\right|_{z_2=0} \tag{59e}$$

$$\left.p_1^{ir}\right|_{z_1=H_1} = \left.p_2^{ir}\right|_{z_2=0} \tag{59f}$$

p_1^{ir} and p_2^{ir} are finite at $r = 0$ (59g)

The method of separation of variables is employed to solve Eqs. (58a) and (58b), and the integration constants are determined from the boundary conditions. Satisfying Eqs. (58a), (59b), and (59g), the function p_1^{ir} takes the form

$$p_1^{ir} = \left[\frac{r}{R}\frac{z_1}{H} + \frac{R}{H}\sum_{n=1}^{\infty}\left(A_n \cosh\left(\lambda_n \frac{z_1}{R}\right) + B_n \sinh\left(\lambda_n \frac{z_1}{R}\right)\right)\frac{J_1\left(\lambda_n \frac{r}{R}\right)}{J_1(\lambda_n)}\right]\rho_1 R \ddot{x}_T(t)\cos\theta \tag{60}$$

and, similarly, on satisfying Eqs. (58b), (59c) and (59g), p_2^{ir} takes the form

$$p_2^{ir} = \left[\frac{r}{R}\frac{(z_2+H_1)}{H} + \frac{R}{H}\sum_{n=1}^{\infty}\left(C_n \cosh\left(\lambda_n \frac{z_2}{R}\right) + D_n \sinh\left(\lambda_n \frac{z_2}{R}\right)\right)\frac{J_1\left(\lambda_n \frac{r}{R}\right)}{J_1(\lambda_n)}\right]\rho_2 R \ddot{x}_T(t)\cos\theta \tag{61}$$

In Eqs. (60) and (61), $\ddot{x}_T(t) = H\ddot{\theta}_b(t)$, J_1 = Bessel function of the first kind of order 1, λ_n = the nth zero of $J'_1(\lambda)$, the first derivative of J_1; and A_n, B_n, C_n and D_n = constants of integration that may be determined from the boundary conditions defined by Eqs. (59a), (59d), (59e) and (59f). After evaluating these constants of integration, and substituting them back into Eqs. (60) and (61), the results are cast into the following forms

$$p_1^{ir}(r,\theta,z_1,t) = C_0^{Ir}(r,z_1)\ \rho_1 R \ddot{x}_T(t)\cos\theta \tag{62}$$

and

$$p_2^{ir}(r,\theta,z_2,t) = C_0^{IIr}(r,z_2)\ \rho_2 R \ddot{x}_T(t)\cos\theta \tag{63}$$

in which $C_0^{Ir}(r,z_1)$ and $C_0^{IIr}(r,z_2)$ = dimensionless coefficients given by

$$C_0^{Ir}(r,z_1) = \frac{r}{R}\frac{z_1}{H} + \frac{R}{H}\sum_{n=1}^{\infty}\frac{G_n}{\Delta_n}\left(A_n\cosh\left(\lambda_n\frac{z_1}{R}\right) + B_n\sin\left(\lambda_n\frac{z_1}{R}\right)\right)\cdot\frac{J_1\left(\lambda_n\frac{r}{R}\right)}{J_1(\lambda_n)} \tag{64}$$

and

$$C_0^{IIr}(r,z_2) = \frac{r}{R}\frac{(z_2 + H_1)}{H} + \frac{R}{H}\sum_{n=1}^{\infty}\frac{G_n}{\Delta_n}\left(C_n\cosh\left(\lambda_n\frac{z_2}{R}\right)\right.$$

$$\left. + D_n\sin\left(\lambda_n\frac{z_2}{R}\right)\right)\frac{J_1\left(\lambda_n\frac{r}{R}\right)}{J_1(\lambda_n)} \tag{65}$$

where

$$A_n = \frac{2}{\lambda_n}\left(\sinh\beta_{1n}\cosh\beta_{2n} + \alpha\cosh\beta_{2n}\sinh\beta_{2n}\right)$$

$$+ (\alpha - 1)\frac{H_1}{r}\cosh\beta_{2i} - \frac{H}{R}\alpha \tag{66a}$$

$$B_n = -\frac{2}{\lambda_n}\left(\cosh\beta_{1n}\cosh\beta_{2n} + \alpha\sinh\beta_{1n}\sinh\beta_{2n}\right) \tag{66b}$$

$$C_n = \frac{2}{\lambda_n}\sinh\beta_{2n} - \frac{H}{R}\cosh\beta_{1n} + (1-\alpha)\frac{H_1}{R}\sinh\beta_{1n}\cosh\beta_{2n} \tag{66c}$$

$$D_n = -\frac{2}{\lambda_n}\cosh\beta_{2n} + (\alpha - 1)\frac{H_1}{R}\sinh\beta_{1n}\cosh\beta_{2n} - \frac{H}{R}\alpha\sinh\beta_{1n} \tag{66d}$$

$$G_n = \frac{2}{\lambda_n^2 - 1} \tag{66e}$$

$$\Delta_n = \cosh\beta_{1n} \cosh\beta_{2n} + \alpha \sinh\beta_{1n} \sinh\beta_{2n} \tag{66f}$$

Convective Component of Solution

The convective component of the hydrodynamic pressures p_1^{cr} and p_2^{cr} must satisfy Laplace's equations

$$\nabla^2 p_1^{cr} = 0 \tag{67a}$$

and

$$\nabla^2 p_2^{cr} = 0 \tag{67b}$$

along with the boundary conditions

$$\left.\frac{\partial p_1^{cr}}{\partial z_1}\right|_{z_1=0} = 0 \tag{68a}$$

$$-\frac{1}{\rho_1}\left.\frac{\partial p_1^{cr}}{\partial r}\right|_{r=R} = 0 \tag{68b}$$

$$-\frac{1}{\rho_2}\left.\frac{\partial p_2^{cr}}{\partial r}\right|_{r=R} = 0 \tag{68c}$$

$$\left.\left(\frac{\partial^2 p_2^{cr}}{\partial t^2} + g\,\frac{\partial p_2^{cr}}{\partial z_2}\right)\right|_{z_2=H_2} = -g\left.\frac{\partial p_2^{ir}}{\partial z_2}\right|_{z_2=H_2} \tag{68d}$$

$$-\frac{1}{\rho_1}\left.\frac{\partial p_1^{cr}}{\partial z_1}\right|_{z_1=H_1} = -\frac{1}{\rho_2}\left.\frac{\partial p_2^{cr}}{\partial z_2}\right|_{z_2=0} \tag{68e}$$

$$\left.\left(\frac{\partial p_1^{cr}}{\partial t^2} + g\,\frac{\partial p_1^{cr}}{\partial z_1} + g\,\frac{\partial p_1^{ir}}{\partial z_1}\right)\right|_{z_1=H_1}$$

$$= \left(\frac{\partial p_2^{cr}}{\partial t^2} + g \frac{\partial p_2^{cr}}{\partial z_2} + g \frac{\partial p_2^{ir}}{\partial z_2} \right) \Bigg|_{z_2=0} \tag{68f}$$

p_1^{cr} and p_2^{cr} are finite at r = 0. (68g)

Again, the method of separation of variables is employed to solve Eqs. (67a) and (67b), and the integration constants are determined from the boundary conditions. Satisfying Eqs. (67a), (67b), (68a), (68b), (68c), (68e) and (68g), the function p_1^{cr} takes the form

$$p_1^{cr}(r,\theta,z_1,t) = \left[\sum_{n=1}^{\infty} D_n(t) \cosh\left(\lambda_n \frac{z_1}{R}\right) \frac{J_1\left(\lambda_n \frac{r}{R}\right)}{J_1(\lambda_n)} \right] \rho_1 R \cos\theta \tag{69a}$$

and p_2^{cr} takes the form

$$p_2^{cr}(r,\theta,z_2,t) = \left[\sum_{n=1}^{\infty} \left(E_n(t) \cosh\left(\lambda_n \frac{z_2}{R}\right) + D_n(t) \sinh\beta_{1n} \sinh\left(\lambda_n \frac{z_2}{R}\right)\right) \frac{J_1\left(\lambda_n \frac{r}{R}\right)}{J_1(\lambda_n)} \right] \rho_2 R \cos\theta \tag{69b}$$

in which $E_n(t)$ and $F_n(t)$ = integration functions that can be determined by satisfying Eqs. (68d) and (68f). Substituting Eqs. (62), (63), (69a) and (69b) into Eq. (68d), one obtains a differential equation

$$\sinh\beta_{1n}\sinh\beta_{2n}\ddot{E}_n + \frac{g\lambda_n}{R}\sinh\beta_{1n}\cosh\beta_{2n}E_n + \cosh\beta_{2n}\ddot{F}_n$$

$$+ \frac{g\lambda_n}{R}\sinh\beta_{2n}F_n = \frac{-g\lambda_n}{R}\frac{R}{H}\frac{G_n}{\Delta_n}\left(\frac{\Delta_n}{\lambda_n} + C_n\sinh\beta_{2n} + D_n\cosh\beta_{2n}\right)\ddot{x}_T(t) \tag{70}$$

and substituting Eqs. (63) and (69b) into Eq. (68f), one obtains another differential equation

$$\cosh\beta_{1n}\ddot{E}_n + (1-\alpha)\frac{g\lambda_n}{R}\sinh\beta_{1n}E_n - \alpha\ddot{F}_n$$

$$= -\frac{g\lambda_n}{R}\frac{R}{H}\cdot\frac{G_n}{\Delta_n}\left[(1-\alpha)\frac{\Delta_n}{\lambda_n} + A_n\sinh\beta_{1n} + B_n\cosh\beta_{1n} - \alpha D_n\right]\ddot{x}_T(t) \tag{71}$$

then, Eqs. (70) and (71) can be solved for $E_n(t)$ and $F_n(t)$.

Taking the Laplace transformation on both sides of Eqs. (70) and (71) and assuming the homogeneous initial conditions for $E_n(t)$ and $F_n(t)$, one obtains two algebraic equations for determination of the Laplace transforms of $E_n(t)$ and $F_n(t)$. The required solutions for $E_n(t)$ and $F_n(t)$ are then obtained by finding the inverse Laplace transforms; then replacing $E_n(t)$ and $F_n(t)$ in Eqs. (69a) and (69b) with the results obtained, one can cast the final expressions into the form presented in Eqs. (11) and (12). The derivation is not difficult but tedious; hence, it is omitted herein. However, the details of the derivation are available in Tang and Chang (1993).

The expressions for $C_{nk}^{Ir}(r, z_1)$ and $C_{nk}^{IIr}(r,z_2)$ for k=1 and 2 are given as follows.

$$C_{n1}^{Ir}(r,z_1) = -\frac{G_n}{\Delta_n}\frac{1}{\Lambda_{n1}}x_1\cosh\left(\lambda_n\frac{z_1}{R}\right)\frac{J_1\left(\lambda_n\frac{r}{R}\right)}{J_1(\lambda_n)} \tag{72a}$$

$$C_{n2}^{Ir}(r,z_1) = \frac{-G_n}{\Delta_n}\cdot\frac{1}{\Lambda_{n2}}y_1\cosh\left(\lambda_n\frac{z_1}{R}\right)\frac{J_1\left(\lambda_n\frac{r}{R}\right)}{J_1(\lambda_n)} \tag{72b}$$

$$C_{n1}^{IIr}(r,z_2) = -\frac{G_n}{\Delta_n} \cdot \frac{1}{\Lambda_{n1}} \left[x_2 \cosh\left(\lambda_n \frac{z_2}{R}\right) + x_1 \sinh\beta_{1n} \sin\left(\lambda_n \frac{z_2}{R}\right)\right]$$

$$\frac{J_1\left(\lambda_n \frac{r}{R}\right)}{J_1(\lambda_n)} \tag{73a}$$

$$C_{n2}^{IIr}(r,z_2) = -\frac{G_n}{\Delta_n} \cdot \frac{1}{\Lambda_{n2}} \left[y_2 \cosh\left(\lambda_n \frac{z_2}{R}\right) + y_1 \sinh\beta_{1n} \sin\left(\lambda_n \frac{z_2}{R}\right)\right]$$

$$\frac{J_1\left(\lambda_n \frac{r}{R}\right)}{J_1(\lambda_n)} \tag{73b}$$

where

$$x_1 = \frac{\Lambda_{n1}\, q_1 - q_2}{\Lambda_{n1} - \Lambda_{n2}} \tag{74a}$$

$$y_1 = \frac{q_2 - \Lambda_{n2}\, q_1}{\Lambda_{n1} - \Lambda_{n2}} \tag{74b}$$

$$x_2 = \frac{\Lambda_{n1}\, q_3 - q_4}{\Lambda_{n1} - \Lambda_{n2}} \tag{74c}$$

and

$$y_2 = \frac{q_4 - \Lambda_{n2}\, q_3}{\Lambda_{n1} - \Lambda_{n2}} \tag{74d}$$

in which

$$q_1 = \frac{s_1 \cosh\beta_{2n} + \alpha s_2}{\Delta_n} \tag{75a}$$

$$q_2 = \frac{s_1 \sinh\beta_{2n}}{\Delta_n} \tag{75b}$$

$$q_3 = \frac{s_2 \cosh\beta_{1n} - s_1 \sinh\beta_{1n} \sinh\beta_{2n}}{\Delta_n} \tag{75c}$$

$$q_4 = \frac{s_2 (1-\alpha) \sinh\beta_{1n} - s_1 \sinh\beta_{1n} \cosh\beta_{2n}}{\Delta_n} \tag{75d}$$

and

$$s_1 = (1-\alpha) \frac{\Delta_n}{\lambda_n} + A_n \sinh\beta_{1n} + B_n \cosh\beta_{1n} - \alpha D_n \tag{76a}$$

$$s_2 = \frac{\Delta_n}{\lambda_n} + C_n \sinh\beta_{2n} + D_n \cosh\beta_{2n} \tag{76b}$$

The characteristic equation for the natural frequencies can be obtained from the associated eigenvalue problem of Eqs. (70) and (71). Letting $\ddot{x}_T(t) = 0$ and $E_n(t) = E_n e^{iwt}$, $F_n(t) = F_n e^{iwt}$ in Eqs. (68) and (69), one obtains two homogeneous equations which are expressed by

$$-\omega^2 \begin{bmatrix} \sinh\beta_{1n} \sinh\beta_{2n} & \cosh\beta_{2n} \\ \cosh\beta_{1n} & -\alpha \end{bmatrix} \begin{Bmatrix} E_n \\ F_n \end{Bmatrix}$$

$$+ \frac{g\lambda_n}{R} \begin{bmatrix} \sinh\beta_{1n} \cosh\beta_{2n} & \sinh\beta_{2n} \\ (1-\alpha)\sinh\beta_{1n} & 0 \end{bmatrix} \begin{Bmatrix} E_n \\ F_n \end{Bmatrix} = 0 \tag{77}$$

It is easy to show that the determinant of Eq. (77) is Eq. (15), the characteristic equation.

APPENDIX II. REFERENCES

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FIGURE CAPTIONS

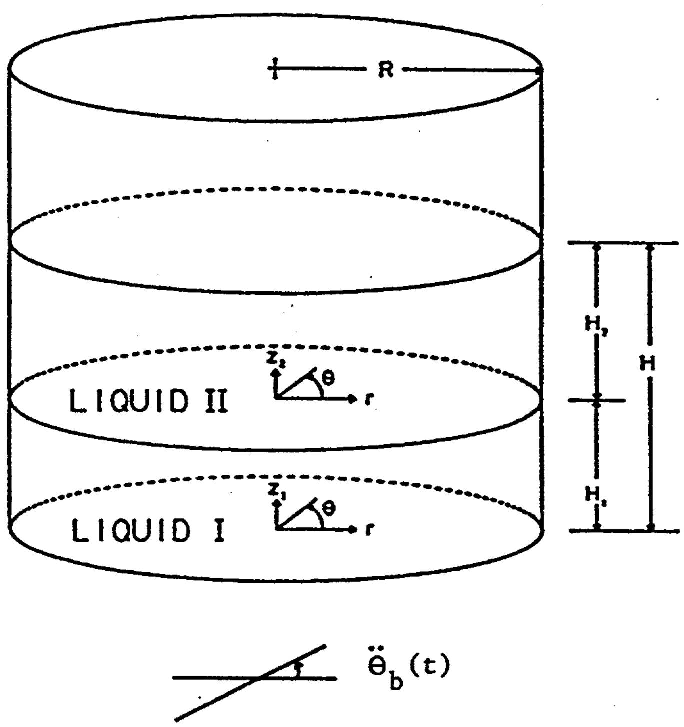

Fig. 1. System Considered

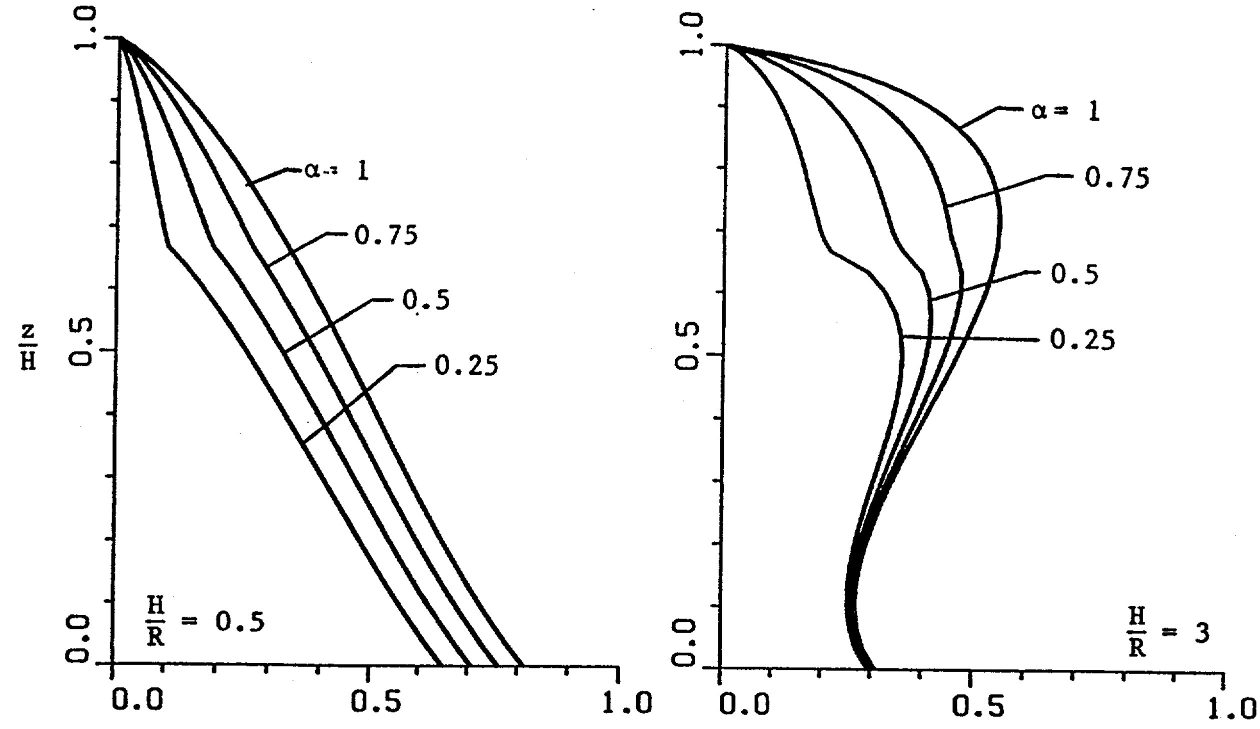

Fig. 2. Impulsive Pressure Exerted on Tank Wall for $H_2/H_1 = 0.5$

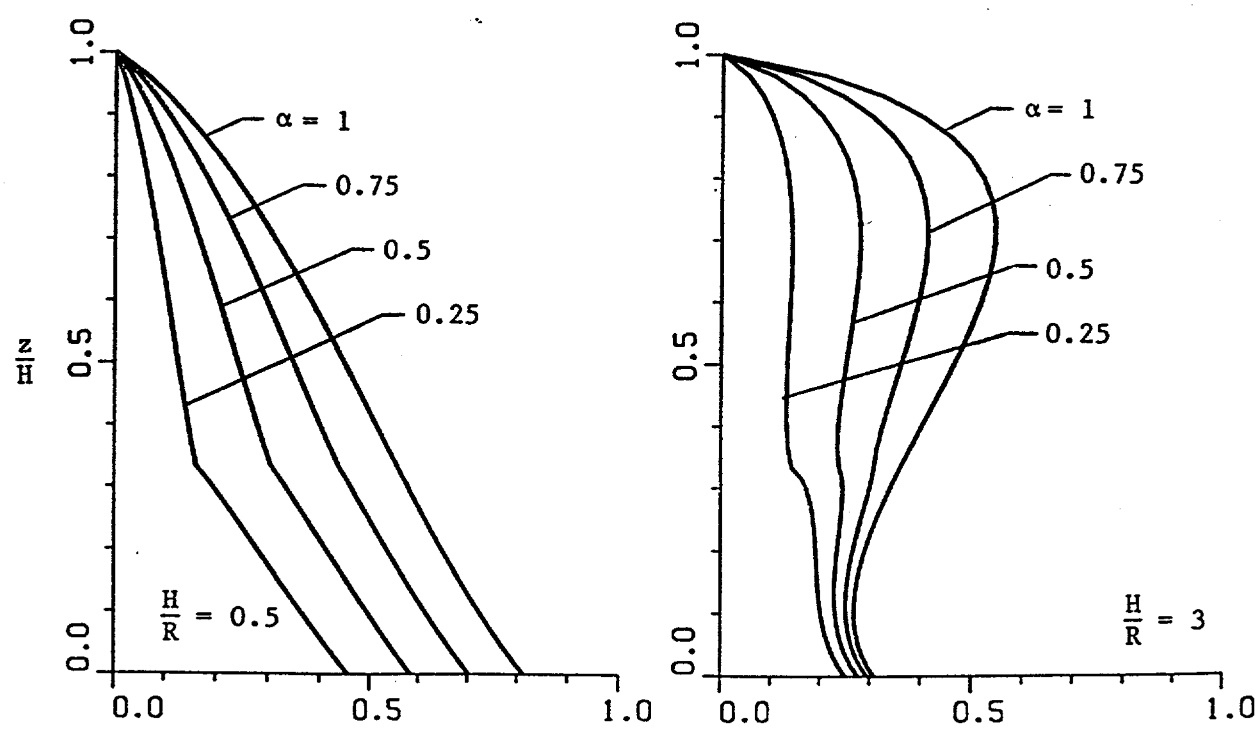

Fig. 3. Impulsive Pressure Exerted on Tank Wall for $H_2/H_1 = 2$

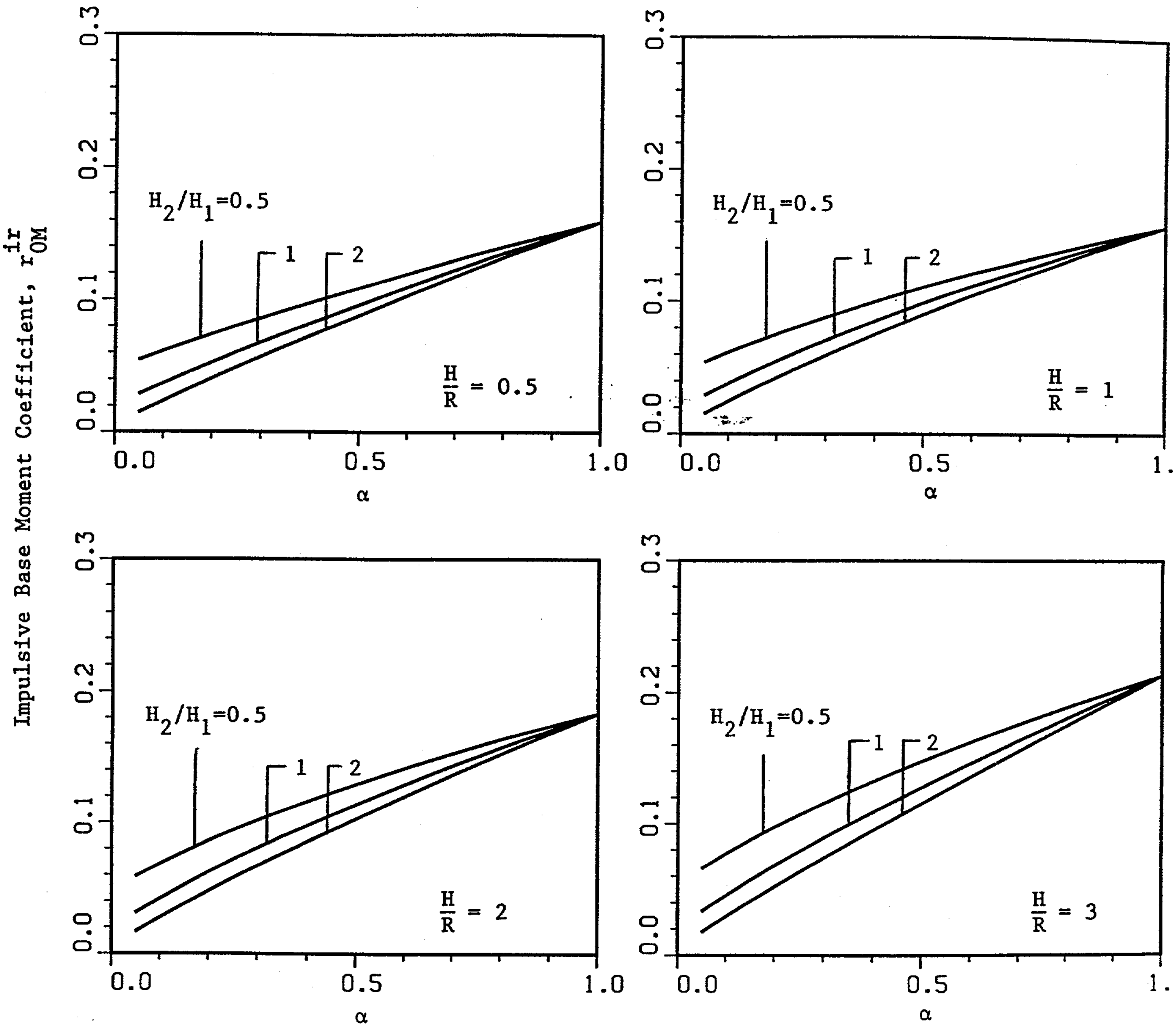

Fig. 4. Effect of Mass Densities on Impulsive Component of Base Moment

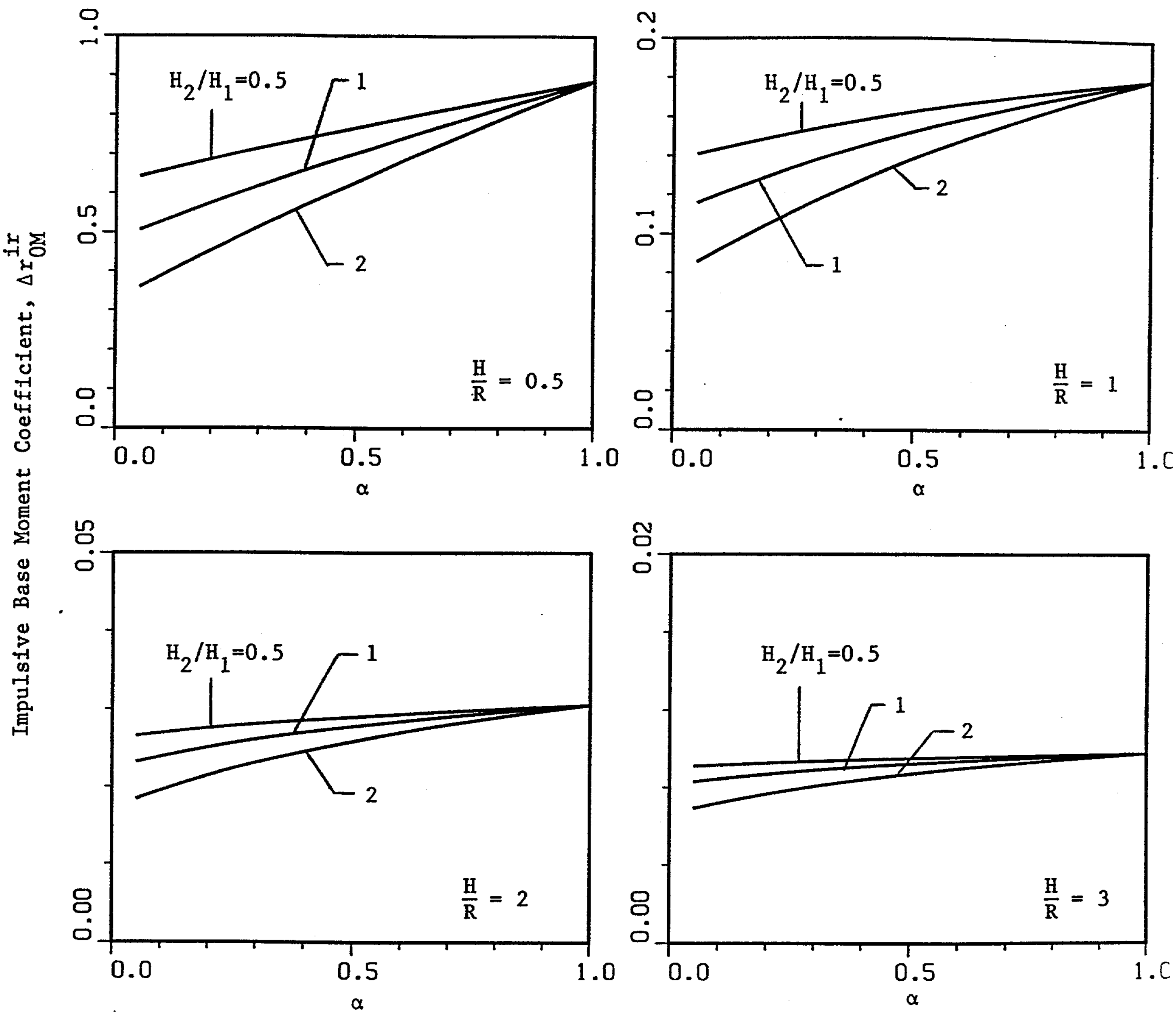

Fig. 5. Effect of Mass Densities on Impulsive Component of Base Moment