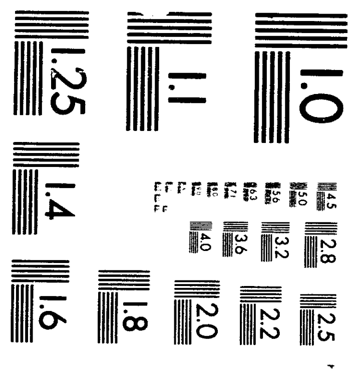
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STOCHASTIC COOLING AND INTRA-BEAM SCATTERING IN RHIC*

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ABSTRACT

During the storage of the heavy ion beam in the Relativistic Heavy Ion Collider (RHIC), the luminosity deterioration due to beam growth and particle loss caused by intra-beam scattering (IBS) is of primary concern. In this paper, we study compensation methods using bunched beam stochastic cooling. With longitudinal and transverse stochastic cooling of 4–8 GHz bandwidth, the longitudinal beam loss resulted from the inadequacy of the rf voltage can be eliminated, and the transverse normalized beam emittance can be confined to about 30π mm·mrad. With such an emittance, the β^* at the crossing point can be lowered under 1 meter without exceeding the transverse aperture limit at the focusing triplets. The achievable luminosity can thus be significantly improved.

1. INTRODUCTION

Hadron beams of species from proton to Au^{79+} will be injected, accelerated, and then stored for 10 hours in the Relativistic Heavy Ion Collider (RHIC), currently under construction at the Brookhaven National Laboratory. For the highly charged ions like Au^{79+}, the luminosity deterioration due to beam growth and particle loss caused by Coulomb intra-beam scattering (IBS)[1] is of primary concern. During the 10-hour storage, beam loss of about 40% is expected due to the IBS beam growth and the inadequacy of the rf voltage. Furthermore, the transverse beam emittance grows from 10π mm·mrad to about 40π mm·mrad. When β^* at the collision point is squeezed down to 1 meter to increase luminosity, transverse beam loss may also occur. Stochastic cooling potentially provides an effective and economic method to reduce the beam size, eliminate beam loss, and thus improve luminosity.

In this paper, we summarize our study on the effects of IBS and the compensation method of bunched-beam stochastic cooling. Expressions for IBS beam growth rates and for transverse cooling rates are presented in sections 2 and 3, respectively. Section 4 discusses the Fokker-Planck approach to evaluate the longitudinal beam loss and beam evolution under IBS and longitudinal stochastic cooling. Conclusions and discussion are given in section 5.

2. INTRA-BEAM SCATTERING BEAM GROWTH

Because of the dispersion that correlates the horizontal closed orbit to the momentum, the scaling behaviour of the IBS growth rates are drastically different at energies low and high compared with the transition energy.[2] At high energies, the rates are approximately independent of the energy. Asymptotically, the horizontal and longitudinal beam amplitudes are linearly related by the average dispersion. At low energies, the beam evolves such that the velocity distribution in the rest frame becomes isotropic in all the directions.

The IBS theories[1] typically assume that the particle distribution remains Gaussian in the six-dimensional (6-D) phase space. Provided that the lattice of the accelerator mainly consists of regular cells, the rates of growth in the rms beam amplitude that are typically expressed in complex integral form, can be simplify into simple analytical form.[2] In particular, the beams in RHIC are stored at energies (γ) much higher than the transition energy (γ_T). Due to injection conditions and transverse coupling, the horizontal and vertical emittances are about the same. In terms of the normalized transverse emittance $\epsilon_{x,y} = \beta\gamma\sigma_{x,y}^2/\beta_{x,y}$ and longitudinal bunch area $S = \pi m_0 c^2\gamma\sigma_s\sigma_p/\beta^3 cA$, the growth rate of the rms

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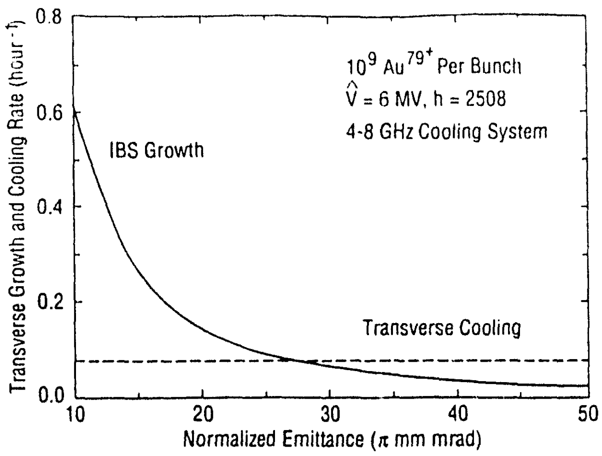

Figure 1: Transverse IBS growth rate and cooling rate as functions of the emittance.

betatron amplitude σ_x and fractional momentum deviation σ_p can be written

$$\begin{bmatrix} \dfrac{1}{\sigma_p}\dfrac{d\sigma_p}{dt} \\ \dfrac{1}{\sigma_x}\dfrac{d\sigma_x}{dt} \end{bmatrix} = \frac{Z^4 N}{A^2}\frac{\pi r_0^2 m_0 c^2 L_c}{16\beta^4\gamma_T\epsilon_x\epsilon_y S}\begin{bmatrix} n_b(1-d^2)/d \\ d/n_c \end{bmatrix} \tag{1}$$

where N is the number of particles per bunch, γ is the Lorentz factor, A is the atomic number, the Coulomb logarithm L_c is about equal to 20, $r_0 = e^2/m_0c^2$, n_b is equal to 1 if the beam is azimuthally bunched, and is equal to 2 if it is not, and n_c is equal to 1 if the horizontal and vertical motions are not coupled, and is equal to 2 if they are fully coupled. The quantity $d = D_p\sigma_p/(\sigma_x^2 + D_p^2\sigma_p^2)^{1/2} < 1$ is the effective ratio between the longitudinal and horizontal total amplitude, where D_p is the average dispersion in the cell.

The growth rates are shown to be linearly proportional to the beam intensity, and are strongly dependent ($\sim Z^4/A^2$) on the charge state Z of the particle. Except for the factor d, the rates are inversely proportional to the 6-D phase-space area. The dependence on the beam energy is usually weak. After the initial stage of storage, the asymptotic configuration $n_b n_c \sigma_x^2 \approx D_p^2\sigma_p^2$ will be approximately reached.

In the case of storage of 10^9 Au^{79+} ions per bunch in RHIC, the beam of initial area 0.3 eV·s/u fills up the entire rf bucket of area 1.28 eV·s/u in less than 30 minutes. Thereafter, beam loss occurs in longitudinal dimension. Without cooling, the transverse emittances grow in 10 hours from 10π mm·mr to about 40π mm·mr. Assuming full transverse coupling, the solid line in Fig. 1 shows the transverse IBS growth rate at different emittances.

3. TRANSVERSE STOCHASTIC COOLING

With a stochastic cooling system of properly chosen bandwidth, the IBS growth in transverse beam emittances can be compensated. The equilibrium state can be reached when the cooling rate is equal to the IBS growth rate.

With the highly charged ions, the system thermal noise is often negligible. The transverse cooling rate can thus be obtained using the Fokker-Planck approach[3, 4]

$$\begin{aligned} \frac{1}{\sigma_{x,y}}\frac{d\sigma_{x,y}}{dt} = & -\frac{f_0^2\sin(\nu_x\theta^{PK})}{2}\sum_{m=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} G(m^{\pm}\omega_0 - l\Omega_i)e^{il\Omega_s\theta^{PK}/\omega_0}J_l^2(m\omega_0\tau_i) \\ & + \frac{\pi f_0^4}{4}\sum_{l=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\left.\frac{\rho(J')}{|l|\left|\frac{d\Omega_s(J')}{dJ'}\right|}\right|_{\Omega_s(J')=k\Omega_s(J)/l}\left\{|G_D(-)|^2 + |G_D(+)|^2 - 2\,[\text{illegible}]\,G_D(+)]\right\}, \end{aligned} \tag{2}$$

where

$$G_D(\pm) = \sum_{m=1}^{\infty} G(m^{\pm}\omega_0 \pm l\Omega_j)J_{\mp l}(m\omega_0\tau_j)J_{\mp k}(m\omega_0\tau_i), \tag{3}$$

θ^{PK} is the distance in azimuthal angle between the pick ups and the kickers, $\omega_0 = 2\pi f_0$ is the revolution frequency, Ω_s is the synchrotron-oscillation frequency, J_l is the Bessel function of lth order, $\rho(J)$ is the density in J, and $2\pi J$ is the longitudinal phase-space area enclosed by the particle trajectory. The quantity $\left|\frac{d\Omega_s}{dJ}\right|$ appeared in Eq. 2 is a monotonically increasing function of J,

$$\frac{d\Omega_s}{dJ} = -\frac{\pi^2 C_W}{8}\frac{1}{k^2\mathrm{K}^3(k)}\left[\frac{\mathrm{E}(k)}{1-k^2} - \mathrm{K}(k)\right] \sim -\frac{C_W}{4}, \text{ with } J = \frac{4}{\pi}\sqrt{\frac{C_\phi}{C_W}}\left[(k^2-1)\mathrm{K}(k) + \mathrm{E}(k)\right] \sim \sqrt{\frac{C_\phi}{C_W}}k^2, \tag{4}$$

where $C_W = h^2\omega_0^2\eta/2E\beta^2$, $C_\phi = Ze\hat{V}/\pi h$, $\hat{V}$ is the peak voltage, h is the harmonic number of the rf system, η is the slip factor, and K and E are complete elliptical integrals of first and second kind. Here in Eq. 4, the sign $\sim$ denotes the value in the small-amplitude limit. The summation on the revolution bands in Eq. 2 is performed over the system bandwidth, while the summation on the synchrotron side-bands is actually performed from $l = 1$ to $m\omega_0\tau$, with $\tau = \arccos\left(1-2k^2\right)/h\omega_0 \sim 2k/h\omega_0$ the oscillation amplitude in time. The factor $e^{il\Omega_s\theta^{PK}/\omega_0}$ represents the phase slip that non-synchronous particles experience during their passage from the pick-up to the kickers. In order to minimize this undesirable "mixing", the distance between the pick-up and the kickers should be chosen such that $\theta^{PK}n\hat{\tau}\Omega_0 \ll 1$, where $\hat{\tau}$ is the half bunch length in time, Ω_0 is the zero-amplitude synchrotron frequency, and n is the average harmonic number of the cooling system. The optimum cooling rate can be obtained from Eq. 2 by using numerically methods. With a bandwidth of 4–8 GHz, the dash line in Fig. 1 shows the optimum cooling rate that can be achieved when the rf bucket is nearly full (Fig. 2b). The transverse emittance at equilibrium is about 30π mm·mr.

For the convenience of order-of-magnitude estimate, the optimum cooling rate is[5] estimated

$$\left.\frac{1}{\sigma_{x,y}}\frac{d\sigma_{x,y}}{dt}\right| = \frac{\langle l\rangle^2}{8\pi\langle\rho(J)\rangle}\left|\frac{d\Omega_s}{dJ}\right| = \frac{nf_0}{\pi^{3/2}hN\langle M\rangle}\frac{\langle J\rangle^{3/2}}{J_{max}^{3/2}} \tag{5}$$

where $2\pi J_{max} = 8\sqrt{C_\phi/C_W}$ is the bucket area, $\langle\ \rangle$ denotes the average over all the particles, and $\langle M\rangle = \omega_0/\langle l\rangle\Omega_0$ is the mixing factor. Cooling becomes very difficult when the bunch area $2\pi\langle J\rangle$ is small compared with the bucket area.

4. BEAM LIFETIME IMPROVEMENT WITH LONGITUDINAL COOLING

Because of the severe boundary limitation and the particle loss in longitudinal direction, the beam evolution can not be adequately described by the growth rates. The transport equation is thus derived to describe the time evolution of the longitudinal density distribution in the presence of IBS and stochastic cooling.

A transport equation in terms of action-angle variables can be obtained by averaging the 6-D Fokker-Planck equation over all transverse variables x, x', y, and y'. Because the time for IBS and stochastic cooling to produce appreciable effect is typically much longer than the synchrotron-oscillation period, which is again much longer than the correlation time of the collision process, this equation can be further reduced by averaging over the angle variable for one synchrotron-oscillation period

$$\frac{\partial\rho}{\partial t} = -\frac{\partial}{\partial J}(F\rho) + \frac{1}{2}\frac{\partial}{\partial J}\left(D\frac{\partial\rho}{\partial J}\right), \quad \text{with} \quad \begin{cases} J = 0: & -F\rho + \dfrac{D}{2}\dfrac{\partial\rho}{\partial J} = 0\,, \\ J = J_{max}: & \rho = 0\,. \end{cases} \tag{6}$$

For intra-beam scattering, the so called coefficient of dynamic friction F and diffusion D are obtained by assuming an 4-D transverse Gaussians distribution and[4] integrating over all the transverse components of the test particle

$$F(J) = \int\frac{2dz}{\pi R}\int_0^{\frac{1}{4}}dQ\left.\frac{\partial W}{\partial J}\right|_\phi^{-1}(Q,J)\int_{J_{min}}^{j}\left.\frac{\partial W}{\partial J}\right|_\phi(Q',J')\left[A_F(\lambda_1) + A_F(\lambda_2)\right]\rho(J')dJ' \tag{7}$$

and

$$D(J) = \int\frac{2dz}{\pi R}\int_0^{\frac{1}{4}}dQ\left[\left.\frac{\partial W}{\partial J}\right|_\phi^{-1}(Q,J)\right]^2\int_{J_{min}}^{j}\left.\frac{\partial W}{\partial J}\right|_\phi(Q',J')\left[A_D(\lambda_1) + A_D(\lambda_2)\right]\rho(J')dJ' \tag{8}$$

where $2\pi R$ is the circumference,

$$A_F(\lambda) = -\frac{2Z^4r_0^2L_cE}{A^2\beta^2\gamma^4}\frac{I_F(\lambda)}{\sigma_x\sigma_y}, \quad A_D(\lambda) = \frac{Z^4r_0^2L_cE^2}{A^2\gamma^3h\omega_0}\frac{I_D(\lambda)}{\sigma_x\sigma_y}, \quad \lambda_{1,2} = \frac{h\omega_0 a}{\gamma\beta^2E}(W \mp W'), \quad a = \frac{1}{2}\sqrt{\frac{6\beta\gamma\beta_{x,y}}{\epsilon_{x,y}}}, \tag{9}$$

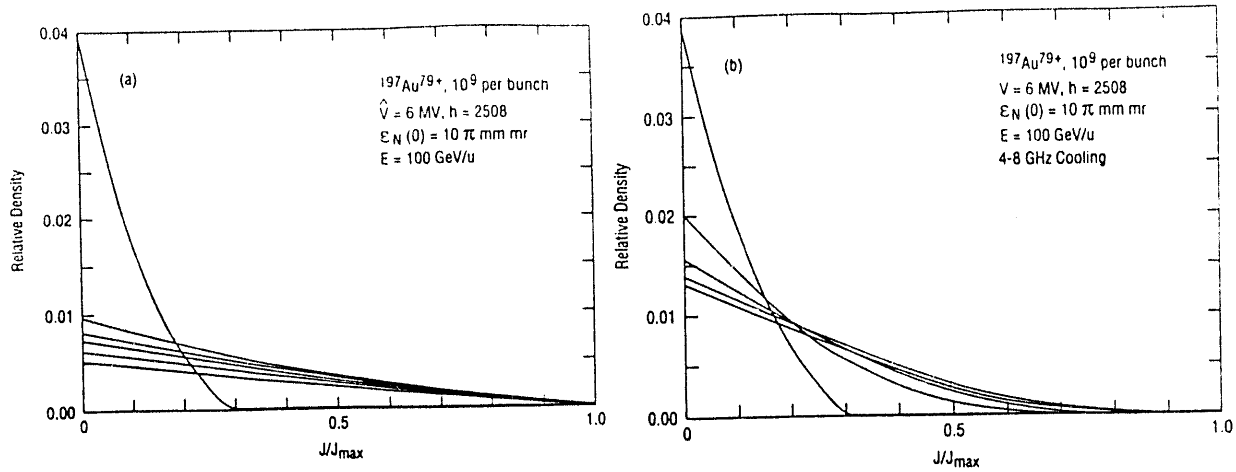

Figure 2: Evolusion of the longitudinal distribution function a) under IBS; b) under IBS when both longitudinal and transverse coolings are applied.

$$\left.\frac{\partial W}{\partial J}\right|_{\phi}^{-1} = 8k\ \mathrm{K}(k)\cos 2\pi Q\ \left[1-4\xi\sin^2 2\pi Q + O(\xi^2)\right],\quad \xi = \exp\left[-\pi \mathrm{K}'(k)/\mathrm{K}(k)\right],\quad \mathrm{K}'(k)=\mathrm{K}(\sqrt{1-k^2}). \tag{10}$$

The first integrals in Eqs. 7 and 8 represents the average over the machine lattice; the second integral represents the average over synchrotron-oscillation period; while the third integral describes particles of different action J' involved in the collision. The integration over J' is performed such that $k(J')\sin 2\pi Q' \approx \sin[\phi(Q,J)/2]$, extending from J_{min} to the bunch edge $\hat{J}$, with $k(J_{min}) \approx [\sin\phi(Q,J)/2]$. For a round beam with $\beta_x D_p' + \alpha_x D_p \sim 0$, we have

$$I_F(\lambda) = 2a^2\mathrm{sgn}(\lambda)\chi\left\{1-\sqrt{\pi}|\lambda|e^{\lambda^2}\left[1-\Phi(\lambda)\right]\right\},\ I_D(\lambda) = a\chi\left\{\sqrt{\pi}(1+2\lambda^2)e^{\lambda^2}\left[1-\Phi(\lambda)\right]-2|\lambda|\right\}, \tag{11}$$

where $\chi = e^{-(D_p\gamma\lambda/2\sigma_x)^2}$, Φ is the error function, and $\mathrm{sgn}(\lambda)$ is 1 if $\lambda \geq 0$, and is -1 if otherwise.

For longitudinal stochastic cooling, the corresponding coefficient of coherent correction F and Schottky diffusion D can be obtained[3, 4]

$$F(J) = \frac{Z^2e^2\omega_0}{\pi}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\frac{l}{m}J_l^2\left[m\omega_0\tau(J)\right]Re\left[G_F(m,+l)-G_F(m,-l)\right] \tag{12}$$

with

$$G_F(m,\pm l) = G\left[mf_0 \pm l\Omega_s(J)\right]\ e^{\pm il\Omega_s(J)\theta^{PK}/\omega_0}, \tag{13}$$

and

$$D(J) = \frac{NZ^4e^4f_0^2}{\pi}\sum_{l=1}^{\infty}\sum_{k=1}^{\infty}\left.\frac{k^2\rho(J')}{l\left|\frac{d\Omega_s(J')}{dJ'}\right|}\right|_{\Omega_s(J')=k\Omega_s(J)/l}\left\{|G_D(k,l)|^2+|G_D(-k,-l)|^2-2Re\left[G_D(k,l)G_D(-k,-l)\right]\right\} \tag{14}$$

where the double summation on l and k represents synchrotron sideband overlapping, and

$$G_D(k,l) = \sum_{m=1}^{\infty}\frac{1}{m}G\left[m\omega_0 + l\Omega_s(J')\right]J_k\left[m\omega_0\tau(J)\right]J_l\left[m\omega_0\tau(J')\right]. \tag{15}$$

With F and D obtained for various mechanisms, Eq. 6 can be solved numerically with any given initial distribution $\rho(J)$ at $t = 0$. Note that at the vicinity of the separatrix, the synchrotron-oscillation period approaches infinity. Consequently, the assumptions that Eq. 6 is based upon is no longer valid (The quantity ξ in Eq. 10 is large compared to 1.). In our numerical calculation, we have conservatively assumed that the particle is lost if its action J exceeds 95% of the bucket value J_{max}. Our study also shows that the results are not sensitive to the closeness of the chosen boundary to the separatrix. This is partly due to the low particle density near the boundary.

Fig. 2a shows the time evolution of $\rho(J)$ during the 10-hour period with 2.5-hour time interval in the absence of cooling. The voltage of the 196 MHz rf system is kept at the maximum value 6 MV. The

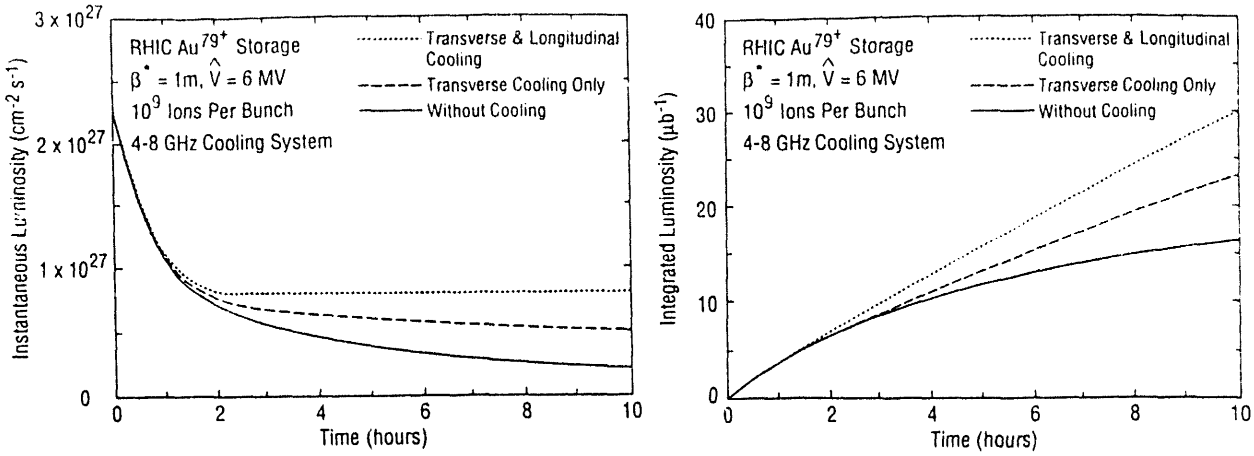

Figure 3: Improvements on a) instantaneous and b) integrated luminosity in RHIC when stochastic cooling is applied.

total beam loss is about 40%. The asymptotic distribution in longitudinal phase space is found to be Gaussian-like, insensitive to the initial conditions.

With both longitudinal and transverse stochastic cooling of 4-8 GHz bandwidth, Fig. 2b shows that the bunch approximately reaches the equilibrium state in several hours. Longitudinal beam loss is essentially eliminated, while the transverse emittance is kept to about 30π mm·mr. With a reasonable number (about 128) of kickers, the peak power needed is of the order of kW. Since the mixing factor is large compared to 1, full-turn delay between pick-ups and the kickers is plausible for this bandwidth.

Figs. 3a and b show the improvement in the instantaneous and integrated luminosity when stochastic coolings in different planes are applied. With both transverse and longitudinal cooling, the instantaneous luminosity can be significantly increased for the later period of the storage. The integrated luminosity is increased by about a factor of 2 over the entire 10-hour period.

5. CONCLUSIONS AND DISCUSSION

Bunched-beam stochastic cooling in both longitudinal and transverse planes provides an effective method to compensate for the beam growth, particle loss, and luminosity deterioration caused by IBS. With longitudinal and transverse cooling of bandwidth 4–8 GHz, the longitudinal beam loss resulted from the inadequacy of the rf voltage can be eliminated, and the transverse normalized beam emittance can be confined to about 30π mm·mrad. With such an emittance, the β^* at the crossing point can be lowered under 1 meter without exceeding the transverse aperture limit at the focusing triplets. The integrated luminosity can be increased by a factor of 2 during the 10-hour storage period.

To accommodate for the future upgrade of the beam intensity in RHIC, a wider cooling bandwidth of 8–16 MHz is desirable. To optimize the performance, the delay between the pick ups and kickers may be chosen to be 1/6 of the machine circumference. More detailed technical aspects, however, are still to be explored.

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