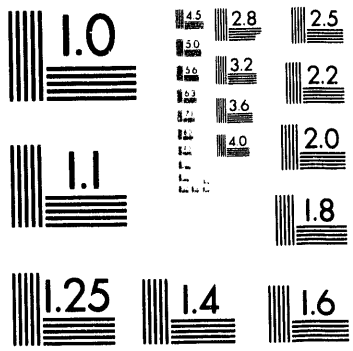
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RESEARCH ON PRIMAL-DUAL INTERIOR POINT ALGORITHMS FOR MATHEMATICAL PROGRAMMING

Final Technical Report

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1 Overview

Today the interior-point methods of choice for general linear programming are primal-dual infeasible interior-point algorithms. These methods are computationally superior to other interior-point methods and frequently outperform the simplex method in solving large-scale problems, as have been demonstrated by a number of numerical studies. However, at the time our proposal was written, there was a big gap between theoretical algorithms and practical algorithms. The major goals of the project were to narrow the gaps between theory and practice of primal-dual interior-point methods. It is extremely satisfying that these goals have been accomplished and our project has made substantial contributions to the recent advances in this area.

The project has been highly productive and fruitful. During the two-year project period, the PI has authored or co-authored a total of sixteen papers (including two essays). Most of these works were the results of joint efforts by the PI and his collaborators. The most significant achievements are the following:

- The PI played a leading role in the joint work with Richard Tapia [1] on constructing the first superlinear and polynomial primal-dual algorithm. This work, together with PI's another work with Richard Tapia and John Dennis, led to a burst of intensive research activities which significantly advanced the state-of-the-art in the field.

- The PI's recent work [11] on infeasible interior-point methods established the first polynomial result for today's interior-point methods of choice. This work again stimulated a new wave of research activities and advanced the field.

- More recently, the PI [15] established polynomial complexities for the Mehrotra-type predictor-corrector methods which have been widely accepted today as the most practically efficient methods. This work has laid a solid theoretical foundation for these methods.

2 Summary of Completed Work

The major theoretical and computational advancements achieved during the project period are summerized below. In each of the following paragraphs the term "we" refers to the author or co-authors of the cited publication(s) in that paragraph.

Theoretical Results:

- We completed the construction of the first primal-dual interior-point algorithm that possesses both polynomial and superlinear convergence [1].
- We established conditions under which the iterates of a primal-dual algorithm converges, and proved that those conditions are attainable [6].
- We proved the the Mizuno-Todd-Ye predictor-corrector algorithm, while possessing $O(\sqrt{n}L)$-iteration complexity, is also 2-step quadratically convergent, without any extra assumptions on the problem or the iteration sequence [7, 9].
- We refined the theory for the superlinear and quadratic convergence of primal-dual interior-point algorithms. The improved theory employs much weaker assumptions than the previous one does [8].
- We constructed the first primal-dual interior-point algorithm, to our best knowledge, that guarantees to converge to the analytic center of the solution set [10].
- We established the first polynomial complexity result for infeasible interior-point algorithms applied to a wide range of problems [11].
- We established one-step Q-superlinear convergence rate arbitrarily close to 2 for infeasible interior-point algorithms for linear programming [12].

- We formulated a primal-dual Newton interior-point method for general nonlinear programming problems and established global and local convergence properties for this method [14].

- We established in [15] the first polynomial complexity results for the Mehrotra-type predictor-corrector methods which have been widely regarded today as the most practically efficient methods. This work has laid a solid theoretical foundation for these methods.

Computational Results:

- We thoroughly studied the computational behavior of various indicators in the context of primal-dual interior-point algorithms and concluded that Tapia's indicator was the most effective one in the primal-dual framework [2]. Indicators play a key role in the finite terminations of interior-point algorithms for linear programming.

- We performed extensive numerical experiments to study various strategies to enforce fast local convergence of primal-dual interior point methods. A technical report By El-Bakry, Tapia and the PI is in preparation, describing our numerical results.

- In collaboration with Tapia, Sorensen and El-Bakry of Rice University, we have devised an iterative procedure for solving linear systems resulted from primal-dual infeasible interior-point algorithms for LP. This procedure is based on applying a variant of Arnoldi's method to a properly scaled symmetric indefinite system with a pre-conditioning scheme. Preliminary testing results have been quite positive. A technical report is in preparation

- In collaboration with Tapia and El-Bakry of Rice University, we have developed

an experimental code and have been experimenting with general nonlinear programming problems. Considerable understanding and insight have been gained from these experiments [14].

Graduate Education:

During the project period, the PI as a co-advisor directed a Ph.D. student, Mr. Ming Chen, who was supported in part by this project as a Research Assistant during the 1991-1992 academic year. Mr. Chen received his Ph.D. degree from the Department of Mathematics and Statistics at UMBC in May 1992 and is currently employed by B-K Dynamics, a computer software company specialized in decision-support systems, as a senior system analyst. In the 1992-1993 academic year, the project supported another doctoral student, Mr. Detong Zhang, through a Research Assistantship. While under the support of this project, he wrote two research papers with the PI.

3 List of Publications

The following list contains the papers authored or co-authored by the PI and completed in the current project period.

1. Y. Zhang and R. A. Tapia. *A superlinearly convergent polynomial primal-dual interior point algorithm for linear programming*, 1991. To appear in *SIAM Journal on Optimization.*

2. A. El-Bakry and R. Tapia and Y. Zhang. *A study on the use of indicators in identifying zero variables for interior point methods*, 1991. Tentatively accepted by *SIAM Review.*

3. R. Tapia, Y. Zhang, M. Saltzman and A. Weiser. *The predictor-corrector interior point method as a composite Newton method.* Technical Report TR90-17 (revised Sept. 1991), Dept. Mathematical Sciences, Rice University, 1991.

4. Y. Ye, R. A. Tapia and Y. Zhang. *A superlinearly convergent $O(\sqrt{n}L)$-iteration algorithm for linear programming.* Technical Report TR91-22, Dept. Mathematical Sciences, Rice University, 1991.

5. J. Ji, F. Potra and R. A. Tapia and Y. Zhang. *An interior-point method for linear complementarity problems with polynomial complexity and superlinear convergence.* Technical Report TR91-23, Dept. Mathematical Sciences, Rice University, 1991.

6. R. A. Tapia, Y. Zhang and Y. Ye. *On the convergence of the iteration sequence in primal-dual interior-point methods.* Technical Report TR91-24, Dept. Mathematical Sciences, Rice University, 1991.

7. Y. Ye, O. Güler, R. A. Tapia and Y. Zhang. *A quadratically convergent $O(\sqrt{n}L)$-iteration algorithm for linear programming*, 1991. To appear in *Mathematical Programming.*

8. Y. Zhang and R. A. Tapia. *Superlinear and quadratic convergence of primal-dual interior point algorithms for linear programming revisited*, 1991. To appear in *Journal of Optimization Theory and Application.*

9. Y. Zhang and R. A. Tapia. *On the convergence of interior-point methods to the center of solution set in linear programming.* Research Report 91-19, Dept. of Mathematics and Statistics, UMBC, 1991.

10. Y. Zhang and A. El-Bakry. *A modified predictor-corrector algorithm for locating weighted centers in linear programming.* Technical Report TR92-19, Dept. Mathematical Sciences, Rice University, 1992. To appear in *JOTA.*

11. Y. Zhang. *On the convergence of a class of infeasible interior-point methods for the horizontal linear complementarity problem*, Research report 92-07, Dept. of Mathematics and Statistics, UMBC, 1992. To appear in *SIAM Journal on Optimization.*

12. Y. Zhang and D. Zhang. *Superlinear Convergence of Infeasible Interior-Point Methods for Linear Programming*, Research Report 92-15. Dept. of Mathematics and Statistics, UMBC, 1992. To appear in *Mathematical Programming.*

13. R. A. Tapia and Y. Zhang. *On the Quadratic Convergence of the Singular Newton's Method*, in *SIAG/OPT Views-and-News, A Forum for the SIAM Activity Group on Optimization*, Issue No. 1, Fall, 1992.

14. A. El-Bakry, R. A. Tapia, T. Tsuchiya and Y. Zhang. *On the formulation of the primal-dual Newton interior-point method for nonlinear programming.* TR92-40, Dept. Computational and Applied Math., Rice University, 1992.

15. Y. Zhang and D. Zhang. *On Polynomiality of the Mehrotra-Type Predictor-Corrector Interior-Point Algorithms*, Research Report 93-12. Dept. of Mathematics and Statistics, UMBC, 1993.

16. Y. Zhang. Interior-point methods for linear programming: the state of the art. In the Proceeding for *The 1993 Conference on Scientific and Engineering Computing for Young Chineses Scientists.* August 16-21, 1993, Beijing China.

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