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7\. Abstract

Availability of software packages to support the engineering evaluation of alternatives activity was investigated. Two potentially useful programs, Expert Choice™ and PCM™, which met the requirements were identified and evaluated. The proper functioning of the two programs was validated by comparing the results from the two programs against each other and against an independent implementation of the underlying algorithm. Both programs produced the correct ranking of options for the validation data set.

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VERIFICATION AND VALIDATION

OF

DECISION SUPPORT SOFTWARE:

EXPERT CHOICE™ AND PCM™

Q. H. Nguyen
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November 4, 1994

Prepared by

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VERIFICATION AND VALIDATION OF DECISION SUPPORT SOFTWARE: EXPERT CHOICE™ AND PCM™

Q. H. Nguyen
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ABSTRACT

This report documents the verification and validation of two decision support programs: EXPERT CHOICE™ and PCM™. Both programs use the Analytic Hierarchy Process (AHP) -- or pairwise comparison technique -- developed by Dr. Thomas L. Saaty. In order to provide an independent method for the validating the two programs, the pairwise comparison algorithm was developed for a standard mathematical program. A standard data set -- selecting a car to purchase -- was used with each of the three programs for validation. The results show that both commercial programs performed correctly.

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VERIFICATION AND VALIDATION OF DECISION SUPPORT SOFTWARE: EXPERT CHOICE™ AND PCM™

1.0 INTRODUCTION

1.1 NEED

Evaluation of alternatives is one of the steps in the systems engineering approach to problem solving and project planning. A number of methods have been used to assist in the evaluation of alternatives process, but they all require some degree of manual calculation. It was considered that a computer based decision support method was needed which would automate judgements and calculations, and produce printed reports suitable for documenting the decision process.

1.2 PLAN

The general plan to fulfill the above need was:

- Establish desirable features and capabilities for the software;
- Determine what commercial software packages were available which might meet the requirements;
- Evaluate available software by comparing literature;
- Acquire one, or more software packages for hands-on evaluation;
- Select one or two software packages for use;
- Validate the selected software.

2.0 REQUIREMENTS

2.1 ESSENTIAL FEATURES

Decision support software to aide in the evaluation of alternatives must have features to automate the process and provide the flexibility to accommodate a broad range of problems. Specific features that the software should have include:

- User data input screens for comparisons;
- Permit input of both subjective and quantitative judgements;
- Allow multiple levels, *e.g.*, Criteria categories, Criteria, Alternatives;
- Produce numerical results suitable for establishing the relative ranking of the alternatives;
- Allow sending reports of comparisons and results to either a printer or a disk file;
- Cost less than $500.

2.2 SYSTEM REQUIREMENTS

The decision support software must run on the standard administrative workstation consisting of:

- Intel® 80x86, or equivalent processor based hardware with 4MB memory;
- MS-DOS®, or equivalent operating system;
- Hard and floppy disk drives.

3.0 SEARCH FOR AVAILABLE SOFTWARE

3.1 THE APPROACH

The search for software packages to support the decision making process followed two courses:

1. Search of a registered commercial software database, and
2. Search of catalogs of distributors of public domain and <u>Shareware</u>™ software.

3.2 FINDINGS

3.2.1 Software Database

The available software database was Data Sources®. This database is on CD-ROM (compact disk, read only memory) and the supporting software allows extensive search and report capabilities. The *category* search mode was used with "Decision Support" as the category. Approximately 160 entries were found, and summary reports were printed for study. The majority of the reports indicated the software was specific to business financial decisions. Others either required a mini or mainframe computer, or were too expensive -- as much as $25,000 per copy. Thirteen candidates remained after the initial culling.

3.2.2 Shareware™

Two Shareware™ distributor catalogs were examined for applicable software packages. The distributors were:

Reasonable Solutions	1221 Disk Drive Medford, Oregon 97501-6636 Phone: 800-876-3475
The Software Labs	100 Corporate Pointe, Suite 195 Culver City, California 90231 Phone: 800-567-7900

The Reasonable Solutions catalog did not list any programs that seemed suitable. The Software Labs catalog had only one listing for applicable programs; the Decision Analysis System (DAS™).

3.3 SELECTED SOFTWARE

3.3.1 Pairwise Comparison Method - PCM™

The disk for the Shareware™ program DAS™ was obtained first. The package consists of two decision support programs; the Matrix Method, MM™, and the Pairwise Comparison Method, PCM™. The on-disk documentation indicated MM™ required quantitative data, while PCM™ used subjective judgements. A quick evaluation of the PCM™ software -- a crippled version of a commercial package -- indicated that it would meet the majority of the requirements. The implemented pairwise comparison technique (or Analytic Hierarchy Process) seemed to be especially applicable to the types of evaluation of alternatives generally encountered in systems engineering.

3.3.2 Expert Choice™

The reports for the thirteen candidate commercial software packages (see Section 3.2.1, above) were reevaluated to see if any of the software packages implemented the pairwise comparison technique. Expert Choice™ was the only package that clearly implemented that technique, so more detailed literature was requested. Evaluation of that literature indicated the program would meet the requirements, and a copy was purchased.

4.0 VERIFICATION

4.1 THE PAIRWISE COMPARISON TECHNIQUE, OR ANALYTIC HIERARCHY PROCESS

The pairwise comparison technique is most beneficial when there is a lack of quantitative data about potential solutions to a problem, *i.e.*, when judgments must be primarily subjective. And, this is the usual situation in most instances of evaluation of engineering alternatives, particularly during early phases of an activity. The technique requires that a problem solution be decomposed into an abstract hierarchical model. The modelling process involves setting up levels where each level contains a set of elements, or nodes that are related to the elements in the immediately higher level, *i.e.*, the resultant model has a tree structure. The process is usually somewhat iterative, but involves the following steps:

- Make a clear and concise statement of the goal;
- Establish the evaluation criteria (which can be more than one level);
- State the alternatives, or options to meeting the goal.

The objective is to arrive at a set of weights or rankings for elements at the last level -- usually the alternatives -- which reflect, as best as possible, their relative impact on fulfilling the goal of the hierarchy. (Note that the pairwise comparison technique is also called the Analytic Hierarchy Process [AHP]).

The mathematical basis for the pairwise comparison technique is described by Thomas L. Saaty in *Multicriteria Decision Making -- The Analytic Hierarchy Process*, (Saaty-1980). The AHP algorithm employs standard matrix mathematics to calculate the intermediate and final weights or priorities. Judgements are derived by comparing each possible pair of elements at each level with respect to the immediately higher level node. The analyst assigns weighting values to each pair comparison that reflects the "goodness" of one pair member relative to the other member. This pairwise comparison is considerably easier to accomplish than attempting to assign a weighting value to one member of a group which is relative to all other members of the group. Another advantage of the technique is that the consistency of judgements can be determined as a measure of the degree of bias in the results. It is difficult to bias the results towards a "preferred" option and still maintain consistency in the pairwise judgements.

4.2 EXPERT CHOICE™

4.2.1 User Documentation

Expert Choice™ comes with a 440 page *User Manual* and a supplemental 112 page *Walk-Through* manual. The *User Manual* has thirteen chapters covering the gamut from introduction to tracking down problems. Modelling is covered in two chapters, and a Tutorial is given in Chapter 12. Examples are provided in the Appendix as well as throughout the chapters.

The manual is generally well written and coverage of the operating details is adequate. Numerous examples make it easier to grasp the intent of the discussion.

4.2.2 Usability

The program presents a graphical interface for the user, with pull down command menus. A mouse is not required, but some operations are easier when a mouse is used. In general, the program is considered easy to use and the screen layouts are logical. However, the user must refer to the manual for some command options because their meaning is not intuitively obvious.

The pairwise comparisons can be accomplished in either the Numerical or the Verbal mode. Comparisons can be on the basis of Preference, Likelihood or Importance, depending on the nature of the elements and the way a person thinks. These options can ease input of comparison rankings, but have no direct affect on numerical calculations.

The basic element for the program is a node. The underlying structure for the input data is a tree, so a node may represent either a branch or a leaf on a branch. The lowest levels may be either criteria or alternatives. A "Ranking" mode can be used when a large number of alternatives are to be evaluated. For this mode, the lowest nodes are criteria categories, *e.g.*, under a Hardware Cost criteria, one might have five categories of >100K$, >50K to 100K$, >25K to 50K$, Up to 25K$, and 0$(On hand).

The program calculates and displays the Consistency Index.

4.2.3 Limitations and Bugs

1) The program allows only seven branches per node. In general this does not present a problem as the model can usually be broken down into more levels. There is a method whereby two major common-element nodes are linked by a common minor node to effectively allow more than seven branches. However, this nodal expansion requires manual recalculation of node weights.

2) The number of characters allowed for node names is limited to eight. However, a built-in glossary capability allows the user to enter a definition for the abbreviated node name.

3) Printout of comparison matrices are not consistent, with some being in the "questionnaire" form while others are in matrix form. Printout of the "questionnaires" is also not consistent, with some being in the "intensity" form while others are in the intended "circle the number" form. These printout inconsistencies were the only bugs noted during validation and during use in an evaluation of alternatives study. These inconsistencies may be fixed in a later release.

4.3 PCM™

NOTE: The Shareware™ edition of PCM™ was evaluated. This is a crippled version in that it does not allow saving of new input data.

4.3.1 User Documentation

The on-disk user manual consists of 37 pages, not counting the order form. While small, the manual is easy to follow and provides sufficient information to use the program. Examples are used to good advantage.

4.3.2 Usability

The program presents a spreadsheet-like primary input screen, with the underlying tree structure hidden. Commands are executed via keyboard function keys, with an on-screen key/function bar. Each level (column) is attached to each element (cell) in the parent level. For some uses, say when there are only two levels, this makes the data input quite straight forward. For three or more levels, the pairwise comparisons may become somewhat difficult when the pair being compared does not have any relationship to the higher level element.

The on-disk example files allow a person to evaluate the usefulness of the program and to see the effect of changes on the results.

The program calculates and displays the Consistency Index.

4.3.3 Limitations and Bugs

1) The program allows a maximum of five levels with no more than sixteen elements per level.

2) The program permits a maximum of eleven characters for element identifiers, and does not have a built-in glossary capability.

3) The program supports only Epson® brand printers. The commercial, or later releases, may support other printers.

No bugs were encountered during the validation process.

4.4 CONFIGURATION CONTROL

Only one copy of the programs are on hand. Updated or upgraded versions of Expert Choice™ and PCM™ may be obtained when they become available.

5.0 VALIDATION

5.1 VALIDATION PLAN

Validation of the EXPERT CHOICE™ and PCM™ programs was accomplished by comparing their results to each other and to the results from an independent implementation of the AHP algorithm -- the heart of the two commercial programs. By using a common data set with each program, correct implementation of the algorithm mathematics is validated by common results.

Rather than write an entirely new program, the general purpose mathematical program Mathematica®[1] was used to provide the independent implementation of the pairwise comparison (AHP) algorithm. The AHP algorithm and its matrix math are summarized in Appendix A. See Saaty-1990 for a thorough treatment and numerous application examples of the process.

5.2 TEST PROCEDURE

A common data set (see Section 5.3 below) was entered for each program. Intermediate and final results from the EXPERT CHOICE™ and PCM™ programs were compared to each other and to the results from Mathematica® for correctness.

5.3 VALIDATION DATA SET

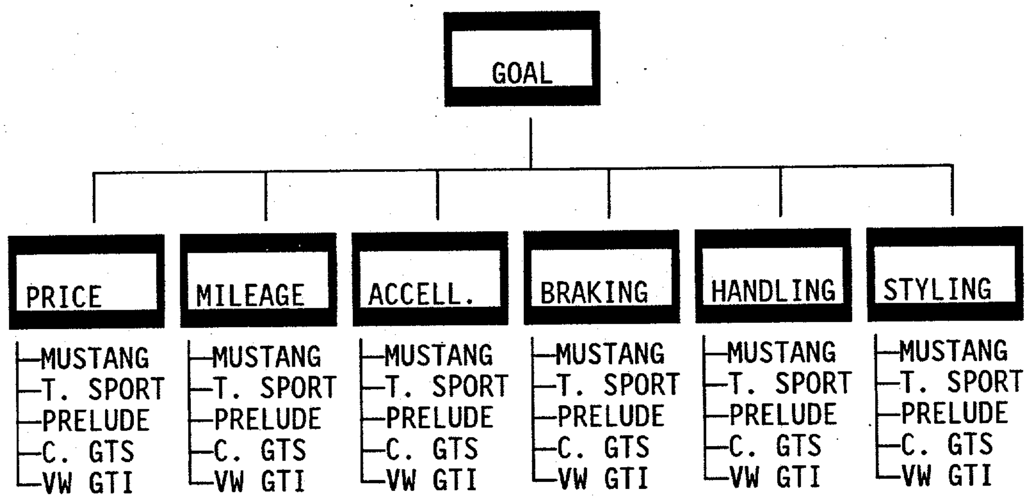
The car purchase example from the DAS™ disk was selected for validation of the PCM™ and Expert Choice™ programs. This evaluation was selected for two reasons; 1) any new evaluation data could not be saved with PCM™, and 2) nearly everyone has been faced with the car selection dilemma. The goal is "To select a car to purchase". Based on the GOAL, criteria that will influence the decision were: Price, Fuel Economy, Acceleration, Braking, Handling, and Styling. The alternatives to be evaluated are the specific car models being considered. Figure 1 illustrates the hierarchy of these factors in graphical form. The detailed data used for validation are provided in Appendix B.

In Figure 1, the GOAL of this test is select a type of car that best meets our criteria. Our evaluation criteria are Level 1, while Level 2 is the type of car being considered. The cars are subsets or branches from each

---

[1]. Mathematica® is a symbolic and numeric math calculation program. Other programs with the same capabilities could have been used.

GOAL: To select a car to purchase



GLOSSARY

ACCELL. --- Acceleration time, zero to 60
BRAKING --- Stopping distance from 60 MPH
C. GTS --- Corolla GTS
HANDLING --- How does the car handle, sluggish or like a sports car
MILEAGE --- Fuel economy, miles per gallon
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
PRICE --- Selling price
STYLING --- How does it look
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

FIGURE 1: Validation Criteria and Alternatives

element in Level 1, *i.e.*, the same set of cars will be evaluated for every criteria under Level 1.

5.4 RESULTS

The final weighting factors from PCM™, Expert Choice™ and Mathematica® are tabulated in Table I. The car that meets the criteria the best has the highest score -- the Tempo Sport with 24.6%, while the car that least meets the criteria has the lowest score.

TABLE I: Final Results from Validation Data Set for Three Programs

CAR	PCM™[1]	EXPERT CHOICE™[2]	MATHEMATICA®[3]
Mustang GT	18.7	0.187	18.69
Tempo Sport	24.6	0.246	24.60
Prelude	12.9	0.129	12.84
Corolla GTS	22.6	0.226	22.65
VW GTI	21.2	0.212	21.22

NOTES: 1. Normalized to a sum of 100 (%).
2. Normalized to a sum of 1.0.
3. Normalized to a sum of 100 (%). Given to four significant digits for round off comparison of the other programs three significant digit reporting.

The detailed final and intermediate results from the programs are given in Appendices C, D and E. Both intermediate and final results were within the error bounds associated with binary mathematical operations on decimal numbers. The apparent round off error in Table I for the Prelude is at least partially due to the need to maintain an integer sum of one (or 100). It is coincidental that both commercial programs produce the same apparent error.

5.5 CONCLUSIONS

Both the PCM™ and Expert Choice™ programs produced the correct intermediate weighting factors and final ranking/priority values for the example data set. The results from any other alternative evaluation data set should also be correct -- provided that all comparisons are reasonable and consistent (inconsistency index for each matrix less than 0.2) -- since the programs use standard matrix mathematics along with the usual addition, subtraction, multiplication and division.

Of the two programs, it is the authors opinion that Expert Choice™ has the greatest flexibility and hence, broadest applicability. Its flexibility derives from its multiple branch tree structure and its alternative "Ranking" mode. These two features allow the greatest flexibility in setting up the abstract evaluation model. However, PCM™ would be the easiest to use for simple, two-level evaluations. The two-level analysis is very simple to input data for since Level 2 is automatically attached to each item in Level 1, whereas for Expert Choice™, the Level 2 items must be attached to one Level 1 item and then manually copied to all the other Level 1 items.

While other programs identified in Section 3.2.1 might also be useful, budgetary and time constraints precluded evaluation of a broader sampling of programs. And, both PCM™ and Expert Choice™ have been shown to produce correct results from the mathematically substantiated Analytic Hierarchy Process evaluation method.

One should keep in mind that these programs are only tools, and that the results depend heavily on the "goodness" of the model, selection of options (or alternatives), and judgmental pair comparison rankings. The results can be used only as guidance, the final decision must be made by the analyst.

6.0 REFERENCES

Saaty-1990 — Saaty, Thomas L., *Multicriteria Decision Making: The Analytic Hierarchy Process*, RWS Publications, Pittsburgh, Pennsylvania, 1990.

EC-1992 — *Expert Choice™*, Version 8.0: Expert Choice, Inc., Pittsburgh, Pennsylvania, 1992.

Armada-1990 — *PCM™*, Version 2.0: Armada Systems, Thornhill, Ontario, Canada, 1989.

Math-1991 — *Mathematica®*, Version 2.0: Stephen Wolfram Inc., Addison-Wesley Publishing Co., Redwood City, California, 1991.

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APPENDICES

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APPENDIX A - ANALYTIC HIERARCHY PROCESS

NOTE: It is not the intent to present a rigorous treatment of the Analytic Hierarchy Process, or the underlying mathematics; but rather to provide an overview. The reader is referred to other works for details.

I. THE PROCESS

A. INTRODUCTION

The Analytic Hierarchy Process (AHP) developed by Dr. Saaty (Saaty-1990) is a disciplined technique providing support in the decision making process when there are multiple criteria and options to be considered. The technique is based on abstract modelling, pairwise comparison of items, and matrix mathematics. It involves the four steps given below.

1. Build a model
2. Do the pairwise comparison of the higher levels
3. Rate the options
4. Evaluate results

B. MODEL BUILDING

The objective of abstract modelling is to break things down into manageable groups of associated items, *i. e.*, hierarchically decompose the system. The three (usually iterative) steps in developing the model are:

1. Establish the goal or objective,
2. Establish the evaluation criteria (which can be more than one level), and
3. Identify the alternatives, or options to meeting the goal.

A clear and concise statement of the goal is essential to the total process. For our car example (Appendix B), the statement "To select a car to purchase" is a clearer statement of the immediate objective than "To purchase a car", which is the ultimate objective.

Establishing the basis for evaluating alternatives is where one can easily become bogged down. It is important to not group items together which do not bear any relationship to each other. On the other hand, items which are too broad may produce misleading results. In general, the old *five plus or minus two* rule is applicable. Some things which should be considered may not become evident until the options are being evaluated. Pencil sketches of the criteria tree can be most beneficial in developing the model.

One should not overlook any possibility when identifying options to fulfilling the goal. Sometimes the seemingly least likely option is the one which should be most seriously considered. Group brainstorming sessions can be helpful in this step.

The resultant model diagram has a tree structure, with the outermost leaves being either alternatives or criteria rating categories.

C. PAIRWISE COMPARISON

Rating the individual criteria is accomplished by comparing each possible pair within a group with regard to its next higher item. This pairwise comparison is considerably easier to accomplish than attempting to assign a weighting value to one member of a group which is relative to all other members of the group.

The analyst assigns weighting values to each pair comparison that reflects the "goodness" of one pair member relative to the other member. The numerical goodness ratings and their corresponding subjective value are given below. Intermediate values can be used as well. For example, a "MODERATELY

NUMERICAL WEIGHT	SUBJECTIVE "GOODNESS" VALUE
1	EQUALLY
3	MODERATELY
5	STRONGLY
7	VERY STRONGLY
9	EXTREMELY

to STRONGLY" subjective rating would have a numerical value of 4. Decimal fraction numerical values are possible, but that fine grained analysis isn't justified. In practice, the five ratings given above are usually as fine as one needs to produce meaningful results.

D. OPTION EVALUATION

The objective of pairwise comparison method, or AHP is to derive a set of quantitative rankings for potential solutions which reflect as best as possible their relative merit for meeting the goal of the hierarchy. The PCM™ program allows only alternatives at the final level, so pairwise comparisons must be made for the alternatives under each of the next higher level criteria. Expert Choice™ also has an optional RANKING mode which makes evaluating a large number of alternatives much easier.

For the Expert Choice™ RANKING mode, the outer leaves on the tree are "criteria categories". For example, under a Hardware Cost criteria, one might have five categories of >100K$, >50K to 100K$, >25K to 50K$, Up to 25K$, and 0$(On hand). The categories are rated through pairwise comparisons to establish their individual weighting value. The alternatives are then assigned a category for each criteria, and the sum of the category weights is the option numerical ranking (which can be normalized).

II. MATHEMATICAL THEORY

A. MATRIX STRUCTURE

The AHP relies on standard matrix mathematics to calculate weighting, or priority values. The matrix is constructed from the pairwise judgement ratings for each group. The initial (reported) matrix for our car purchase example (Appendix B) looks like:

	PRICE	MILEAGE	ACCELL.	BRAKING	HANDLING	STYLING
PRICE		3.0	4.0	4.0	4.0	2.0
MILEAGE			2.0	2.0	2.0	(2.0)
ACCELL.				1.0	(2.0)	(3.0)
BRAKING					(2.0)	(2.0)
HANDLING						(2.0)
STYLING						

(The comparison is row item to column item. When the column item is preferred over the row item, the judgement value is enclosed in parentheses.) Note that we initially have to fill in only the upper right portion of the matrix. The program takes this input and completes the matrix.

The completed matrix looks like this.

	PRICE	MILEAGE	ACCELL.	BRAKING	HANDLING	STYLING
PRICE	1.0	3.0	4.0	4.0	4.0	2.0
MILEAGE	1/3	1.0	2.0	2.0	2.0	1/2
ACCELL.	1/4	1/2	1.0	1.0	1/2	1/3
BRAKING	1/4	1/2	1.0	1.0	1/2	1/2
HANDLING	1/4	1/2	2.0	2.0	1.0	1/2
STYLING	1/2	2.0	3.0	2.0	2.0	1.0

When the column item is preferred over the row item, the judgement value is to the left of the diagonal. (Although inverted, the matrix location of the input values can also be seen in the Level 1 judgement report given in Appendix B, page B-2.) The values filled in for the corresponding diagonal elements are the reciprocals of the input.

B. MATRIX CALCULATIONS[1]

For each pairwise comparison matrix at each level do the following.

---

[1] Interested readers should refer to any advanced mathematics text for detailed matrix theory. For example, see Menzel-1961, Part II, Section 29, Introduction to the theory of matrices.

1. Calculate the Eigenvalues. The result is a 1 x n matrix with real or complex (real + imaginary) terms.

2. Find the largest absolute value Eigenvalue.

3. Calculate the Eigenvector matrix for the largest Eigenvalue. The result is a 1 x n matrix with real or complex terms. These are our raw "weighting factors".

4. Calculate the sum of the absolute value of the Eigenvector terms.

5. Divide the absolute value of the Eigenvectors by the sum found in Step 4.

The last two steps accomplish two things; 1) complex numbers are converted to real, and 2) the weighting factors are normalized to a sum of 1.0. If percentage results are desired, multiply the quotient by 100.

The expressions (instructions) for the equivalent operations in the Mathematica® program are as follows. Note that some operations are easier to perform on the full matrix rather than on a single row as implied above. The literal interpretation of the instruction is indented below the instruction.

Ev = Eigenvalues[N[matrix-name]]
  Calculate the Eigenvalue matrix, Ev, of [matrix-name] using Numeric values (rather than Symbolic).

M = Eigenvectors[N[matrix-name]]
  Calculate the Eigenvector matrix, M, for [matrix-name] using Numeric values. The result is a matrix of all Eigenvalues. However, for our example calculations the first row corresponds to the largest Eigenvalue and is the only row of interest -- it is our raw "weighting factors".

Total = Sum[M[[1,t]], {t,n}]
 where n is the size of the matrix
  Sum the absolute values of the elements in the first row of M.

M1 = M[[1]]/Total
  Form a matrix of the element absolute value divided by the Total for the first row of M.

C. CONSISTENCY INDEX

For each pairwise comparison matrix, a consistency index (CI) can also calculated and used as an indication of consistency of the judgements (Saaty-1990). The CI value for a matrix is calculated as follows:

CI = (Ev[[1]] - n)/(n - 1)

where n is the size of the matrix, and Ev[1] is the largest Eigenvalue.

Ideally, the CI is zero, but this is usually not practical. A CI value greater than, say, 0.2 indicates there was an inconsistency in one or more of the associated pairwise comparisons and they should be reevaluated.

D. MULTILEVEL MATHEMATICS

For analyses where the lowest level is options, their ranking (or rating) is calculated as given below.

1. Multiply each normalized Eigenvector matrix term at a level by the normalized Eigenvector of the parent element. (Hint: start at the top and work down.)

2. Normalize the resultant vector for the alternatives.

III. REFERENCES

Menzel-1961 Menzel, Donald H., *Mathematical Physics*, Dover Publications, Inc., New York, 1961.

Saaty-1990 Saaty, Thomas L., *Multicriteria Decision Making: The Analytic Hierarchy Process*, RWS Publications, Pittsburgh, Pennsylvania, 1990.

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APPENDIX B - VALIDATION DATA SET

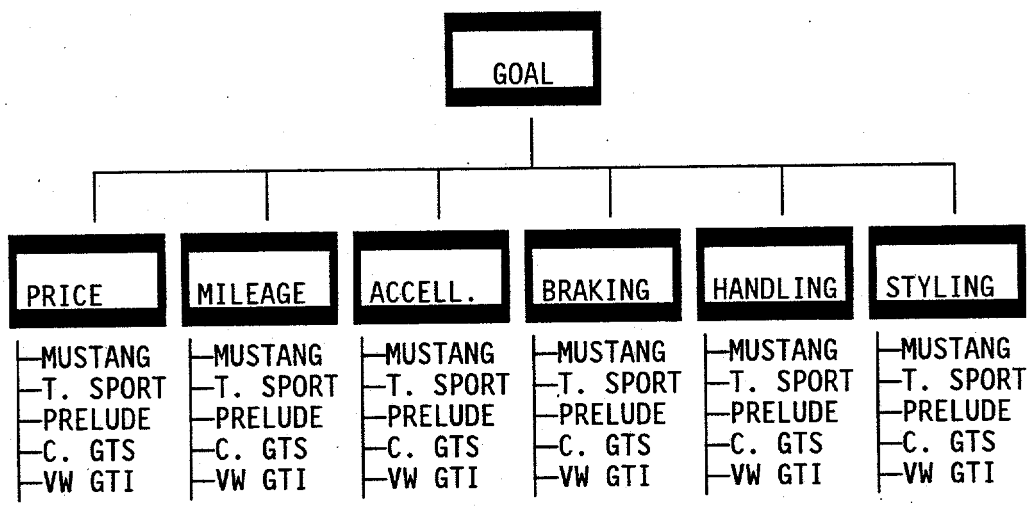
The data set used for validation of the decision support programs PCM™ and Expert Choice™ is on the following pages, with the model given below. The data (an example from the PCM™ disk) is given in the form of completed questionaires printed from Expert Choice™ because they show the individual judgements more clearly than a straight numerical table would.

In the model diagram below, the GOAL is Level 0, the evaluation criteria is at Level 1, and the alternatives are Level 2. The cars being considered appear under each category.

In the tables which follow, the blocked out number is the pairwise comparison value (or weight), which can range from 1 to 9. The use of IMPORTANCE, PREFERENCE and LIKELIHOOD was somewhat arbitrary and has no effect on the matrix calculations. Glossary definitions for each comparison item are given below the tables.

THE MODEL:

GOAL: To select a car to purchase



Verbal judgments of IMPORTANCE with respect to:
GOAL
Node: 0

1	PRICE	9 8 7 6 5 4 ■ 2	1	2 3 4 5 6 7 8 9	MILEAGE
2	PRICE	9 8 7 6 5 ■ 3 2	1	2 3 4 5 6 7 8 9	ACCELL.
3	PRICE	9 8 7 6 5 ■ 3 2	1	2 3 4 5 6 7 8 9	BRAKING
4	PRICE	9 8 7 6 5 ■ 3 2	1	2 3 4 5 6 7 8 9	HANDLING
5	PRICE	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	STYLING
6	MILEAGE	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	ACCELL.
7	MILEAGE	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	BRAKING
8	MILEAGE	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	HANDLING
9	MILEAGE	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	STYLING
10	ACCELL.	9 8 7 6 5 4 3 2	■	2 3 4 5 6 7 8 9	BRAKING
11	ACCELL.	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	HANDLING
12	ACCELL.	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	STYLING
13	BRAKING	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	HANDLING
14	BRAKING	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	STYLING
15	HANDLING	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	STYLING

1=EQUAL 3=MODERATE 5=STRONG 7=VERY STRONG 9=EXTREME

GOAL: To purchase a car

ACCELL. --- Acceleration time, zero to 60
BRAKING --- Stopping distance from 60 MPH
HANDLING --- How does the car handle, sluggish or like a sports car
MILEAGE --- Fuel economy, miles per gallon
PRICE --- Selling price
STYLING --- How does it look

Verbal judgments of LIKELIHOOD with respect to:
PRICE < GOAL

Node: 10000

1	MUSTANG	9 8 7 6 5 4 3 2	1	2 3 ■ 5 6 7 8 9	T. SPORT
2	MUSTANG	9 8 7 6 5 4 3 2	■	2 3 4 5 6 7 8 9	PRELUDE
3	MUSTANG	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	C. GTS
4	MUSTANG	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	VW GTI
5	T. SPORT	9 8 7 6 ■ 4 3 2	1	2 3 4 5 6 7 8 9	PRELUDE
6	T. SPORT	9 8 7 6 5 4 ■ 2	1	2 3 4 5 6 7 8 9	C. GTS
7	T. SPORT	9 8 7 6 5 4 ■ 2	1	2 3 4 5 6 7 8 9	VW GTI
8	PRELUDE	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	C. GTS
9	PRELUDE	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	VW GTI
10	C. GTS	9 8 7 6 5 4 3 2	■	2 3 4 5 6 7 8 9	VW GTI

1=EQUAL 3=MODERATE 5=STRONG 7=VERY STRONG 9=EXTREME

GOAL: To purchase a car

C. GTS --- Corolla GTS
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
PRICE --- Selling price
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

Verbal judgments of PREFERENCE with respect to:
MILEAGE < GOAL Node: 20000

1	MUSTANG	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	T. SPORT
2	MUSTANG	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	PRELUDE
3	MUSTANG	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	C. GTS
4	MUSTANG	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	VW GTI
5	T. SPORT	9 8 7 6 5 4 3 2	■	2 3 4 5 6 7 8 9	PRELUDE
6	T. SPORT	9 8 7 6 5 4 3 2	■	2 3 4 5 6 7 8 9	C. GTS
7	T. SPORT	9 8 7 6 5 4 3 2	■	2 3 4 5 6 7 8 9	VW GTI
8	PRELUDE	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	C. GTS
9	PRELUDE	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	VW GTI
10	C. GTS	9 8 7 6 5 4 3 2	■	2 3 4 5 6 7 8 9	VW GTI

1=EQUAL 3=MODERATE 5=STRONG 7=VERY STRONG 9=EXTREME

GOAL: To purchase a car

C. GTS --- Corolla GTS
MILEAGE --- Fuel economy, miles per gallon
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

Verbal judgments of LIKELIHOOD with respect to:
ACCELL. < GOAL

Node: 30000

1	MUSTANG	9 8 7 6 ■ 4 3 2	1	2 3 4 5 6 7 8 9	T. SPORT
2	MUSTANG	9 8 7 6 ■ 4 3 2	1	2 3 4 5 6 7 8 9	PRELUDE
3	MUSTANG	9 8 7 6 ■ 4 3 2	1	2 3 4 5 6 7 8 9	C. GTS
4	MUSTANG	9 8 7 6 5 ■ 3 2	1	2 3 4 5 6 7 8 9	VW GTI
5	T. SPORT	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	PRELUDE
6	T. SPORT	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	C. GTS
7	T. SPORT	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	VW GTI
8	PRELUDE	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	C. GTS
9	PRELUDE	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	VW GTI
10	C. GTS	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	VW GTI

1=EQUAL 3=MODERATE 5=STRONG 7=VERY STRONG 9=EXTREME

GOAL: To purchase a car

ACCELL. --- Acceleration time, zero to 60
C. GTS --- Corolla GTS
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

Verbal judgments of PREFERENCE with respect to:
BRAKING < GOAL Node: 40000

1	MUSTANG	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	T. SPORT
2	MUSTANG	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	PRELUDE
3	MUSTANG	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	C. GTS
4	MUSTANG	9 8 7 6 5 4 3 2	1	2 3 4 ■ 6 7 8 9	VW GTI
5	T. SPORT	9 8 7 6 5 4 3 2	1	2 3 ■ 5 6 7 8 9	PRELUDE
6	T. SPORT	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	C. GTS
7	T. SPORT	9 8 7 6 5 4 3 2	1	2 3 4 5 ■ 7 8 9	VW GTI
8	PRELUDE	9 8 7 6 5 4 ■ 2	1	2 3 4 5 6 7 8 9	C. GTS
9	PRELUDE	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	VW GTI
10	C. GTS	9 8 7 6 5 4 3 2	1	2 3 4 ■ 6 7 8 9	VW GTI

1=EQUAL 3=MODERATE 5=STRONG 7=VERY STRONG 9=EXTREME

GOAL: To purchase a car

BRAKING --- Stopping distance from 60 MPH
C. GTS --- Corolla GTS
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

Verbal judgments of PREFERENCE with respect to:
HANDLING < GOAL

Node: 50000

1	MUSTANG	9 8 7 6 5 ■ 3 2	1	2 3 4 5 6 7 8 9	T. SPORT
2	MUSTANG	9 8 7 6 5 4 ■ 2	1	2 3 4 5 6 7 8 9	PRELUDE
3	MUSTANG	9 8 7 6 5 4 3 2	1	■ 3 4 5 6 7 8 9	C. GTS
4	MUSTANG	9 8 7 6 5 4 3 ■	1	2 3 4 5 6 7 8 9	VW GTI
5	T. SPORT	9 8 7 6 5 4 3 2	■	2 3 4 5 6 7 8 9	PRELUDE
6	T. SPORT	9 8 7 6 5 4 3 2	1	2 3 4 ■ 6 7 8 9	C. GTS
7	T. SPORT	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	VW GTI
8	PRELUDE	9 8 7 6 5 4 3 2	1	2 3 4 ■ 6 7 8 9	C. GTS
9	PRELUDE	9 8 7 6 5 4 3 2	1	2 ■ 4 5 6 7 8 9	VW GTI
10	C. GTS	9 8 7 6 5 ■ 3 2	1	2 3 4 5 6 7 8 9	VW GTI

1=EQUAL 3=MODERATE 5=STRONG 7=VERY STRONG 9=EXTREME

GOAL: To purchase a car

C. GTS --- Corolla GTS
HANDLING --- How does the car handle, sluggish or like a sports car
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

Verbal judgments of PREFERENCE with respect to:
STYLING < GOAL Node: 60000

1	MUSTANG	9 8 7 6 5 4 █ 2	1	2 3 4 5 6 7 8 9	T. SPORT
2	MUSTANG	9 8 7 6 5 4 █ 2	1	2 3 4 5 6 7 8 9	PRELUDE
3	MUSTANG	9 8 7 6 5 4 3 2	█	2 3 4 5 6 7 8 9	C. GTS
4	MUSTANG	9 8 7 6 5 4 3 █	1	2 3 4 5 6 7 8 9	VW GTI
5	T. SPORT	9 8 7 6 5 4 3 2	█	2 3 4 5 6 7 8 9	PRELUDE
6	T. SPORT	9 8 7 6 5 4 3 2	1	2 █ 4 5 6 7 8 9	C. GTS
7	T. SPORT	9 8 7 6 5 4 3 2	1	█ 3 4 5 6 7 8 9	VW GTI
8	PRELUDE	9 8 7 6 5 4 3 2	1	2 █ 4 5 6 7 8 9	C. GTS
9	PRELUDE	9 8 7 6 5 4 3 2	1	█ 3 4 5 6 7 8 9	VW GTI
10	C. GTS	9 8 7 6 5 4 3 2	█	2 3 4 5 6 7 8 9	VW GTI

1=EQUAL 3=MODERATE 5=STRONG 7=VERY STRONG 9=EXTREME

GOAL: To purchase a car

C. GTS --- Corolla GTS
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
STYLING --- How does it look
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

APPENDIX C - JUDGEMENTS AND CALCULATIONS FROM EXPERT CHOICE™

The intermediate and final results from the Expert Choice™ program for the car selection example data set (Appendix B) are given on the following pages. Both tabular and graphical results are shown for some of the comparison matrices (these provide an indication of the types of reports supported by the program). Only Level 1 input judgements are included here (Page C-2). Level 2 judgements are not repeated since those reports from Expert Choice™ are given in Appendix B. Final judgements are given on Pages C-3 through C-5.

In the Level 1 input matrix (Page C-2), note that when a judgement value is enclosed in parentheses, the column item is preferred over the row item. Also note that the PRIORITIES at Level 1 are normalized to a sum of 1.0, while those at Level 2 (Page C-3) have been normalized and then multiplied by the priority of the criteria to give the overall priority. The INCONSISTENCY RATIO is the Consistency Index discussed in Appendix A.

JUDGMENTS WITH RESPECT TO GOAL

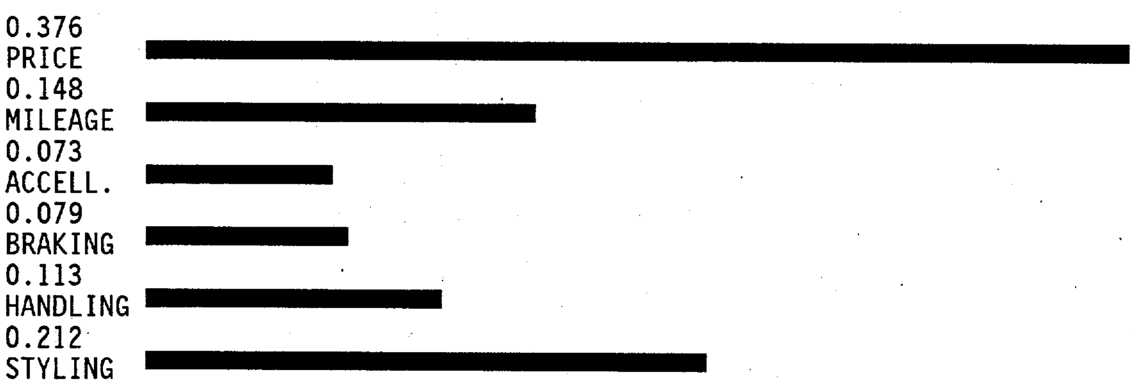
	PRICE	MILEAGE	ACCELL.	BRAKING	HANDLING	STYLING
PRICE		3.0	4.0	4.0	4.0	2.0
MILEAGE			2.0	2.0	2.0	(2.0)
ACCELL.				1.0	(2.0)	(3.0)
BRAKING					(2.0)	(2.0)
HANDLING						(2.0)
STYLING						

Matrix entry indicates that ROW element is
1 EQUALLY 3 MODERATELY 5 STRONGLY 7 VERY STRONGLY 9 EXTREMELY
more IMPORTANT than COLUMN element unless enclosed in parenthesis.

GOAL: To purchase a car

ACCELL. --- Acceleration time, zero to 60
BRAKING --- Stopping distance from 60 MPH
HANDLING --- How does the car handle, sluggish or like a sports car
MILEAGE --- Fuel economy, miles per gallon
PRICE --- Selling price
STYLING --- How does it look

PRIORITIES



INCONSISTENCY RATIO = 0.020.

To purchase a car

Details for Synthesis of Leaf Nodes with respect to GOAL

DISTRIBUTIVE MODE

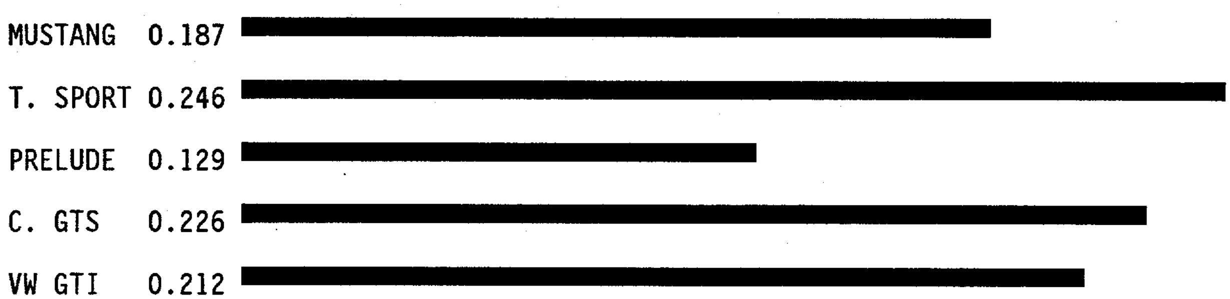
LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5
PRICE =0.376				
.	MUSTANG =0.035			
.	T. SPORT =0.173			
.	PRELUDE =0.029			
.	C. GTS =0.070			
.	VW GTI =0.070			
MILEAGE =0.148				
.	MUSTANG =0.011			
.	T. SPORT =0.034			
.	PRELUDE =0.045			
.	C. GTS =0.029			
.	VW GTI =0.029			
ACCELL. =0.073				
.	MUSTANG =0.038			
.	T. SPORT =0.006			
.	PRELUDE =0.005			
.	C. GTS =0.009			
.	VW GTI =0.015			
BRAKING =0.079				
.	MUSTANG =0.007			
.	T. SPORT =0.005			
.	PRELUDE =0.021			
.	C. GTS =0.009			
.	VW GTI =0.037			
HANDLING =0.113				
.	MUSTANG =0.028			
.	T. SPORT =0.008			
.	PRELUDE =0.008			
.	C. GTS =0.051			
.	VW GTI =0.018			
STYLING =0.212				
.	MUSTANG =0.067			
.	T. SPORT =0.021			
.	PRELUDE =0.021			
.	C. GTS =0.059			
.	VW GTI =0.044			

To purchase a car

Synthesis of Leaf Nodes with respect to GOAL

DISTRIBUTIVE MODE

OVERALL INCONSISTENCY INDEX = 0.02



C. GTS --- Corolla GTS
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

GOAL: To purchase a car

GOAL
L 1.000
G 1.000

	PRICE	MILEAGE	ACCELL.	BRAKING	HANDLING	STYLING
L	0.376	0.148	0.073	0.079	0.113	0.212
G	0.376	0.148	0.073	0.079	0.113	0.212
MUSTANG L	0.093	0.076	0.523	0.088	0.251	0.318
MUSTANG G	0.035	0.011	0.038	0.007	0.028	0.067
T. SPORT L	0.460	0.227	0.087	0.060	0.068	0.099
T. SPORT G	0.173	0.034	0.006	0.005	0.008	0.021
PRELUDE L	0.076	0.304	0.066	0.266	0.072	0.099
PRELUDE G	0.029	0.045	0.005	0.021	0.008	0.021
C. GTS L	0.186	0.197	0.123	0.117	0.449	0.277
C. GTS G	0.070	0.029	0.009	0.009	0.051	0.059
VW GTI L	0.186	0.197	0.201	0.468	0.160	0.206
VW GTI G	0.070	0.029	0.015	0.037	0.018	0.044

ACCELL. --- Acceleration time, zero to 60
BRAKING --- Stopping distance from 60 MPH
C. GTS --- Corolla GTS
HANDLING --- How does the car handle, sluggish or like a sports car
MILEAGE --- Fuel economy, miles per gallon
MUSTANG --- Ford Mustang GT
PRELUDE --- Honda Prelude
PRICE --- Selling price
STYLING --- How does it look
T. SPORT --- Tempo Sport
VW GTI --- Volkswagen GTI

L --- LOCAL PRIORITY: PRIORITY RELATIVE TO PARENT
G --- GLOBAL PRIORITY: PRIORITY RELATIVE TO GOAL

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APPENDIX D - JUDGEMENTS AND CALCULATIONS FROM PCM™

Comparison matrices, and intermediate and final results from the PCM™ program for the car purchase example (Appendix B) are given on the following pages. The primary user interface "spreadsheet" structure is shown below. Level 1 input and weighting factors are given on Page D-2, and those for Level 2 on Pages D-3 through D-5. The final rankings are on Page D-6. These reports also serve to illustrate the output capabilities of the program.

In the pairwise comparison matrices, the row element is rated relative to the column element. For example, in the Level 1 matrix (page E-2) row B (Fuel econ.) column D (Braking), fuel economy is rated Equal-to-Moderately more important than braking ability. A minus sign (-) indicates that the comparison is inverse, *i.e.*, the column element is ranked higher than the row element, it does not indicate a negative value. Keep in mind that the negative sign only signifies that the reciprocal of the numerical value is entered into the matrix element (see Appendix A). Also note that in the tables which follow, the WEIGHTS have been normalized to a sum of 100.0 (percent).

Pairwise Comparison Method File 'C:CAR .PCM' (C) ARMADA SYSTEMS 1986, 1990
Decision Tree Hierarchy
GOAL: To purchase a car.

	Level 1	Level 2	Level 3	Level 4	Level 5
1	Price	Mustang GT			
2	Fuel econ.	Tempo Sport			
3	Acceleraton	Prelude			
4	Braking	Corolla GTS			
5	Handling	VW GTI			
6	Styling				
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					

```
Pairwise Comparison Data for level 1, with respect to: GOAL
1: Equal      3: Moderate      5: Strong      7: Very Strong      9: Extreme
================================================================================
  A  B  C  D  E  F                   WEIGHTS
A    3  4  4  4  2  A  Price         37.6 *********
B       2  2  2 -2  B  Fuel econ.    14.8 ****
C          1 -2 -3  C  Acceleraton    7.3 **
D            -2 -2  D  Braking        7.9 **
E               -2  E  Handling      11.3 ***
F                   F  Styling       21.2 *****
```

```
Bar Graph of Preference Weights for level 1, with respect to: GOAL
Inconsistency=  2.0% (acceptable)

Price        37.6 **************************************************************
Fuel econ.   14.8 ************************
Acceleraton   7.3 ************
Braking       7.9 *************
Handling     11.3 *******************
Styling      21.2 ***********************************
```

Pairwise Comparison Data for level 2, with respect to: Price
1: Equal 3: Moderate 5: Strong 7: Very Strong 9: Extreme

	A	B	C	D	E			WEIGHTS
A		-4	1	-2	-2	A	Mustang GT	9.3
B			5	3	3	B	Tempo Sport	46.0
C				-3	-3	C	Prelude	7.6
D					1	D	Corolla GTS	18.6
E						E	VW GTI	18.6

Bar Graph of Preference Weights for level 2, with respect to: Price
Inconsistency= 1.3% (acceptable)

```
Mustang GT   9.3 *************
Tempo Sport 46.0 ***************************************************************
Prelude      7.6 **********
Corolla GTS 18.6 *************************
VW GTI      18.6 *************************
```

Pairwise Comparison Data for level 2, with respect to: Fuel econ.
1: Equal 3: Moderate 5: Strong 7: Very Strong 9: Extreme

	A	B	C	D	E			WEIGHTS
A		-3	-3	-3	-3	A	Mustang GT	7.6
B			1	1	1	B	Tempo Sport	22.7
C				2	2	C	Prelude	30.4
D					1	D	Corolla GTS	19.7
E						E	VW GTI	19.7

Bar Graph of Preference Weights for level 2, with respect to: Fuel econ.
Inconsistency= 1.7% (acceptable)

```
Mustang GT   7.6 ***************
Tempo Sport 22.7 ***********************************************
Prelude     30.4 ***************************************************************
Corolla GTS 19.7 *****************************************
VW GTI      19.7 *****************************************
```

Pairwise Comparison Data for level 2, with respect to: Acceleraton

1: Equal 3: Moderate 5: Strong 7: Very Strong 9: Extreme

	A	B	C	D	E			WEIGHTS
A		5	5	5	4	A	Mustang GT	52.3
B			2	-2	-3	B	Tempo Sport	8.7
C				-2	-3	C	Prelude	6.6
D					-2	D	Corolla GTS	12.3
E						E	VW GTI	20.1

Bar Graph of Preference Weights for level 2, with respect to: Acceleraton

Inconsistency= 3.9% (acceptable)

```
Mustang GT  52.3 ***************************************************************
Tempo Sport  8.7 **********
Prelude      6.6 ********
Corolla GTS 12.3 ***************
VW GTI      20.1 ************************
```

Pairwise Comparison Data for level 2, with respect to: Braking

1: Equal 3: Moderate 5: Strong 7: Very Strong 9: Extreme

	A	B	C	D	E			WEIGHTS
A		2	-3	-2	-5	A	Mustang GT	8.8
B			-4	-2	-6	B	Tempo Sport	6.0
C				3	-2	C	Prelude	26.6
D					-5	D	Corolla GTS	11.7
E						E	VW GTI	46.8

Bar Graph of Preference Weights for level 2, with respect to: Braking

Inconsistency= 2.2% (acceptable)

```
Mustang GT   8.8 ************
Tempo Sport  6.0 ********
Prelude     26.6 ***********************************
Corolla GTS 11.7 ***************
VW GTI      46.8 **************************************************************
```

Pairwise Comparison Data for level 2, with respect to: Handling
1: Equal 3: Moderate 5: Strong 7: Very Strong 9: Extreme

	A	B	C	D	E			WEIGHTS
A		4	3	-2	2	A	Mustang GT	25.1
B			1	-5	-3	B	Tempo Sport	6.8
C				-5	-3	C	Prelude	7.2
D					4	D	Corolla GTS	44.9
E						E	VW GTI	16.0

Bar Graph of Preference Weights for level 2, with respect to: Handling
Inconsistency= 2.3% (acceptable)

```
Mustang GT  25.1 ***********************************
Tempo Sport  6.8 *********
Prelude      7.2 **********
Corolla GTS 44.9 ***************************************************************
VW GTI      16.0 **********************
```

Pairwise Comparison Data for level 2, with respect to: Styling
1: Equal 3: Moderate 5: Strong 7: Very Strong 9: Extreme

	A	B	C	D	E			WEIGHTS
A		3	3	1	2	A	Mustang GT	31.8
B			1	-3	-2	B	Tempo Sport	9.9
C				-3	-2	C	Prelude	9.9
D					1	D	Corolla GTS	27.7
E						E	VW GTI	20.6

Bar Graph of Preference Weights for level 2, with respect to: Styling
Inconsistency= 0.9% (acceptable)

```
Mustang GT  31.8 ***************************************************************
Tempo Sport  9.9 *******************
Prelude      9.9 *******************
Corolla GTS 27.7 *******************************************************
VW GTI      20.6 *****************************************
```

Pairwise Comparison Method File 'C:CAR .PCM' (C) ARMADA SYSTEMS 1986, 1990
Decision Tree Hierarchy
GOAL: To purchase a car.

	Level 1		Level 2		Level 3	Level 4	Level 5
1	Price	38	Mustang GT	19			
2	Fuel econ.	15	Tempo Sport	25			
3	Acceleraton	7	Prelude	13			
4	Braking	8	Corolla GTS	23			
5	Handling	11	VW GTI	21			
6	Styling	21					
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

Overall average inconsistency= 0.18 (acceptable)

Bar Graph of Preference Weights for level 2
Inconsistency= 1.6% (acceptable)

```
Mustang GT  18.7 ***********************************************
Tempo Sport 24.6 **************************************************************
Prelude     12.9 ********************************
Corolla GTS 22.6 *********************************************************
VW GTI      21.2 *****************************************************
```

AFTER ISSUING THE "ORDER" COMMAND:

Bar Graph of Preference Weights for level 2
Inconsistency= 1.6% (acceptable)

```
Tempo Sport 24.6 **************************************************************
Corolla GTS 22.6 *********************************************************
VW GTI      21.2 *****************************************************
Mustang GT  18.7 ***********************************************
Prelude     12.9 ********************************
```

APPENDIX E - JUDGEMENTS AND CALCULATIONS FROM MATHEMATICA®

Input matrices and calculation results from the Mathematica® program for the car purchase example (Appendix B) are given on the following pages. The first level is on Page E-2, Level 2 judgements on Pages E-3 through E-8, and final rankings are on Page E-9. See Appendix A for a discussion of the implemented mathematics.

Each of the seven matrices for Levels 1 and 2 were input manually as complete (all elements filled in) matrices. The program instructions for Mathematica® for each input matrix -- Levels 1 and 2 -- are:

1. Ev = Eigenvalues[N[matrix-name]]

   Where "matrix-name" is the specific name given to the input matrix.

2. CI = (Ev[[1]]-6)/(6-1)   -- The consistency index

3. M = Eigenvectors[N[matrix-name]]

4. Total = Sum[M[[1,t]], {t,6}]

5. Mx = M[[1]]*100/Total   -- Normalized (to 100%) weighting factors

   Where x = 1 through 7 for the seven input matrices.

The program instructions for the overall (global) ranking for the car selection are given on Page F-9.

NOTE: On the following pages **bold** faced text are annotations and normal text is program output. The output from the program has been annotated to make it easier for the reader to follow.

Pairwise Comparison Data for Level 1, with respect to GOAL.

THE INPUT MATRIX:

$$\{\{1, 3, 4, 4, 4, 2\}, \{\frac{1}{3}, 1, 2, 2, 2, \frac{1}{2}\}, \{\frac{1}{4}, \frac{1}{2}, 1, 1, \frac{1}{2}, \frac{1}{3}\}, \{\frac{1}{4}, \frac{1}{2}, 1, 1, \frac{1}{2}, \frac{1}{2}\}, \{\frac{1}{4}, \frac{1}{2}, 2, 2, 1, \frac{1}{2}\}, \{\frac{1}{2}, 2, 3, 2, 2, 1\}\}$$

EIGENVALUES:
{6.12452, 0.000569825 + 0.817914 I, 0.000569825 - 0.817914 I,
-0.0440959 + 0.297542 I, -0.0440959 - 0.297542 I, -0.0374669}

CI = 0.0249038

EIGENVECTORS:
{{-0.779673, -0.307613, -0.151316, -0.16328, -0.233728, -0.439656},
{1.00339 - 0.598015 I, 0.470112 + 0.641111 I,
-0.159532 - 0.131145 I, -0.239139 - 0.280633 I,
-0.449002 + 0.481126 I, 0.733334 - 0.391184 I},
{1.00339 + 0.598015 I, 0.470112 - 0.641111 I,
-0.159532 + 0.131145 I, -0.239139 + 0.280633 I,
-0.449002 - 0.481126 I, 0.733334 + 0.391184 I},
{0.970046 - 0.916692 I, -0.0872601 - 0.373335 I,
-0.366516 - 0.093354 I, 0.215531 - 0.0474362 I,
0.149399 + 0.218751 I, -0.235971 + 1.02695 I},
{0.970046 + 0.916692 I, -0.0872601 + 0.373335 I,
-0.366516 + 0.093354 I, 0.215531 + 0.0474362 I,
0.149399 - 0.218751 I, -0.235971 - 1.02695 I},
{-1.0813, 0.631719, -0.329243, 0.486753, -0.259129, -0.183437}}

TOTAL = -2.07527

NORMALIZED WEIGHTING FACTORS:
PRICE, FUEL ECON., ACCELERATION, BRAKING, HANDLING, STYLING
{37.5698, 14.8228, 7.2914, 7.86792, 11.2626, 21.1855}

Pairwise Comparison data for Level 2, with respect to PRICE

THE INPUT MATRIX:

$\{\{1, \frac{1}{4}, 1, \frac{1}{2}, \frac{1}{2}\}, \{4, 1, 5, 3, 3\}, \{1, \frac{1}{5}, 1, \frac{1}{3}, \frac{1}{3}\}, \{2, \frac{1}{3}, 3, 1, 1\},$

$\{2, \frac{1}{3}, 3, 1, 1\}\}$

EIGENVALUES:
{5.05619, -0.0125847 + 0.532148 I, -0.0125847 - 0.532148 I, -0.0310232, 0.}

CI = 0.0140481

EIGENVECTORS:
{{0.171021, 0.847008, 0.140109, 0.341833, 0.341833},
{0.0833655 - 0.444057 I, 3.03985 - 0.328241 I,
-0.411542 - 0.269837 I, -0.19653 + 0.845906 I,
-0.19653 + 0.845906 I}, {0.0833655 + 0.444057 I,
3.03985 + 0.328241 I, -0.411542 + 0.269837 I,
-0.19653 - 0.845906 I, -0.19653 - 0.845906 I},
{-0.666926, 2.34631, 0.357631, -0.256591, -0.256591},
{$6.29125\ 10^{-20}$, $-5.00521\ 10^{-19}$, $2.59379\ 10^{-21}$, -0.707107, 0.707107}}

TOTAL = 1.8418

NORMALIZED WEIGHTING FACTORS:
PRICE, FUEL ECON., ACCELERATION, BRAKING, HANDLING, STYLING
{9.28552, 45.988, 7.60719, 18.5597, 18.5597}

Pairwise Comparison Data for Level 2 with respect to FUEL ECONOMY

THE INPUT MATRIX:

$\{\{1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}, \{3, 1, 1, 1, 1\}, \{3, 1, 1, 2, 2\}, \{3, 1, \frac{1}{2}, 1, 1\}, \{3, 1, \frac{1}{2}, 1, 1\}\}$

EIGENVALUES:

$\{5.07757, -3.76036\ 10^{-19}, -0.0387871 + 0.626406\ I, -0.0387871 - 0.626406\ I, 0.\}$

CI = 0.0193936

EIGENVECTORS:

$\{\{-0.158586, -0.475757, -0.638391, -0.412893, -0.412893\},$
$\{0.345229, -1.03569, 3.79471\ 10^{-19}, -1.49078\ 10^{-19}, -2.71051\ 10^{-19}\}, \{0.198184 + 0.216214\ I, 0.594551 + 0.648642\ I, -1.78223 + 0.534388\ I, 0.0818791 - 0.742201\ I, 0.0818791 - 0.742201\ I\}, \{0.198184 - 0.216214\ I, 0.594551 - 0.648642\ I, -1.78223 - 0.534388\ I, 0.0818791 + 0.742201\ I, 0.0818791 + 0.742201\ I\},$
$\{0., 3.61401\ 10^{-20}, -1.10119\ 10^{-21}, -0.707107, 0.707107\}\}$

TOTAL = -2.09852

NORMALIZED WEIGHTING FACTORS:

PRICE, FUEL ECON., ACCELERATION, BRAKING, HANDLING, STYLING

{7.55702, 22.6711, 30.421, 19.6754, 19.6754}

Pairwise Comparison Data for Level 2, with respect to ACCELERATION

THE INPUT MATRIX:

$$\{\{1, 5, 5, 5, 4\}, \{\frac{1}{5}, 1, 2, \frac{1}{2}, \frac{1}{3}\}, \{\frac{1}{5}, \frac{1}{2}, 1, \frac{1}{2}, \frac{1}{3}\}, \{\frac{1}{5}, 2, 2, 1, \frac{1}{2}\}, \{\frac{1}{4}, 3, 3, 2, 1\}\}$$

EIGENVALUES:
{5.17546, 0.0122741 + 0.937289 I, 0.0122741 - 0.937289 I, -0.100006 + 0.156072 I, -0.100006 - 0.156072 I}

CI = 0.0438662

EIGENVECTORS:
{{-2.00047, -0.334554, -0.253292, -0.469224, -0.766885},
{-3.26372 - 2.61508 I, 0.292432 + 0.573701 I,
-0.364678 + 0.332018 I, 0.630419 - 0.198371 I, 0.720969 - 1.0032 I}
, {-3.26372 + 2.61508 I, 0.292432 - 0.573701 I,
-0.364678 - 0.332018 I, 0.630419 + 0.198371 I, 0.720969 + 1.0032 I}
, {-3.372 + 2.42239 I, -1.00769 + 0.0477829 I,
0.609369 - 0.312278 I, 1.35732 + 1.03137 I, -0.365954 - 1.75632 I}\
, {-3.372 - 2.42239 I, -1.00769 - 0.0477829 I,
0.609369 + 0.312278 I, 1.35732 - 1.03137 I, -0.365954 + 1.75632 I}}

TOTAL = -3.82443

NORMALIZED WEIGHTING FACTORS:
PRICE, FUEL ECON., ACCELERATION, BRAKING, HANDLING, STYLING
{52.3077, 8.74782, 6.62301, 12.2691, 20.0523}

Pairwise Comparison Data for Level 2, with respect to BRAKING

THE INPUT MATRIX:

$$\{\{1, 2, \frac{1}{3}, \frac{1}{2}, \frac{1}{5}\}, \{\frac{1}{2}, 1, \frac{1}{4}, \frac{1}{2}, \frac{1}{6}\}, \{3, 4, 1, 3, \frac{1}{2}\}, \{2, 2, \frac{1}{3}, 1, \frac{1}{5}\},$$
$$\{5, 6, 2, 5, 1\}\}$$

EIGENVALUES:
{5.0988, 0.00424292 + 0.696841 I, 0.00424292 - 0.696841 I, -0.0536434 + 0.127257 I, -0.0536434 - 0.127257 I}

CI = 0.0247002

EIGENVECTORS:
{{-0.157056, -0.107393, -0.473957, -0.208283, -0.834137},
{-0.152296 - 0.618422 I, 0.311781 - 0.171617 I,
0.256923 + 1.16629 I, -0.881291 + 0.228039 I, 1.57017 + 1.75061 I},
{-0.152296 + 0.618422 I, 0.311781 + 0.171617 I,
0.256923 - 1.16629 I, -0.881291 - 0.228039 I, 1.57017 - 1.75061 I},
{0.123296 + 0.415021 I, -0.140243 - 0.396664 I,
-1.83013 + 2.04123 I, 0.0514951 - 0.72591 I, 3.4103 + 0.271398 I},
{0.123296 - 0.415021 I, -0.140243 + 0.396664 I,
-1.83013 - 2.04123 I, 0.0514951 + 0.72591 I, 3.4103 - 0.271398 I}}

TOTAL = -1.78082

NORMALIZED WEIGHTING FACTORS:
PRICE, FUEL ECON., ACCELERATION, BRAKING, HANDLING, STYLING
{8.81926, 6.0305, 26.6145, 11.6959, 46.8399}

Pairwise Comparison Data for Level 2, with respect to Handling

THE INPUT MATRIX:

$$\{\{1, 4, 3, \frac{1}{2}, 2\}, \{\frac{1}{4}, 1, 1, \frac{1}{5}, \frac{1}{3}\}, \{\frac{1}{3}, 1, 1, \frac{1}{5}, \frac{1}{3}\}, \{2, 5, 5, 1, 4\}, \{\frac{1}{2}, 3, 3, \frac{1}{4}, 1\}\}$$

EIGENVALUES:
{5.1016, -0.0509557 + 0.699 I, -0.0509557 - 0.699 I, 0.000155598 + 0.164797 I, 0.000155598 - 0.164797 I}

CI = 0.0254001

EIGENVECTORS:
{{-0.45893, -0.123658, -0.131155, -0.819433, -0.292267},
{0.415098 - 0.194285 I, -0.15666 + 0.0373273 I,
-0.183288 - 0.0102186 I, 0.617613 - 1.30765 I,
0.283628 + 0.514755 I}, {0.415098 + 0.194285 I,
-0.15666 - 0.0373273 I, -0.183288 + 0.0102186 I,
0.617613 + 1.30765 I, 0.283628 - 0.514755 I},
{-0.250641 + 0.980826 I, -0.250973 - 0.0589476 I,
0.244882 + 0.0682627 I, 0.160499 - 1.12052 I,
0.138981 - 0.215357 I}, {-0.250641 - 0.980826 I,
-0.250973 + 0.0589476 I, 0.244882 - 0.0682627 I,
0.160499 + 1.12052 I, 0.138981 + 0.215357 I}}

TOTAL = -1.82544

NORMALIZED WEIGHTING FACTORS:
PRICE, FUEL ECON., ACCELERATION, BRAKING, HANDLING, STYLING
{25.1407, 6.77414, 7.18481, 44.8896, 16.0107}

Pairwise Comparison Data for Level 2, with respect to STYLING

THE INPUT MATRIX:

$\{\{1, 3, 3, 1, 2\}, \{\frac{1}{3}, 1, 1, \frac{1}{3}, \frac{1}{2}\}, \{\frac{1}{3}, 1, 1, \frac{1}{3}, \frac{1}{2}\}, \{1, 3, 3, 1, 1\},$

$\{\frac{1}{2}, 2, 2, 1, 1\}\}$

EIGENVALUES:

$\{5.03938, -0.0196887 + 0.445028\ I, -0.0196887 - 0.445028\ I,$
$-1.18927\ 10^{-19}, -1.49436\ 10^{-20}\}$

CI = 0.00984433

EIGENVECTORS:

$\{\{-0.649003, -0.202418, -0.202418, -0.565505, -0.420777\},$
$\{0.417453 + 0.696455\ I, 0.0809893 - 0.00683931\ I,$
$0.0809893 - 0.00683931\ I, 0.068483 - 0.737491\ I,$
$-0.645016 + 0.127069\ I\}, \{0.417453 - 0.696455\ I,$
$0.0809893 + 0.00683931\ I, 0.0809893 + 0.00683931\ I,$
$0.068483 + 0.737491\ I, -0.645016 - 0.127069\ I\},$
$\{-1.49071, 0.372678, 0.372678, -0.745356, -4.80857\ 10^{-19}\},$
$\{0.345171, -0.7934, 0.620814, 0.172586, -2.9484\ 10^{-20}\}\}$

TOTAL = -2.04012

NORMALIZED WEIGHTING FACTORS:
PRICE, FUEL ECON., ACCELERATION, BRAKING, HANDLING, STYLING
{31.812, 9.92186, 9.92186, 27.7192, 20.6251}

Overall Results

PROGRAM INSTRUCTIONS:

```
M = M1.{M2, M3, M4, M5, M6, M7};
     Multiply the Level 2 matrices by the Level 1 matrix.

Total = Sum[ M[[t]], {t,5}];

FinalRating = M*100/Total
```

FINAL RANKING:
Mustang GT, Tempo Sport, Prelude, Corolla GTS, VW GTI
{18.6876, 24.6153, 12.8554, 22.6323, 21.2095}

NOTE: These final results are expressed in percent.

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