DOE/PC/90185--T7

2

DOE/PC/90185--T7

DE93 004933

Quarterly Progress Report

July 1, 1992 - September 30, 1992

Principal Investigator: Mark W. Richman

Contract Number: DE-AC22-91PC90185

Worcester Polytechnic Institute.

Introduction

In this quarter, we continued our study of the effects of vibrating boundaries on granular assemblies. We extended our earlier work on *isotropically* vibrating boundaries by employing formal methods of statistical averaging to calculate the rates at which momentum and energy are transferred from *anisotropically* fluctuating bumpy boundaries to dense granular assemblies.

The assemblies consist of identical, smooth, nearly elastic spheres that are thermalized by repeated collisions with the boundaries, but experience no mean motion as a consequence of these collisions. The boundaries vibrate with velocities that are governed by a tri-axial Gaussian distribution function that depends on both the normal and tangential mean square fluctuation speeds of the boundaries. Using the transfer rates calculated, we have written down conditions that ensure that momentum and energy are balanced at such boundaries, and have employed these conditions with a corresponding kinetic constitutive theory to analyze steady, gravity-free, thermalized states of granular assemblies between parallel, vibrating, bumpy boundaries.

This work is described in detail in attached paper, "The Effects of Anisotropic Boundary Vibrations on Confined, Thermalized, Granular Assemblies," prepared for and presented at the Joint NSF/DOE Workshop on the Flow of Particulates and Fluids, in Gaithersburg, MD, September 17-18, 1992.

THE EFFECTS OF ANISOTROPIC BOUNDARY VIBRATIONS ON CONFINED, THERMALIZED, GRANULAR ASSEMBLIES

Mark W. Richman and Richard E. Martin

Mechanical Engineering Department
Worcester Polytechnic Institute
Worcester, Massachusetts 01609

Abstract

In this paper, we employ formal methods of statistical averaging to calculate the rates at which momentum and energy are transferred from anisotropically fluctuating bumpy boundaries to dense granular assemblies. The assemblies consist of identical, smooth, nearly elastic spheres that are thermalized by repeated collisions with the boundaries, but experience no mean motion as a consequence of these collisions. The boundaries vibrate with velocities that are governed by a tri-axial Gaussian distribution function that depends on both the normal and tangential mean square fluctuation speeds of the boundaries. Using the transfer rates calculated, we write down conditions that ensure that momentum and energy are balanced at such boundaries, and employ these conditions with a corresponding kinetic constitutive theory to analyze steady, gravity-free, thermalized states of granular assemblies between parallel, vibrating, bumpy boundaries. We find that, as the boundaries become bumpier, vibrations that are tangent the boundaries become more effective and vibrations that are normal to the boundaries become less effective at transferring energy to the assemblies.

Introduction

Of interest here are the effects of vibrating boundaries on the granular flows with which they interact. Such boundaries may be employed with great advantage to induce granular flows that would otherwise not occur, and to enhance flows that would otherwise be driven by less effective means. For this reason, the effects of vibrating boundaries on granular flows have been the subject of much recent work, including the experiments of Savage [1988], Thomas, et. al. [1989], Jaeger et. al. [1989], Evesque and Rajchenbach [1989], Akiyama and Shimomura [1991], and Liu and Nagel [1992]; the theoretical work of Jackson [1991], Potanin [1992], and Richman [1992]; and the computer simulations of Rosato [1992]. In particular, Richman [1992] employed formal methods of statistical averaging to obtain expressions for the rates at which momentum and energy are exchanged between a granular flow of identical, smooth, nearly elastic spheres and a bumpy boundary that fluctuates isotropically.

In this paper, we focus on dense assemblies of identical spheres that interact with vibrating boundaries that induce no mean motion as a consequence of the interactions. In this manner, we eliminate the effects of slip work at the boundaries and stress power throughout the assemblies, and isolate the effects of boundary vibrations on the resulting thermalized states. In particular, we consider bumpy boundaries that vibrate anisotropically with fluctuation velocities that are governed by a tri-axial Gaussian distribution function. Employing this velocity distribution for the boundaries and a Maxwellian as the lowest order distribution for the assembly particles, we calculate the statistically averaged rates at which momentum and energy are transferred from the boundaries to the assemblies. The resulting expressions depend explicitly on the bumpiness and mean normal and tangential fluctuation speeds of the boundaries. Finally, we employ the expressions for the transfer rates in conditions that ensure that momentum and energy are balanced at the bumpy boundaries of interest.

As an application of the boundary conditions, we employ them and a specialization of the constitutive theory of Jenkins and Richman [1985] to calculate the solid fraction and granular temperature variations throughout dense, gravity-free, assemblies of identical, smooth, inelastic spheres that are confined between and thermalized by the vibrations of parallel, bumpy boundaries. In presenting the results, we pay special attention to the separate effects of normal vibrations, tangential vibrations, and boundary bumpiness on the thermalized states induced.

Rates of Momentum and Energy Transfer

We are concerned here with the rates at which momentum and energy are transferred from vibrating bumpy boundaries to assemblies of identical, smooth, inelastic spheres of diameter σ and mass m. These transfer rates depend on the geometry, dissipative nature, and vibratory motion of the boundaries. Here we focus on flat surfaces to which identical, smooth, hemispherical particles of diameter d are randomly attached at an average distance s apart in such a way that it is not possible for the flow particles to collide with the flat surfaces. The bumpiness of the boundaries is measured by the angle $\theta \equiv \sin^{-1}(d+s)/(d+\sigma)$, which increases from 0 to $\pi/2$ as the boundaries evolve from perfectly flat to extremely bumpy.

The distributions governing the velocities $\mathbf{c}$ of flow particles and the velocities $\mathbf{C}$ of boundary particles are $f(\mathbf{c}, \mathbf{r})$ and $p(\mathbf{C})$, defined such that $f(\mathbf{c}, \mathbf{r})d\mathbf{c}$ gives the number of flow particles per unit volume at $\mathbf{r}$ with velocities $\mathbf{c}$ within the range $d\mathbf{c}$, and $p(\mathbf{C})d\mathbf{C}$ gives the probability that a boundary particle has velocity $\mathbf{C}$ within $d\mathbf{C}$. The mean velocity $\langle\mathbf{C}\rangle$ of the boundary, and the full second moment $\langle\mathbf{C}\otimes\mathbf{C}\rangle$ of its velocity are defined by,

$$\langle C \rangle \equiv \int C\, p(C) dC \quad \text{and} \quad \langle C \otimes C \rangle \equiv \int C \otimes C\, p(C) dC \quad , \tag{1}$$

where the integrations are carried out over all velocities **C**. We restrict our attention to boundaries that vibrate about zero mean velocity; i.e. $\langle C \rangle = 0$.

At impact between a boundary particle and a flow particle, the unit vector directed from the center of the former to the center of the latter is **k**, the boundary particle center is located at **x**, and the distance between centers is $\delta \equiv (\sigma + d)/2$. The frequency per unit area of flat surface of collisions between flow particles (with velocities **c** within d**c**) and boundary particles (with velocities **C** within d**C**) that meet within solid angle d**k** centered about **k** is then given by,

$$\frac{\chi}{\pi \sin^2\theta} f(\mathbf{c}, \mathbf{x}+\delta\mathbf{k}) p(\mathbf{C}) (\mathbf{g} \cdot \mathbf{k}) d\mathbf{k}\, d\mathbf{c}\, d\mathbf{C} \quad , \tag{2}$$

where $\mathbf{g} \cdot \mathbf{k}$ must be positive for a collisions to occur. The factor χ accounts for the effects of excluded volume and the shielding of flow particles from boundary particles by other flow particles.

At the instant just prior to collision, the impact velocity as observed from the boundary particle is $\mathbf{g} \equiv \mathbf{C} - \mathbf{c}$. If e_w is the coefficient of restitution that characterizes the energy lost in such a collision and the velocity of the boundary is unaffected by the collision, then the change in mometum experienced by the flow particle is,

$$m(\mathbf{c}^* - \mathbf{c}) = m(1+e_w)(\mathbf{g} \cdot \mathbf{k})\mathbf{k} \quad , \tag{3}$$

where $\mathbf{c}^*$ is the velocity of the flow particle just after impact; the corresponding change in energy is,

$$\frac{m}{2} (\mathbf{c}^* \cdot \mathbf{c}^* - \mathbf{c} \cdot \mathbf{c}) = m(1+e_w)(\mathbf{g} \cdot \mathbf{k}) \left[(\mathbf{C} \cdot \mathbf{k}) - \frac{1}{2}(1-e_w)(\mathbf{g} \cdot \mathbf{k}) \right] \quad . \tag{4}$$

As expected, the contribution to the change in energy from the inelasticity of the particles is always negative. However, the contribution from the absolute motion of the boundary may be positive or negative, depending on whether the boundary particle moves towards $(\mathbf{C} \cdot \mathbf{k} > 0)$ or away from $(\mathbf{C} \cdot \mathbf{k} < 0)$ the flow particle as they strike.

The net rates at which momentum and energy are transferred from the boundaries to the assemblies are calculated as statistical averages of the appropriate changes experienced by a flow particle in a single collision with the boundary. If, for example, **M** is the rate per unit area of flat surface at which momentum is supplied to the flow by the boundary, then, from equation (3), **M** is the average of the momentum change $m(1+e_w)(\mathbf{g}\cdot\mathbf{k})\mathbf{k}$ weighted by the collision frequency (2) and integrated over all points of contact **k** that are accessible to flow particles and all incoming velocities **c** and **C** for which $\mathbf{g}\cdot\mathbf{k}$ is positive. Similarly, if F is the rate per unit area at which energy is supplied to the flow due to the vibratory motion of the boundary, and D is the rate per unit area at which it is absorbed due to dissipative collisions with the boundary, then, from equation (4), F and D are the corresponding weighted averages of the contributions $m(1+e_w)(\mathbf{g}\cdot\mathbf{k})(\mathbf{C}\cdot\mathbf{k})$ and $m(1-e_w^2)(\mathbf{g}\cdot\mathbf{k})^2/2$ to the total energy change. The net rate at which the boundary supplies energy to the flow is equal to the difference F-D.

In order to carry out the averaging procedure, it is necessary to write down expressions for the velocity distribution functions. To this end, we introduce an x_1-x_2-x_3 Cartesian coordinate system in which the x_2-direction is normal to the flat parts of the bumpy boundaries, and focus on fluctuating boundaries whose velocities are described statistically by the tri-axial Gaussian,

$$p(\mathbf{C}) = \frac{1}{(2\pi)^{3/2} v_1 v_2 v_3} \exp\left[\frac{-1}{2}\left(\frac{C_1^2}{v_1^2} + \frac{C_2^2}{v_2^2} + \frac{C_3^2}{v_3^2}\right)\right] \quad . \tag{5}$$

Calculated according to integral definition (1), the x_1-x_2-x_3 components of the full second moment are then simply,

$$\langle \mathbf{C}\otimes\mathbf{C} \rangle = \begin{bmatrix} v_1^2 & 0 & 0 \\ 0 & v_2^2 & 0 \\ 0 & 0 & v_3^2 \end{bmatrix} \quad . \tag{6}$$

In this manner, we further restrict our attention to boundaries with second moments that are diagonal in Cartesian coordinate systems that are themselves aligned with the boundaries. Furthermore, we approximate the flow particle velocity distribution $f(\mathbf{c}, \mathbf{r})$ as Maxwellian, which, in the absence of any mean flow velocity, is given in terms of the particle number density n and granular temperature w^2 by,

$$f(\mathbf{c}, \mathbf{r}) = \frac{n}{(2\pi w^2)^{3/2}} \exp\left(\frac{-\mathbf{c}\cdot\mathbf{c}}{2w^2}\right) \quad . \tag{7}$$

For thermalized assemblies of nearly elastic particles that experience no mean motion, the largest corrections to the Maxwellian that we have neglected are proportional to gradients of granular temperature.

When the mean velocity vanishes and the gradients of temperature and solid fraction are normal the boundary, the transfer rates are insensitive to permutations of the tangential x_1- and x_3-directions. For this reason, we carry out the averaging procedure for the special cases in which the tangential fluctuation speeds v_1 and v_3 are equal. Under these circumstances, the lowest order expression for the rate of momentum supplied to the assembly due to nearly elastic collisions with the boundary is given by,

$$\mathbf{M} = \rho\chi\left[(w^2+v_2^2) + \frac{1}{2}(v_1^2-v_2^2)\sin^2\theta\right]\mathbf{N} \quad , \tag{8}$$

where $\rho \equiv nm$ is the mass density of the flow, and $\mathbf{N}$ is the unit inward normal to the boundary. Because the x_1-, x_2-, and x_3-directions are the principal directions of $\langle \mathbf{C}\otimes\mathbf{C}\rangle$, only momentum normal to the boundaries is supplied to the assemblies. Interestingly, as the bumpiness of the boundary increases, the rate at which momentum is supplied may increase or decrease, depending on whether the tangential fluctuation speed v_1 is greater or less than the normal fluctuation speed v_2.

In terms of the ratio $R \equiv (v_2^2-v_1^2)/(w^2+v_1^2)$, the lowest order expression for the rate of energy absorbed due to dissipative collisions is,

$$D = \left(\frac{2}{\pi}\right)^{1/2} 2\rho\chi(1-e_w)(w^2+v_2^2)^{3/2}\frac{\csc^2\theta}{8(1+R)^{3/2}}\left[H(R,\theta) + 3I(R,\theta)\right] \quad , \tag{9}$$

in which the functions H and I are defined by,

$$H(R,\theta) \equiv (1+R)^{1/2}(5+2R) - (1+R\cos^2\theta)^{1/2}(5+2R\cos^2\theta)\cos\theta \quad , \tag{10}$$

and

$$I(R,\theta) \equiv \begin{cases} \dfrac{1}{\sqrt{R}}\left[\sinh^{-1}\sqrt{R} - \sinh^{-1}(\sqrt{R}\cos\theta)\right] & \text{when } R>0 \\ \\ \dfrac{1}{\sqrt{-R}}\left[\sin^{-1}\sqrt{-R} - \sin^{-1}(\sqrt{-R}\cos\theta)\right] & \text{when } R<0 \ . \end{cases} \tag{11}$$

The corresponding expression for the rate of energy supplied due to the vibratory motion of the boundary is,

$$F = \left(\frac{2}{\pi}\right)^{1/2} 4\rho\chi(w^2+v_2^2)^{1/2} \frac{\csc^2\theta}{(1+R)^{1/2}} \left\{\frac{1}{2}\left[\frac{(w^2+v_1^2)}{4} - v_1^2\right] J(R, \theta) - \frac{(w^2+v_1^2)}{4} K(R, \theta)\right\} \quad , \qquad (12)$$

where J and K are defined by,

$$J(R, \theta) \equiv (1+R\cos^2\theta)^{1/2}\cos\theta - (1+R)^{1/2} - I(R, \theta) \quad , \qquad (13)$$

and

$$K(R, \theta) \equiv (1+R\cos^2\theta)^{3/2}\cos\theta - (1+R)^{3/2} \quad . \qquad (14)$$

Expressions for **M**, D and F have been obtained by Richman [1992] when the boundary vibrations are isotropic, and depend on mean square fluctuation speed $v^2 \equiv tr(\langle C \otimes C \rangle)/3$ rather than on the individual components of $\langle C \otimes C \rangle$. In this special case, the tangential and normal velocities v_1 and v_2 are equal, the ratio R vanishes, and expressions (8), (9), and (12) for **M**, D, and F reduce to those obtained by Richman [1992]. If the boundaries are flat, then θ is equal to zero, and even for anisotropic vibrations **M**, D, and F are insensitive to v_1. In this limit, only after we replace v_2 by v wherever it occurs, do our results reduce to those obtained by Richman [1992].

Thermalized States of Confined Assemblies

We focus attention on steady, gravity-free, thermalized states of assemblies of identical, smooth, inelastic spheres that are confined between two infinite, parallel bumpy boundaries that randomly vibrate with no mean velocities. The spheres are of mass density α, and coefficient of restitution e. The boundaries are separated by a fixed distance 2L, have mean square fluctuation velocities $3v^2 \equiv (2v_1^2+v_2^2)$, and vibrate in the manner described in the previous section. Under these circumstances, the velocity field and the slip velocity vanish, and the variations in granular temperature w^2 and solid fraction ν are induced entirely by the motions of the boundaries.

We employ an x_1-x_2-x_3 Cartesian coordinate system oriented as described in the previous section. The boundaries are located at $x_2=+L$ and $x_2=-L$ and extend infinitely in the x_1- and x_3-directions. The solid fraction ν and the dimensionless measure $W \equiv w/v$ of granular temperature depend only on the dimensionless distance $y \equiv x_2/\sigma$ from the midplane between the boundaries. The dimensionless perpendicular distance between the boundaries is 2β, where β is the ratio L/σ.

In these agitated states, the balance of mass is satisfied identically. Furthermore, if $-P_{12}$ and P_{22} are the shear stress and normal pressure throughout the assembly and $S \equiv -P_{12}/\alpha v^2$ and $P \equiv P_{22}/\alpha v^2$ are their dimensionless counterparts, the x_1- and x_2-components of the balance of momentum are simply,

$$S' = 0 \quad \text{and} \quad P' = 0 \quad , \tag{15}$$

where primes denote differentiation with respect to y. Because only a normal force is required to keep the distance between the boundaries fixed, equations (15) demonstrate that the shear stress vanishes throughout, and the normal pressure is everywhere equal to the pressure applied at the boundaries. Finally, if Q_2 is the x_2-component of the energy flux, γ is the collisional rate of energy dissipation, and $q \equiv -Q_2/\alpha v^3$ and $\Gamma \equiv \sigma\gamma/\alpha v^3$ are their dimensionless counterparts, then the energy equation reduces to,

$$q' - \Gamma = 0 \quad . \tag{16}$$

In these thermalized states, energy is conducted to balance the rate at which it is dissipated.

To compliment the balance equations, we employ the kinetic constitutive theory of Jenkins and Richman [1985], which applies to flows of nearly elastic spheres in which the transfer of momentum and energy occurs entirely by particle transport and particle collisions. For simplicity, we focus attention on dense flows and ignore the effects of particle transport. In this limit, the normal pressure P is given by,

$$P = 4\nu G W^2 \quad , \tag{17}$$

where G is the function of solid fraction given by $\nu(2-\nu)/2(1-\nu)^3$; the energy flux is given by,

$$q = \frac{2MPW'}{\sqrt{\pi}} \quad , \tag{18}$$

where M is equal to $1+9\pi/32$; and the energy dissipation is given by,

$$\Gamma = \frac{6(1-e)PW}{\sqrt{\pi}} \quad . \tag{19}$$

In principle, for fixed normal pressures P, equations (16), (17), (18), and (19) determine $W(y)$, $\nu(y)$, $q(y)$, and $\Gamma(y)$ to within two constants of integration.

To eliminate q and Γ, we employ constitutive relations (18) and (19) in equation (16). In this manner, the energy equation reduces to,

$$W'' - \lambda^2 W = 0 \quad , \tag{20}$$

where $\lambda^2 \equiv 3(1-e)/M$. The profile W(y) is therefore given by the solution,

$$W = \frac{\Omega \cosh \lambda y}{\cosh \lambda \beta} \quad , \tag{21}$$

in which, because of symmetry, we have ensured that $W'(y=0)=0$. Only the constant $\Omega \equiv W(y=0)$ remains to be determined. Its value is fixed by appropriate conditions at the bumpy boundaries ($y=\pm\beta$).

At the vibrating surfaces that induce the thermalized states of interest, the normal pressure is determined by the rate M_2 per unit area at which momentum in the x_2-direction is supplied to the assembly by the boundaries, and the energy flux is determined by the competition between the rate F per unit area at which energy is supplied to the assembly by vibrations of the boundaries and the rate D per unit area at which it is absorbed from the assembly due to dissipative collisions with the boundaries. If $\mathbf{M} \equiv -M_2/\alpha v^2$, $\mathbf{F} \equiv F/\alpha v^3$, and $\mathbf{D} \equiv D/\alpha v^3$ are the corresponding dimensionless transfer rates, then, at the upper ($y=\beta$) vibrating boundary the balance of momentum requires that,

$$\mathbf{M} = P \quad , \tag{22}$$

and the balance of energy requires that,

$$\mathbf{F} - \mathbf{D} = q \quad . \tag{23}$$

When neither the shear stress nor the slip velocity vanishes, the energy balance at the boundary also includes a contribution from slip work, which is an additional mechanism by which the boundaries may supply energy to the flows.

In order to nondimensionalize M_2, F, and D, we introduce the dimensionless fluctuation speeds $V_t \equiv v_1/v$ and $V_n \equiv v_2/v$ that satisfy the relation $V_n^2+2V_t^2=3$. Defined in this manner $V_n^2/3$ is the fraction of total boundary energy that is due to normal motion. In particular, the boundary's energy is divided isotropically in the three coordinate directions when $V_n^2=1$, and is divided evenly between normal and tangential motion when $V_n^2=3/2$.

We employ the x_2-component of the rate (8) of momentum transfer in condition (22) to obtain,

$$\nu\chi(\Omega^2 + V_n^2) = P\left[1 - \frac{R\sin^2\theta}{2(1+R)}\right]^{-1} \quad , \tag{24}$$

which determines the factor χ so that the solid fraction at the boundary is a free parameter. In terms of dimensionless quantities, the ratio R is given by $(V_n^2 - V_t^2)/(\Omega^2 + V_t^2)$. Finally, we employ energy transfer rates (12) and (9), and constitutive relation (18), to replace **F**, **D**, and q in condition (23). If equation (24) is used to eliminate χ and solution (21) is used to eliminate W′ from the intermediate result, then the equation that determines the unknown temperature Ω is given by,

$$\frac{M\lambda\tanh\lambda\beta}{\sqrt{2}\csc^2\theta}\left[1 - \frac{R\sin^2\theta}{2(1+R)}\right]\Omega(\Omega^2+V_n^2)^{1/2}(1+R)^{1/2} =$$

$$= \left[\frac{(\Omega^2+V_t^2)}{4} - V_t^2\right]J - \frac{(\Omega^2+V_t^2)}{2}K - (1-e_w)(\Omega^2+V_n^2)\frac{(H+3I)}{8(1+R)} \quad , \tag{25}$$

where the functions J(R,θ), K(R,θ), H(R,θ), and I(R,θ) are defined by equations (13), (14), (10), and (11). Because we have focused on dense assemblies, ν appears in neither the energy equation (20) nor the energy flux boundary condition (25), and the granular temperature profile is independent of solid fraction.

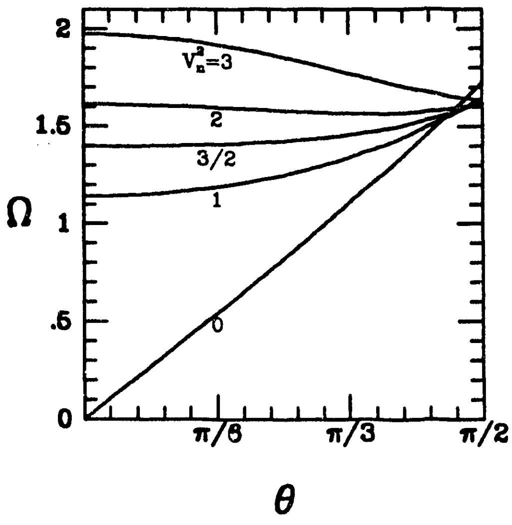
With β, e, e_w, θ, V_n, and V_t prescribed, the granular temperature Ω at y=β is determined by equation (25), and the granular temperature profile is fixed by equation (21). Then, with P prescribed, the solid fraction profile and its depth-averaged value,

$$\bar{\nu} \equiv \frac{1}{2\beta}\int_{-\beta}^{\beta}\nu(y)dy \quad , \tag{26}$$

are determined by inverting constitutive relation (17). If, instead, $\bar{\nu}$ is prescribed, then we guess at a value of P and iterate on the guess until $\bar{\nu}$ calculated according its definition (26) agrees with its prescribed value. In either case, the solid fraction profiles are symmetric about y=0 and increase monotonically from the boundaries to the midplane.

Results and Discussion

Of primary interest are the effects of the boundaries' motion, geometry, and dissipative character on the thermalized states that their vibrations induce. For this reason, in carrying out the solution procedure described



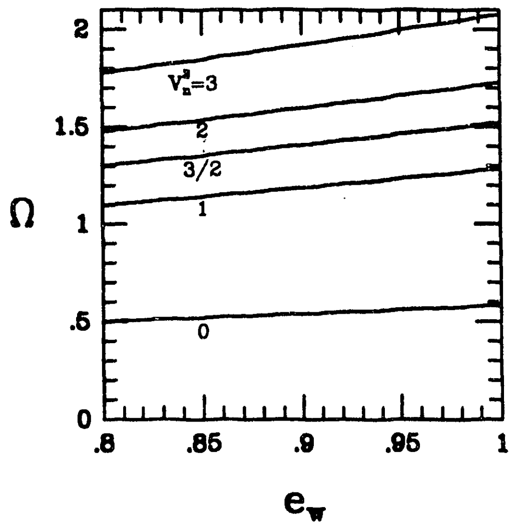


Figure 1: The variations of Ω with θ for V_n^2=0, 1, 3/2, 2, and 3, when $e=e_w$=.9, and β=5.

Figure 2: The variations of Ω with e_w for V_n^2=0, 1, 3/2, 2, and 3, when $\theta=\pi/6$, e=.9, and β=5.

above, we have varied only V_n, θ, and e_w. All the results presented here are for e=.9, and β=5.

In Figure 1, we plot the variations of the granular temperature Ω at $y=\beta$ with bumpiness θ for normal fluctuation speeds V_n^2=0, 1, 3/2, 2, and 3, when e_w=.9. When the vibrations are due entirely to tangential motion (V_n^2=0), the temperatures *increase* from zero as the boundaries evolve from perfectly flat (θ=0) to extremely bumpy ($\theta=\pi/2$), as expected. However, when the vibrations are due entirely to normal motion (V_n^2=3), the temperatures actually *decrease* as the boundaries becomes bumpier and experience fewer normal and more oblique impacts. These two extreme cases demonstrate the competing dynamics that determine the influence of bumpiness on vibrationally induced granular temperatures. Interestingly, the increase in Ω with θ when V_n^2=0 is far more pronounced than the corresponding decrease in Ω when V_n^2=3. Consequently, about two-thirds of the boundaries' energy must be in normal motion (V_n^2=2) for these competing effects to roughly cancel over the full range of θ.

Figure 1 also demonstrates that, for fixed values of θ between 0 and 1.41, the granular temperatures throughout the assemblies *increase* monotonically as the energy of tangential vibration is converted to energy of normal vibration. However, the differences between the temperatures induced by pure tangential motion and those induced by pure normal motion diminish as θ increases in this range. This is because tangential vibrations become more effective and normal vibrations become less effective at tranferring energy to the spheres as the boundaries become bumpier. In fact, the theory predicts that when the boundaries are extremely bumpy (θ>1.41),

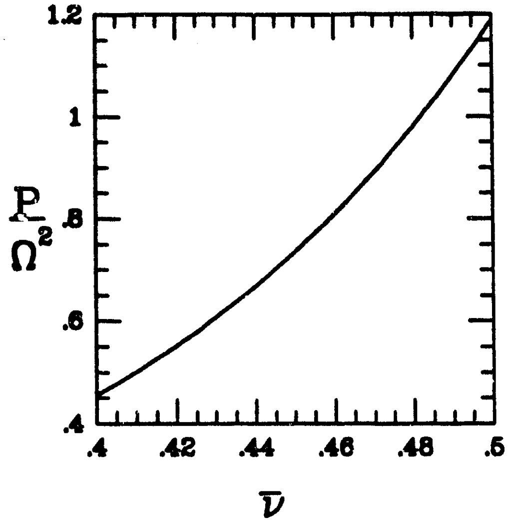


Figure 3: The variation of P/Ω^2 with ν for e=.9 and β=5.

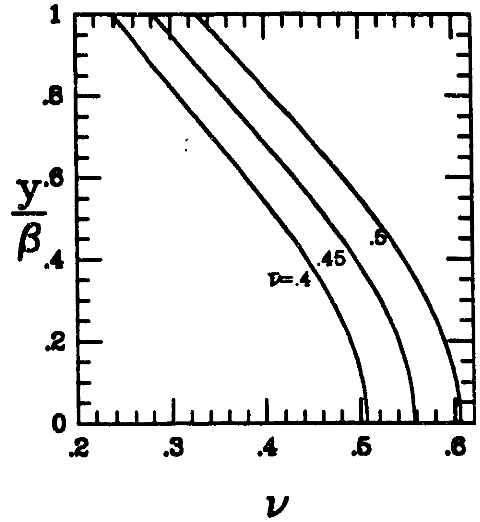


Figure 4: The variations of ν with y/β for $\bar{\nu}$=.4, .45, and .5, when e=.9 and β=5.

energy is transferred most effectively by tangential vibrations. Under these circumstances, the granular temperatures actually *decrease* as the energy of tangential vibrations is converted to energy of normal vibrations.

In Figure 2 we show the variations of Ω with e_w between .8 and 1.0 for V_n^2=0, 1, 3/2, 2, and 3, when $\theta=\pi/6$. As expected, for fixed values of V_n^2, the granular temperatures throughout the assemblies increase monotonically as the collisions between boundary and flow particles become less dissipative. This trend becomes somewhat more pronounced as V_n^2 increases because e_w is related to the energy lost in normal impacts, which occur more frequently as tangential vibrations are converted to normal vibrations.

In the dense assemblies of interest here, the profiles W(y), given in closed form by solution (21), depend on V_n, θ, e_w, e, and β, but are independent of solid fraction. Conversely, according to constitutive relation (17), both the ratio P/Ω^2 and the solid fraction profile ν(y) depend on e, β and ν, but are independent of V_n, θ, and e_w. In Figure 3, for example, we show the dependence of P/Ω^2 on $\bar{\nu}$ between .4 and .5 for e=.9 and β=5. Independent of boundary effects, the constitutive behavior of the assemblies dictates that as $\bar{\nu}$ increases from .4 to .5, the normal pressure nearly triples. In Figure 4, we plot the solid fraction profiles corresponding to $\bar{\nu}$=.4, .45, and .5. Because the temperature *decreases* monotonically from the boundaries to the midplane, the solid fraction must *increase* to ensure that the normal pressure is constant throughout the assemblies.

As a numerical example, we consider the case in which $\theta=\pi/6$, e_w=e=.9, and β=5. Under these circumstances, as V_n^2 increases from 0 to 3, the boundary value Ω of W increases by a factor of 3.55 from .54 to 1.92, and the midplane value of W increases from .15 to .51. If, in addition, $\bar{\nu}$=.45, then regardless of V_n^2, the ratio P/Ω^2 is equal to .74, the boundary value of ν is .28, and the midplane value of ν is .56. As V_n^2 increases from 0 to 3, the pressure P increases by more than a factor of 12 from .22 to 2.73, while the solid fraction profile remains unchanged.

For assemblies that are less dense than those considered here, the energy equation, the energy flux boundary condition, and the temperature profiles all depend on the solid fraction. However, just as in the dense assemblies considered here, both the ratio P/Ω^2 and the solid fraction profile are independent of the parameters V_n, θ, and e_w that describe the boundary.

Acknowledgements

The authors are grateful to the Pittsburgh Energy Technology Center of the U.S. Department of Energy for their support of this work under contract DE-AC22-91PC90185.

References

Akiyama, T., Shimomura, T., 1991, Investigation of Wall Shear Stress in Vibrating Particle Beds, Powd. Tech., Vol. 66, pp. 243-247.

Evesque, P., Rajchenbach, J., 1989, Instability in Sand Heaps, Phys. Rev. Lett., Vol. 62, pp. 44-46.

Jackson, R., 1991, Final Project Report, US DOE, Contract DE-FG22-86PC90518.

Jaeger, H.M., Liu, C., Nagel, S.R., 1989, Relaxation at the Angle of Repose, Phys. Rev. Lett., Vol. 62, pp. 40-43.

Jenkins, J.T., Richman, M.W., 1985, Grad's 13-Moment System for a Dense Gas of Inelastic Spheres, Arch. Rat. Mech. Anal., Vol. 87, pp. 355-377.

Liu, C., Nagel, S., 1992, Sound in Sand, Phys. Rev Lett., under review.

Mehta, A., Barker, G.C., 1991, Vibrated Powders: A Microscopic Approach, Phys. Rev. Lett., Vol. 67, pp. 394-397.

Potanin, A.A., 1992, On Models of Granular Material Flow Under Dynamic Conditions, Powd. Tech., Vol. 69, pp. 107-117.

Richman, M.W., 1992, Boundary Conditions for Granular Flows at Randomly Fluctuating Bumpy Boundaries, Mech. Mat'ls., in press.

Rosato, A, 1992, Computer Simulations of Unconfined Granular Assemblies Agitated by Flat, Horizontal, Vibrating Surfaces, in preparation.

Savage, S.B., 1988, Streaming Motions in a Bed of Vibrationally Fluidized Dry Granular Material, J. Fluid Mech., Vol. 194, pp. 457-478.

Thomas, B., Mason, M.O., Liu, Y.A., Squires, A.M., 1989, Identifying States in Shallow Vibrated Beds, Powder Tech., Vol. 57, pp. 267-280.

DATE
FILMED
01/20/93