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Quarterly Progress Report

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Introduction

In this quarter, we studied the effects of isotropic boundary vibrations on unconfined granular materials that are compressed by gravity and dilated by the agitation of horizontal, vibrating boundaries. Because the vibrations are isotropic, the assemblies experience no mean motion. Consequently, the resulting boundary value problem is simpler than the boundary value problem that results when the vibrating surface is inclined. However, in both cases, we confront difficulties associated the occurrence of a free surface, and must implement recently derived boundary conditions for vibrating boundaries. During the next quarter, we will extend this work to isotropically vibrating inclines, and during the following quarter we will begin work on anisotropically vibrating boundaries.

In what follows, we calculate the steady state solid fraction and granular temperature profiles throughout assemblies of identical, smooth, inelastic spheres that are unconfined from above and thermalized from below by bumpy, horizontal surfaces that randomly vibrate about zero mean velocity. The analysis is based upon a kinetic constitutive theory and conditions that account for the effects of boundary vibrations on the momentum and energy transferred to the spheres. In presenting the results, we pay special attention to the thermalized states of fixed masses subjected to varying boundary vibrations and varying masses subjected to fixed vibrations. In the first study, we find that as the vibration of the boundary intensifies, the assemblies of fixed mass become deeper, more dilute, and more thermalized. In the second, we find that as the assemblies become more massive, fixed boundary vibrations effectively thermalize decreasing fractions of the total mass.

Boundary Value Problem

We are concerned here with the steady states of dry granular assemblies that are unconfined from above and thermalized from below by horizontal, bumpy boundaries that vibrate isotropically with no mean velocity. The assemblies consist of identical, smooth, inelastic spheres of diameter σ, mass density α, and coefficient of restitution e. Although the spheres are agitated by the vibrating boundary, they experience no mean motion.

We establish an x_1-x_2-x_3 Cartesian coordinate system such that the x_2-direction is perpendicular to, and the x_1-x_3 plane coincides with the flat part of the bottom boundary. In this coordinate system, the material extends infinitely without change in the x_1- and x_3-directions, and is constrained by the gravitational acceleration g in the negative x_2-direction. The top of the assembly is located at the plane x_2=L. Throughout the material, the mean velocity vanishes, while the solid fraction ν and measure $w \equiv (T/\sigma g)^{1/2}$ of the granular temperature T vary with the dimensionless distance $y \equiv (L - x_2)/\sigma$ only.

In this simple agitated state, the shear stress vanishes throughout the assembly, and the balance equations of mass and momentum in the x_1- and x_3-directions are satisfied identically. If P_{22} is the x_2-x_2 component of the pressure tensor and $N \equiv P_{22}/\alpha\sigma g$ is its dimensionless counterpart, then the x_2-component of the balance of momentum is given by,

$$N' = \nu \quad , \tag{1}$$

where a prime denotes differentiation with respect to y. Furthermore, if Q_2 is the x_2-component of the energy flux, γ is the rate of energy dissipation, and $q \equiv Q_2/\alpha(\sigma g)^{3/2}$ and $\Gamma \equiv \gamma/\alpha\sigma^{1/2}g^{3/2}$ are their dimensionless counterparts, then the balance of energy reduces to,

$$q' - \Gamma = 0 \quad . \tag{2}$$

According to equations (1) and (2), the normal pressure increases with depth due to the weight of the material, while energy must be conducted to compensate for the rate at which it is dissipated.

In order to complete the system of equations that determine the variations of solid fraction and granular temperature, we employ the constitutive theory derived by Jenkins and Richman [1985]. Under the circumstances of interest here, the normal pressure is given by,

$$N = 4\nu G F w^2 \quad , \tag{3}$$

where G is the product of ν and the equilibrium radial distribution function at impact, and $F \equiv 1 + 1/4G$; the energy flux is given by,

$$q = \frac{2MNw'}{\sqrt{\pi}\, F} \quad , \tag{4}$$

where $M \equiv 1 + 9\pi(1 + 5/12G)^2/32$; and the rate of energy dissipation is given by,

$$\Gamma = \frac{6(1-e)Nw}{\sqrt{\pi}\,F} \quad . \tag{5}$$

Here we take $G \equiv v/(1-v/v_m)^{5v_m/2}$. This form of G is based on the radial distribution function at impact proposed by Lun and Savage [1986], which has the correct value (=1) and slope (=5/2) in the dilute limit, agrees well with that proposed by Carnahan and Starling [1969] for all values of v up to .5, and becomes unbounded as v approaches v_m so that collisions between particles may support high pressures at low granular temperatures. The results presented in the next section are based on the random close packed value v_m=.65.

Finally, we employ constitutive relations (4) and (5) to eliminate q and Γ from equation (2). In this manner, the energy equation becomes,

$$\frac{w''}{w} + \frac{(1-2H)N'}{N}\frac{w'}{w} + 4H\left(\frac{w'}{w}\right)^2 - \frac{3(1-e)}{M} = 0 \quad , \tag{6}$$

where H is the function of solid fraction defined by,

$$2H \equiv \frac{-d[\ln(M/F)]/dv}{d[\ln(vGF)]/dv} \quad , \tag{7}$$

that decreases monotonically from 1/2 to 0 as v increases from 0 to its maximum value v_m. Equations (1), (3), and (6) determine N, v, and w to within three constants of integration.

At the top of the assembly, y is equal to 0, and both the normal pressure and energy flux vanish; that is,

$$N(0) = 0 \qquad \text{and} \qquad w'(0) = 0 \quad . \tag{8}$$

At the vibrating base, y is equal to $\beta \equiv L/\sigma$, and the energy flux is determined by the competition between the rate S per unit area at which energy is supplied to the assembly by vibrations of the boundary and the corresponding rate D at which it is absorbed from the flow due to dissipative collisions with the boundary. If $S \equiv S/\alpha(\sigma g)^{3/2}$ and $D \equiv D/\alpha(\sigma g)^{3/2}$ are the dimensionless transfer rates corresponding to S and D, then the balance of energy at the vibrating base requires that,

$$S - D = q \quad . \tag{9}$$

A term corresponding to the slip work done at the boundary does not appear in energy balance (9) because, in the simple thermalized state of interest here, both the slip velocity and the shear stress vanish. For steady states to be maintained, the rate S must exceed D to compensate exactly for the dissipation from inelastic collisions throughout the entire assembly.

Expressions for the transfer rates S and D depend, in general, on the geometry, and dissipative character of the boundary. Here we are concerned with flat surfaces to which smooth hemispherical particles of diameter d are randomly attached at an average distance s apart, constructed to prevent flow particles from colliding with the flat surfaces. A convenient measure of the bumpiness of the boundaries is the angle θ defined by the relation $\sin\theta \equiv (d+s)/(d+\sigma)$. As θ increases from 0 to $\pi/2$, the boundaries evolve from perfectly flat to extremely bumpy. Collisions between boundary particles and assembly particles are nearly elastic and the coefficient of restitution between them is e_w.

The transfer rates also depend on the vibratory motion of the boundary. Here we focus on randomly vibrating bases whose velocities are governed by Gaussian distributions with root mean square fluctuation speeds V that are fixed by the dimensionless measure $v \equiv V/(\sigma g)^{1/2}$. At these boundaries and in the thermalized states of concern here, the expressions for S and D obtained by Richman [1992] reduce to:

$$S = \left(\frac{2}{\pi}\right)^{1/2} 4v^2(w^2 + v^2)^{-1/2} N(1-\cos\theta)\csc^2\theta \quad ; \tag{10}$$

and

$$D = \left(\frac{2}{\pi}\right)^{1/2} 2(1-e_w)(w^2 + v^2)^{1/2} N(1-\cos\theta)\csc^2\theta \quad . \tag{11}$$

As expected, S vanishes when v=0, and D vanishes when e_w=1.

Finally, we employ equations (10), (11), and (4) to eliminate S, D, and q from energy balance (9), and obtain a second boundary condition on w and its gradient:

$$\frac{w'}{w} = \frac{\sqrt{2}\,F(1-\cos\theta)\csc^2\theta}{Mw\sqrt{w^2+v^2}}\left[2v^2 - (1-e_w)(w^2+v^2)\right] \quad , \tag{12}$$

in which all functions of y are evaluated at y=β. The first and second terms on the right-hand-side of equation (12) are measures of the energy supplied to the assembly by the fluctuations of the boundary and energy absorbed in dissipative collisions with the boundary.

Equations (1), (3), and (6) determine N(y), v(y), and w(y) to within three constants of integration. These constants and the dimensionless depth β are determine by conditions (8) at y=0, condition (12) at y=β, and a fixed mass hold-up m_t calculated as the integral,

$$m_t = \int_0^\beta v\,dy \quad . \tag{13}$$

In principle, the solution procedure is as follows. For fixed values of e, e_w, and θ, we prescribe the mass hold-up m_t and the fluctuation speed v of the boundary, guess at the granular temperature w(0) at the top of the assembly, and numerically integrate equations (1), (3), and (6) from y=0 (where both N and w′ vanish) to the depth at which the mass hold-up assumes its prescribed value. Condition (12), which may be written as a quadratic equation for v^2, then determines the value of v that sustains that thermalized state. Finally, we iterate on the guess for w(0) until the value of v calculated in this manner agrees with its prescribed value. The distance β is the depth of the assembly in the final iteration.

In practice, the solution procedure is somewhat less straightforward. In order for the normal pressure to vanish at y=0, the solid fraction must equal zero there because the granular temperature does not. Consequently, not only does w′ vanish at y=0, but, according to equations (1), (3), (6), so too do N′, v′, and w". Integrations initiated from y=0 therefore yield no spatial variations in N, v, and w. According to the theory, then, the exact solutions for N, v, and w approach their values at y=0 asymptotically and infinitely far from the vibrating base. In order to overcome this difficulty, we follow Oyediran et. al. [1992], who relaxed very slightly the normal stress condition (8) by allowing v(0) to assume a very small nonzero value. In this manner, integrations may be initiated from y=0, and yield solid fraction and granular temperature profiles that are insensitive to values of v(0) less than 10^{-2}.

Results and Discussion

In all that follows, we set the coefficients of restitution e and e_w equal to .9 and the boundary parameter θ equal to $\pi/6$, unless otherwise indicated. This value of θ includes the special case of equal particle diameters (σ=d) and no boundary particle spacing (s=0). In order to present the results compactly, we introduce the normalized function of y,

$$I(y) \equiv \frac{1}{m_t} \int_y^{\beta} \nu dy \quad , \tag{14}$$

which gives the fraction of the total mass m_t below any location y, and varies from 0 (when y=β) at the base to 1 (when y=0) at the top of the assembly.

In presenting our results, we first study the effects of vibrational speed v on the thermalized states throughout assemblies of fixed mass hold-ups m_t. In Figure 1, for example, we show the variations of w(I) with v for I=0, .05, .1, .25, .5, .75, and 1 when m_t=2.5, 5, 10, and 20. As expected, the granular temperatures, which are induced entirely by basal vibrations, increase from zero as v increases from zero. For any pair of v and m_t, the temperature decreases monotonically from the base (I=0) to the top of the assembly (I=1). This is because energy must be conducted *into* the mass above any location y to balance the collisional dissipation that occurs within that mass.

Furthermore, because the energy flux at the top of the assembly vanishes, the energy flux at the boundary must balance the total dissipation in the assembly. Consequently, the granular temperature typically varies far more widely throughout the lower half of the mass than it does in the upper half, and hardly varies at all through the upper quarter of the mass. These observations become even more striking as the mass hold-up increases. According to third and fourth panels of Figure 1, the measure w of granular temperature is less than .01 in the upper portions of the assemblies for values of v less than .81 when m_t=10 and for all values of v (between 0 and 5) when m_t=20. Under these circumstances, the energy supplied by the vibrating boundary is, in effect, insufficient to thermalize the entire assembly. In fact, for m_t=20 and all values of v shown in the fourth panel of Figure 1, the measure w of thermalization is less than .01 throughout more than half the total mass.

In Figure 2, we show the variations of w and ν with dimensionless distance Y≡β-y from the base for v=.25, 2, and 5 when m_t=5. The solid dots on the profiles indicate the heights at which I=.99. These profiles are typical. As the fluctuation speed of the boundary increases, the assembly becomes deeper, more dilute, and more thermalized. The granular temperature increases monotonically from the top to the bottom of the assembly, whereas the solid fraction increases from zero at the top to its maximum at an intermediate location and then decreases to a smaller value at the base. In fact, N' and w' are positive everywhere; but according to constitutive relation (3), ν' is positive only where N'/N is greater than $2w'/w$.

Next, we study the effects of varying the mass hold-up m_t on the thermalized states induced by boundaries whose vibrational speeds v are fixed. In Figure 3, for example, we plot the variations of w(I) with m_t for I=0, .05, .1, .25, .5, .75, and 1 when v=.25, 2, and 5. As expected, for fixed values of vibrational speed v and mass fraction I, the granular temperature typically decreases as the total mass hold-up m_t increases. Interestingly, this effect on w(I) diminishes as I decreases, until at the base (I=0) the granular temperature is virtually insensitive to mass hold-ups m_t beyond about 2.5. Figure 3 also demonstrates that as vibrational speed increases, so too does the mass that can be effectively thermalized. As v varies from .25 to 2.5 to 5, for example, the maximum value of m_t for which w is everywhere greater than .01 increases from 8.20 to 11.10 to 11.95.

In Figure 4 we show the variations of w and ν with $Y \equiv \beta - y$ for m_t=2.5, 5, 10, and 20 when v=2. Again the solid dots indicate the locations at which I=.99. Although the effect of increasing the mass of the assembly from m_t=2.5 to 20 is to decrease the granular temperatures everywhere, the decrease is far more pronounced near the top of the assembly than at the base. At the two lower values (m_t=2.5 and 5) of mass hold-up, the boundary vibrations are sufficient to fully thermalize the assemblies and to disperse them at solid fractions that are everywhere significantly less than the random close packed value. However, at the two higher values (m_t=10 and 20), the assemblies are, in effect, only partially thermalized. These more massive assemblies consist of an upper nearly passive region, in which the solid fraction is essentially constant and nearly equal to its maximum value, supported by a more dilute thermalized region that extends approximately 11 particle diameters above the base.

Finally, in Figure 5, we study the effects of boundary bumpiness on the induced thermalized states by plotting the profiles of w(Y) and ν(Y) for θ=0, $\pi/3$, and $\pi/2$ when v=2 and m_t=5. The effects demonstrated here, as the boundary evolves from perfectly flat (θ=0) to extremely bumpy ($\theta=\pi/2$), are typical. As θ increases from zero, so too does the energy imparted to the assembly by the tangential components of the isotropic bounday vibrations. For this reason, as the boundary becomes bumpier, the granular temperatures increase and the solid fractions decrease throughout the assembly. However these effects are moderated by the fact that the energy imparted to the assembly by the normal component of the boundary vibrations actually *decreases* as the boundary becomes bumpier and experiences fewer normal and more oblique impacts with the assembly. Consequently, as the boundary becomes bumpier it becomes only moderately more effective at transferring energy to the assembly. The effect of varying θ on the resulting thermalized states is therefore relatively minor.

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Figure Captions

Figure 1: The variations of w(I) with v for m_t=2.5 (first panel), 5 (second panel), 10 (third panel), and 20 (fourth panel), when I=0, .05, .1, .25, .5, .75, and 1, $\theta=\pi/3$, and $e=e_W=.9$.

Figure 2: The variations of w and v with $Y\equiv\beta$-y for v=.25, 2, and 5 when m_t=5, $\theta=\pi/3$, and $e=e_W=.9$.

Figure 3: The variations of w(I) with m_t for v=.25 (first panel), 2 (second panel), and 5 (third panel), when I=0, .05, .1, .25, .5, .75, and 1, $\theta=\pi/3$, and $e=e_W=.9$.

Figure 4: The variations of w and v with $Y\equiv\beta$-y for mt=2.5, 5, 10, and 20, when v=2, $\theta=\pi/3$, and $e=e_W=.9$.

Figure 5: The variations of w and v with $Y\equiv\beta$-y for $\theta=0$, $\pi/3$, and $\pi/2$, when v=2, m_t=5, ane $e=e_W=.9$.

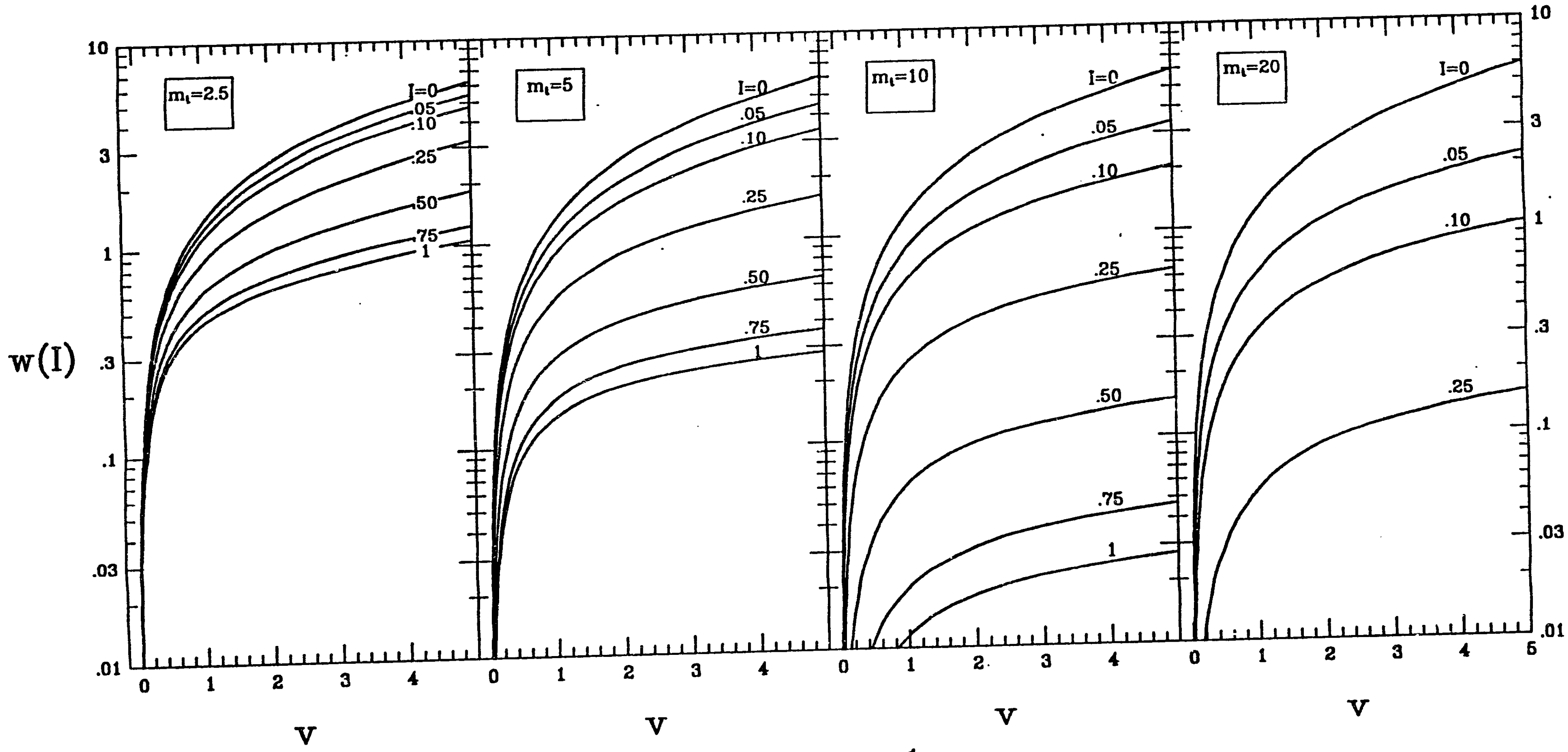

Figure 1

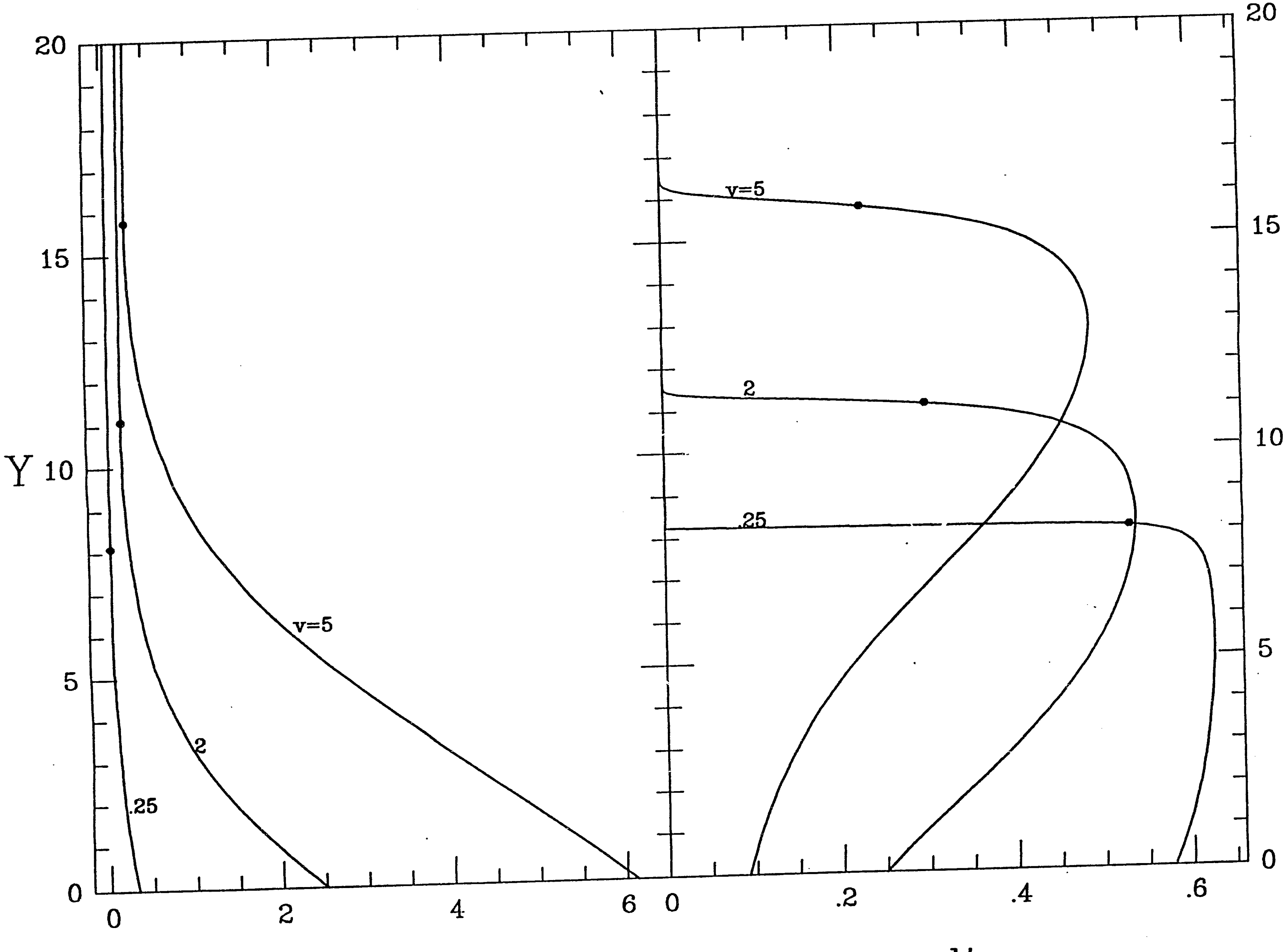

Figure 2

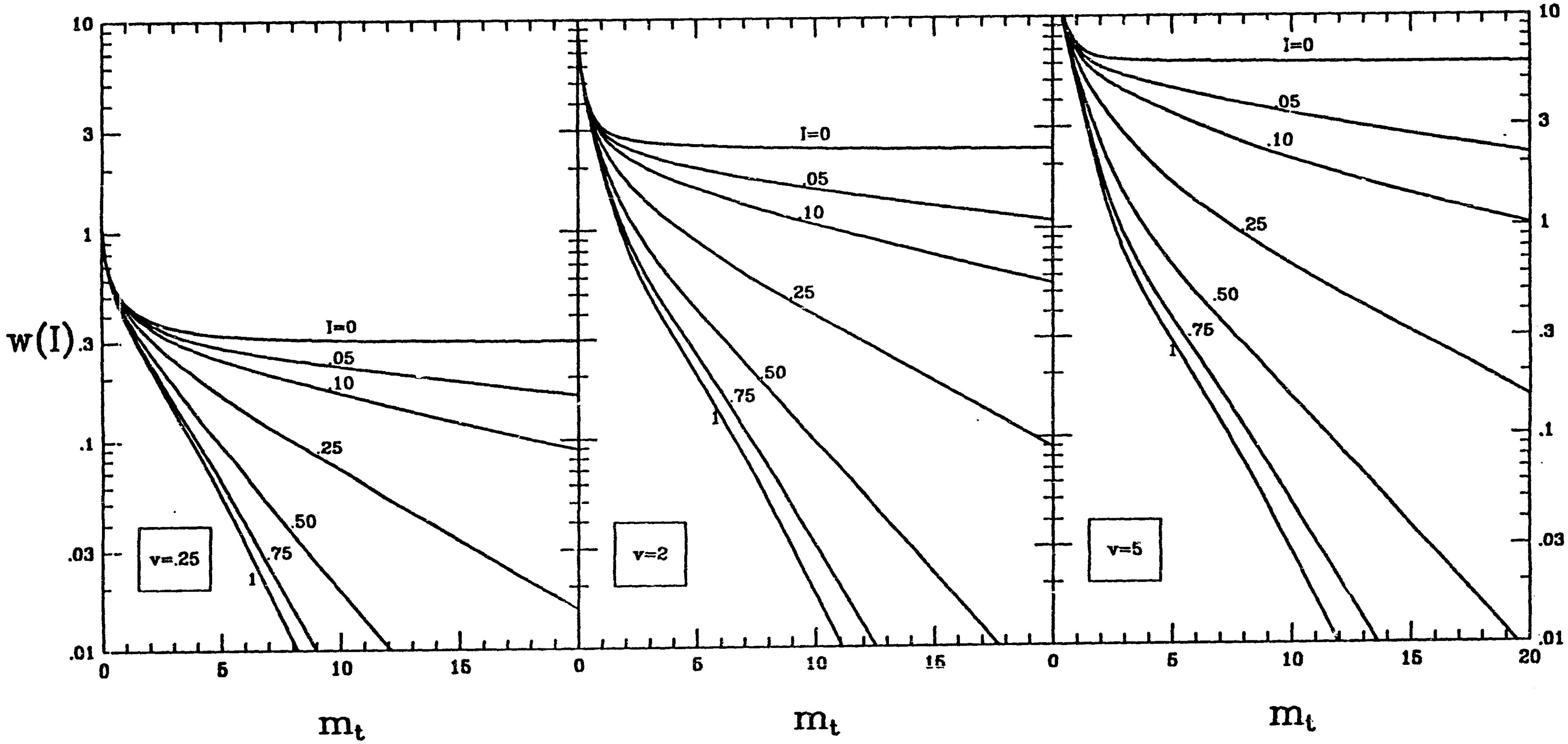

Figure 3

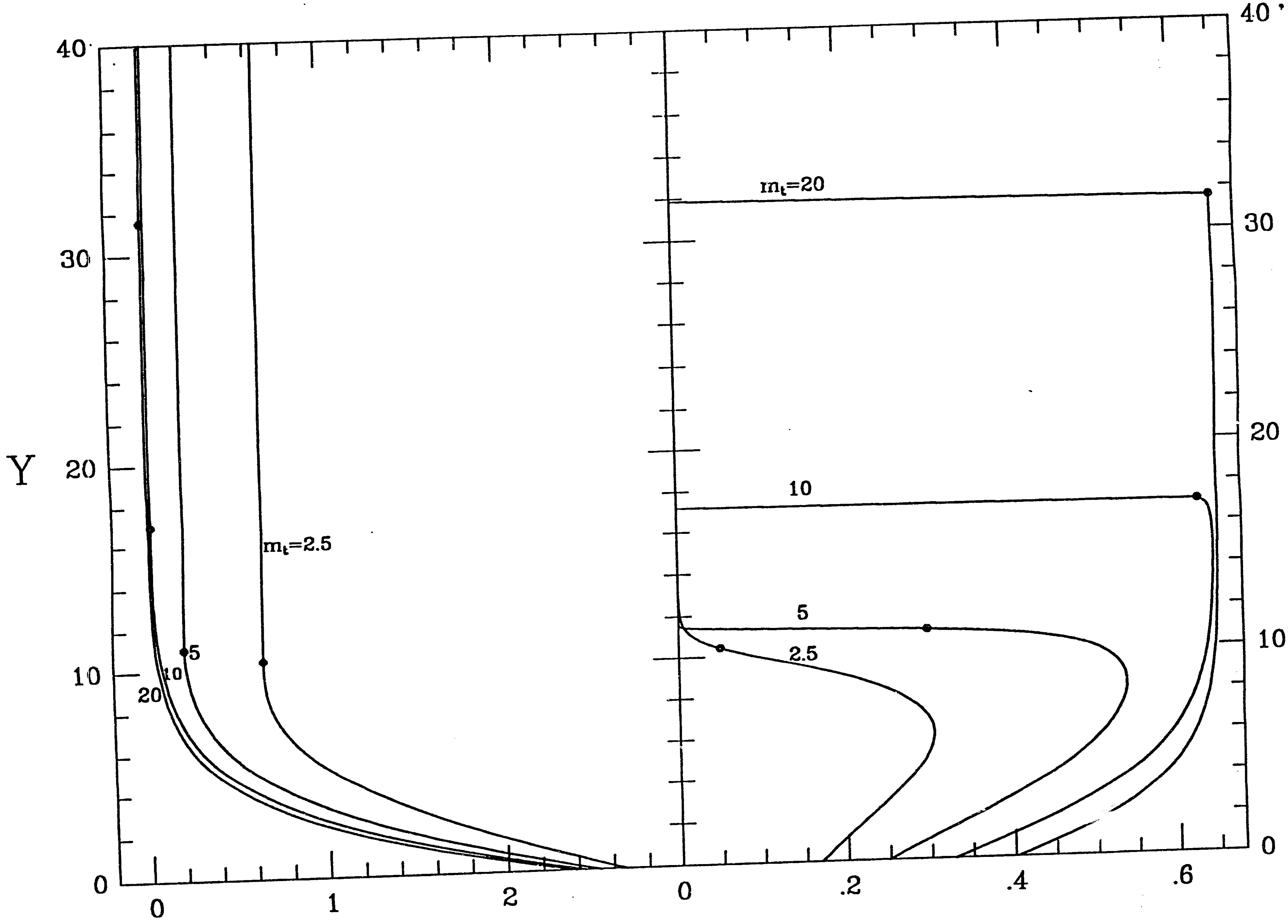

Figure 4

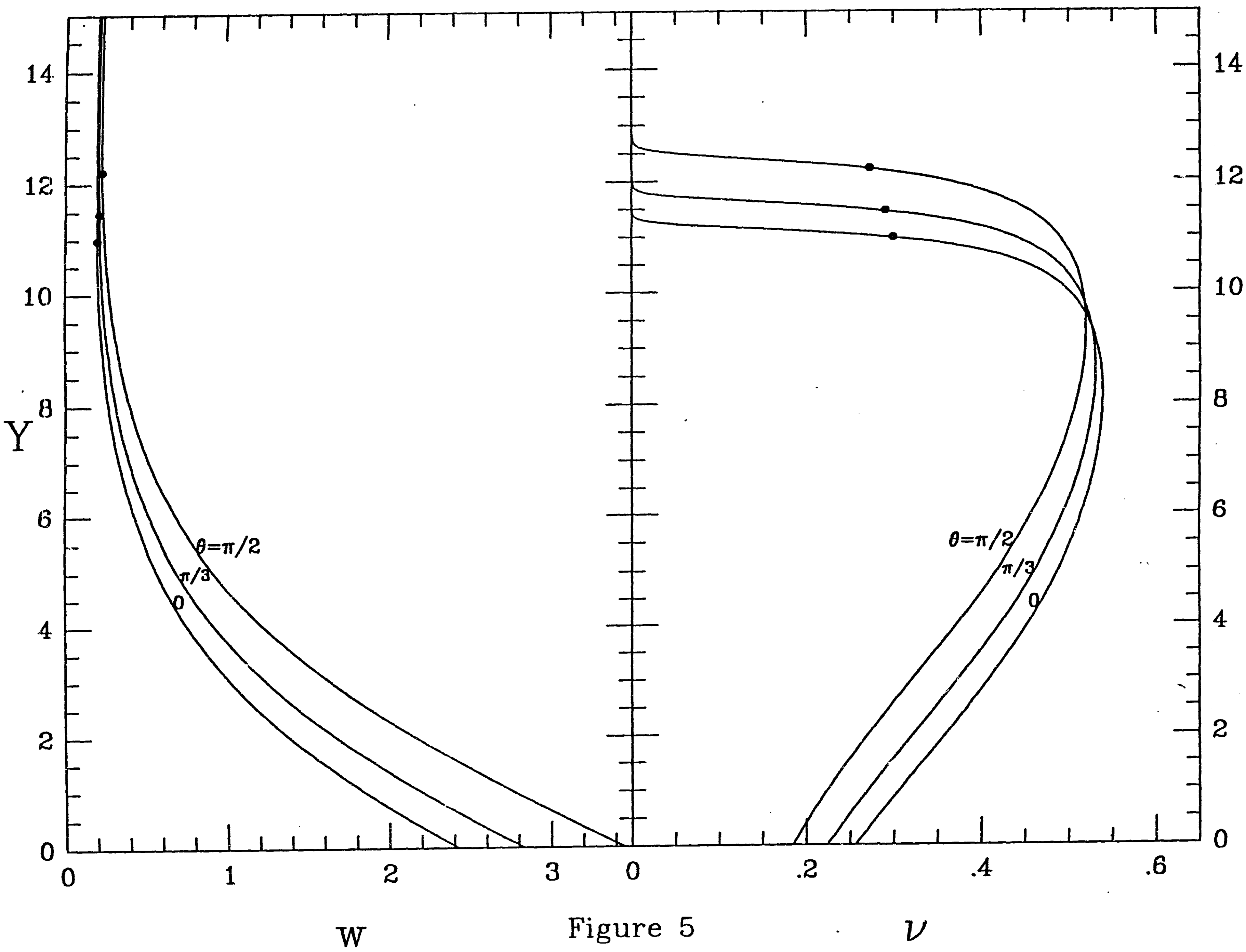

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Figure 5

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