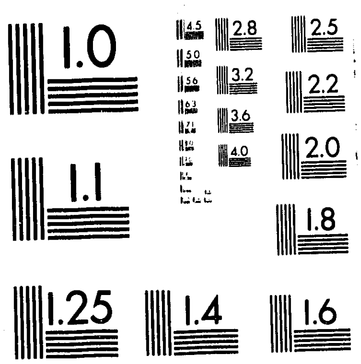
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GRAPHICAL REPRESENTATION OF ROBOT GRASPING QUALITY MEASURES

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Abstract

When an object is held by a multi-fingered hand, the values of the contact forces can be multivalued. An objective function, when used in conjunction with the frictional and geometric constraints of the grasp, can however, give a unique set of finger force values. The selection of the objective function in determining the finger forces is dependent on the type of grasp required, the material properties of the object, and the limitations of the robot fingers. In this paper several optimization functions are studied and their merits highlighted. A graphical representation of the finger force values and the objective function is introduced that enable one in selecting and comparing various grasping configurations. The impending motion of the object at different torque and finger force values are determined by observing the normalized coefficient of friction plots.

1 Introduction

An object can be grasped using force and position feedback. If the robot is to perform tasks using the grasped object, determination of the quality of the grasp is necessary. Using the grasp quality information, the object is manipulated within the robot fingers and oriented to a different grasp configuration.This paper develops a new graphical representation of grasping quality measures. Grasp quality measures developed by other researchers ([8], [12] and [3]) can neither effectively account for friction nor study the influence of external disturbances on the grasp. The measure developed here overcomes these shortcomings.

The grasped object in the robot end-effector should satisfy force and moment balance to be in stable equilibrium. Direction of application of the contact forces is used to classify the grasp into two categories: Form Closure and Force Closure [7]. In Form Closure the relative motion of the object and the end-effector is constrained by physical contacts. The magnitude of the applied force has no effect on maintaining the contact required for the grasp. In a Force Closure, on the other hand, the contact between the object and the gripper is maintained by the application of suitable finger forces. Form Closure is a preferred grasp; however, in many practical cases, only a Force Closure is feasible. Before attempting to grasp the object, it is important to determine the number of contact points required to achieve a particular grasp. For a Form Closure, four point contacts are required in planar cases while seven point contacts are required in **3D**. This conjuncture is proved in Mishra et al. [10] and Markenscoff et al. [9]. Depending on the surface topology there are objects that cannot be grasped using point contacts. Selig [15] gave a classification of surfaces that cannot be grasped using frictionless point contacts.

Quality of a grasp is defined by the ability of the finger forces to resist the external applied forces and moments. Given a grasp, the required friction at the finger contacts can be used to represent the quality of the grasp ([1] and [18]). If the required value of friction coefficient to prevent the finger contact from slipping is large, the grasp quality is poor. Wolter et al. [18] gave the grasp quality measure as a combination of the object's centroid and the value of friction at the contact locations. This measure was applied to the grasping of polygonal objects with parallel jaw grippers. A similar geometrical grasping measure function was developed by Park et al. [13] to determine grasp quality. Positioning of the fingers with respect to the centroid, the spacing of the fingers, and topology of the surface of contact were used to develop such a function. The minimum value of the function was determined by evaluating it at all possible feasible locations of the contact.

A grasp matrix which is the mapping between the finger forces and external forces and moments can also be used to determine the grasp quality. Directions of the external force at which the grasp is destabilized are known as singular directions of the grasp. Magnitude of the singular values give a measure of the distance the grasp is away from singularity. The smallest singular value of a given grasp can hence be used to indicate its quality measure ([8]). Since two completely different grasps can have the same minimum singular value, a volume measure can be used in such cases. The shortcoming of this approach is that the frictional constraints, being non-linear, cannot be satisfied by the grasp matrix representation alone. A subspace of the actual space of the finger forces is required to satisfy the frictional constraint which is computationally very difficult. Therefore, although a grasp matrix is a good representation, it cannot be directly used to solve grasping problems with frictional contacts without external constraints.

An optimization criterion is used to generate a unique set of finger forces for a multi-fingered grasp. The value of the finger forces depends on the objective function used. The norm of the forces is a good intuitive choice for an objective function. Nakamura et el. [12] used the total finger forces at the contact points in the calculation of the minimum norm of a grasp.The minimax of the finger forces and the minimum value of the sum of finger forces were used as grasp quality measures by Ferrari et al. [3]. A quality measure is defined as the distance of the nearest facet of the convex hull constructed from the wrenches corresponding to the contact locations ([3]). The minimum norm solution can be modified by adding the homogeneous solution of the grasp matrix to minimize the maximum value of the friction angle at the points of contact. Mukherjee et. al. [11] developed a closed form solution of the finger forces for the above minimax formulation.

Once the object is within the robot end-effector, it might be necessary to know the quality of the grasp. A scalar value is usually used to represent the grasp quality measure. If two grasps have the same quality measure, it is difficult to discriminate between the grasps. If a graphical representation of the variation of the finger forces to change in the direction of the external force is used to represent the quality of a grasp, the above shortcomings are rectified. Depending on the direction of the external force/moment acting on the object while the robot performs a task, a grasp can be selected using the graphical representation. A graph showing the relationship of the variation of the frictional coefficients to variation of the external force can also be used to determine slippage that can occur at the contact locations. Three optimization functions and their influences on the finger force distributions are studied in this paper. The forces at the contacts are the resultant of the normal and the frictional forces. Since the normal force necessary at the contacts can be easily controlled by the torque generated by the tandem driven finger linkages, it is better to use normal force as opposed to resultant force, which is dependent on friction. Hence, the objective functions in this paper use normal force rather than the actual resultant contact force.

2 Grasp Force Formulation

Consider a rigid object held by a multi-fingered robot hand (Figure 1). The external disturbing load acting on the body can either be a force, a moment or both. Since any force and moment on the body can be translated into an equivalent force and moment at the center of mass of the object, the disturbing forces are assumed to be acting at the center of mass of the object.

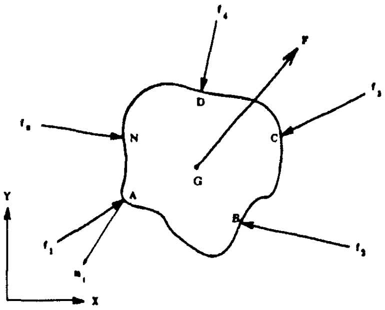

Figure 1: Rigid Body Grasped by a Multifingered Robot Hand

The finger/object contacts can typically be modeled as point contacts with no friction, point contacts with friction, or soft contacts. For a point contact with no friction, the unknown variable is the applied normal force by the finger. For a point contact with friction, there are three variables accounting for the normal contact force, and the frictional forces acting in the tangent plane that pass through the point of contact. For a soft contact there is a small area of contact between the two bodies and frictional forces acting in this area can generate a frictional torque along the contact normal. Hence soft contacts are modeled with four variables. This paper considers only point contacts with friction. To address point contact with no friction, frictional forces are not considered and for the soft contact model the frictional constraint equation has to be modified to account for the contact frictional torque.

We next look at the equations that need to be satisfied to maintain a stable grasp and avoid slippage at the contact locations between the object and the finger. To maintain a stable grasp, force and moment balances have to be satisfied. The resulting static equilibrium equations can be expressed as:

$$\underline{\mathbf{F}} = \underline{\mathbf{G}} \cdot \underline{\mathbf{f}} \tag{1}$$

where $\underline{\mathbf{G}}$ is the grasp mapping matrix, $\underline{\mathbf{F}}$ is a vector composed of the external forces and moments and $\underline{\mathbf{f}}$ is a vector of the finger contact forces. The matrix $\underline{\mathbf{G}}$ can be viewed as a transformation from the finger force space to the external force space and is represented by:

$$\underline{\mathbf{G}} : \underline{\mathbf{f}} \longrightarrow \underline{\mathbf{F}}$$

where $\underline{\mathbf{f}} \in \mathbf{R}^{3n}$, $\underline{\mathbf{F}} \in \mathbf{R}^{6}$, $\underline{\mathbf{G}} \in \mathbf{R}^{6x3n}$ and n denotes the number of finger contacts of the multifingered grasp. From Equation (1) it is clear that the vector $\underline{\mathbf{G}} \cdot \underline{\mathbf{f}}$ denotes the set of external forces and moments (F) that can be resisted by the finger forces $\underline{\mathbf{f}}$, while $\underline{\mathbf{G}}^{-1}\,\underline{\mathbf{F}}$ denotes the set of finger forces that can resist the external force and moment $\underline{\mathbf{F}}$. $\mathbf{G}^{-1}(\in \mathbf{R}^{3nx6})$ is a pseudo inverse of the matrix $\mathbf{G}$. The development of the $\underline{\mathbf{G}}$ matrix is discussed fully in Section 4.

The general solution to Equation (1) is obtained by a linear combination of the homogeneous solution ($\underline{\mathbf{f}}_h$) and particular solution ($\underline{\mathbf{f}}_p$).

$$\underline{\mathbf{f}} = \lambda \underline{\mathbf{f}}_h + \underline{\mathbf{f}}_p.$$

The vectors $\underline{\mathbf{f}}_h \in ker(\underline{\mathbf{G}})$ and $\underline{\mathbf{f}}_p \in R(\underline{\mathbf{G}})$,where $ker(.)$ denotes the kernal or the nullspace of $\underline{\mathbf{G}}$ and $R(.)$ denotes the range space of $\underline{\mathbf{G}}$.

To avoid slippage at the contacts, frictional forces at each contact location should be within the friction cone. Let $\underline{\mathbf{f}}_1$ in Figure 1 denote the total force (sum of the normal and frictional forces) acting on the rigid body at the contact location **A**. $\underline{\mathbf{n}}_1$ is the surface normal at the point of contact. The portion of the finger force which acts normal to the surface ($\underline{\mathbf{f}}_{11}$) is given by:

$$\underline{\mathbf{f}}_{11} = (\underline{\mathbf{f}}_1 \cdot \underline{\mathbf{n}}_1)\,\underline{\mathbf{n}}_1$$

Let $\underline{\mathbf{f}}_{\mu 1}$ denote the total frictional force generated at the contact and let $\underline{\mathbf{f}}_{\perp 2}$ and $\underline{\mathbf{f}}_{13}$ denote the two orthogonal components of this force in the tangent plane at the contact. That is:

$$\underline{\mathbf{f}}_{\mu 1} = \underline{\mathbf{f}}_{12} + \underline{\mathbf{f}}_{13}$$

To satisfy the frictional constraint at the contact the following inequality needs to be satisfied:

$$\|\, \underline{\mathbf{f}}_{12} + \underline{\mathbf{f}}_{13} \| \le \mu_s \|\underline{\mathbf{f}}_{11}\| \qquad (2)$$

where μ_s is the static coefficient of friction between the contact surfaces.

Replacing μ_s with the actual friction coefficient at the contact location ($\mu : 0 \le \mu \le \mu_s$), the above equation can be recast as:

$$\frac{\|\ \underline{\mathbf{f}}_{\mu 1}\ \|}{\|\ \underline{\mathbf{f}}_{11}\ \|} = \mu. \qquad (3)$$

Knowing the static coefficient of friction and the value of the friction coefficient from Equation (3), the impending motion of the body at the contact location due to increase in magnitude of the external force can be predicted. This is useful in determining the modes of motion of a grasped body under the influence of an external disturbing force.

3 Objective Function

Consider again the grasp of an object with a multifingerd robot hand as shown in Figure 1. The grasping problem can be perceived through a control volume where, the finger forces ($\mathbf{f}_i$) are the inputs and the external load ($\mathbf{F}$) is the output. The ratio of the input to the output can be used to qualify the grasp. This ratio is analogous to the ratio suggested by Kerr and Roth [6] when calculating the optimal grasp directions. For the case where the Input/ Output ratio is composed of the second norms of the finger and external forces, it can be expressed as:

$$\frac{\text{Input}}{\text{Output}} = \frac{\mathbf{f}^t \cdot \mathbf{f}}{\mathbf{F} \cdot \mathbf{F}}$$

The minimum values of this ratio may constitute a potential measure of the quality of the grasp.

Solving the grasping problem using Equations (1) and (2) results in multiple solutions when $ker(\underline{\mathbf{G}})$ has nonzero elements. An objective function that minimizes or maximizes the value of an appropriate criterion may constitute a measure for the quality of the grasp and will result in a unique finger force solution. Nakamura et. al. [12] showed that excessive finger forces result in unstable grasps. It is also true that object's fragility and actuators limitations impose upper bounds on the applied finger forces. These considerations lead to three possible objective functions that are tested as quality measures of a grasp in the following sections. The objective functions together with the grasping Equations(1) and (2) are the mathematical formulations of the quality of a grasp. The three objective functions considered are:

(a) Minimization of the norm (L_2) of the finger forces
(b) Minimizing the maximum value (L_∞) of the finger forces (Minimax of the finger forces)
(c) Minimization of Entropy of the finger forces

The objective functions are formed using the applied normal finger forces and not the resultant finger forces as given by Nakamura et. al. [12] and Ferrari and Canny [3]. Since the normal forces applied by the robots are more readily measured and controlled, there is more merit in optimizing the normal finger forces as opposed to the resultant forces at the contacts.

3.1 Minimization of the Norm

If the fingertips are modeled as springs, the sum of the square of the finger forces indicate the potential energy stored due to the deformation of the tissues. Minimization of the norm of the finger forces reduces the energy required to grasp an object. This criterion intuitively suggests the minimization of the energy supplied by the actuators to grasp an object.

Mathematically it translates to:

$$S(\underline{f}) = \mathbf{Min}(\underline{\mathbf{f}}_{\mathbf{n}}{}^{t}.\underline{\mathbf{f}}_{\mathbf{n}})^{1/2}$$

where $\mathbf{f_n}$ is the vector containing the applied normal finger forces.

3.2 Minimization of the Maximum Finger Forces

During grasping, under ideal conditions, the external load should be evenly distributed among all fingers. If the object is fragile, an excessive force at contacts can be damaging. Taking this into consideration, the minimax optimization criterion tries to minimize the maximum of the forces at the fingers, which results in a close to equal distribution of the forces. For a fragile object which has to be grasped at specified contact locations, minimax criteria can determine whether an object can be grasped without excessive stresses at the contact points. Mathematically this translates to:

$$S(\underline{f}) = \mathbf{Min}(\mathbf{Max}(\underline{\mathbf{f}}_{\mathbf{ni}})) \qquad \text{for i = 1... n}$$

Examples of using this function are given in [17].

3.3 Minimization of the Entropy of the Finger Forces

The L_2 and the minimax objective functions may result in grasps that are generated by non uniform finger forces. If a well distributed and uniform finger forces is a concern, the entropy of the finger forces may be used as an objective function.

The negative of the Shannon's Entropy Function $(-x log(x))$ is used extensively in information theory and it may be applied to grasping mechanics. Minimization of the function $\sum_{i=1}^{n} f_i \ \log(f_i)$ subject to the constraint $\sum_{i=1}^{n} f_i = Constant$, yields an even distribution of the variable f_i [4],[2]. When the *Constant* = 1, the minimum is obtained when $f_i = 1/n$.

Since f_i in the grasping formulation is an independent variable and its sum cannot be constrained, the formulation of the entropy function is modified as follows [16]:

$$\sum_{i=1}^{n} f_i \ \log(f_i) - \sum_{i=1}^{n} f_i + n \tag{4}$$

For a single variable case, the entropy function yields $f * \log(f) - f + 1$. This function obtains its minimum when $f = 1$. The two variable case the minimum is obtained when $f_1 = f_2 = 1$ which differs from the $f_1 = f_2 = 1/2$ obtained when minimizing the original entropy function, yet it conserves the uniform distribution property. Equation 4 therefore yields uniform finger force solutions that one may seek in quality grasping. Notice that, the magnitude of the minimum entropy can be used to measure the uneveness of the distribution of f_i. The minimization of the entropy function guarantees the positiveness of the finger forces f_i because of the logarithmic terms in the function. When norm or minimax optimization function is used, the positiveness of the finger forces had to be imposed by external constraints.

Mathematically the minimization of the entropy of the finger normal forces can be written as:

$$S(\underline{f}) = \mathbf{Min}(\sum_{\mathbf{i=1}}^{\mathbf{n}}(\underline{\mathbf{f}}_{\mathbf{ni}} \cdot \ \log(\underline{\mathbf{f}}_{\mathbf{ni}})) - \sum_{\mathbf{i=1}}^{\mathbf{n}}(\mathbf{f}_{\mathbf{ni}}) + \mathbf{n})$$

$S(\underline{f})$ is one of the objective functions that is used to study the quality of a grasp in this paper.

4 Development of Grasping Matrix

In the development of the grasp matrix by Salisbury and Roth [14], Li and Sastry [8], and Kerr and Roth [5] the matrix elements are constants which account for the finger force directions and contact positions. Consider a rectangular object held by three finger contacts as shown in Figure 2. The finger forces are applied normal to the sides of the rectangle. f_{11}, f_{21}, and f_{31} are the normal forces and f_{12}, f_{22}, and f_{32} are the frictional forces at the contacts. l_1 and h_1, l_2 and h_2, and l_3 and h_3 are the contact locations of fingers 1, 2, and 3, respectively, measured from the center of mass of the object. $\underline{F}$ is a unit external force applied at the center of mass of the rectangle. For the planar case the external force is given by the equation:

$$\underline{F} = \cos\theta \ \mathbf{i} + \sin\theta \ \mathbf{j}$$

Equation (1) for the force and moment balances of the three-fingered grasp can be written as:

$$\begin{bmatrix} -\cos\theta \\ -\sin\theta \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ h_1 & l_1 & -h_2 & l_2 & l_3 & h_3 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{21} \\ f_{22} \\ f_{31} \\ f_{32} \end{bmatrix} \tag{5}$$

The elements of the matrix are constants and do not account for the change in the external force direction. When using Equation (5), the frictional constraints at the contact locations **A**, **B**, **andC** have to be satisfied by external constraint equations.

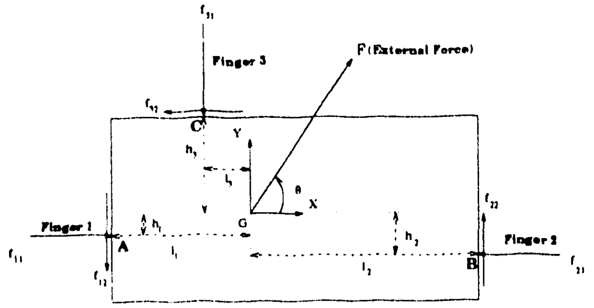

Figure 2: A Three Fingered Grasp of a Rigid Body

5 Quality Measure of a Grasp

Grasp quality measures given by several researchers use a single number to describe the grasp ([8],[18] and [3]). Using the criteria described by Li and Sastry [8], Wolter et. al. [18], and Ferrari and Canny [3] many grasps can give the same quality measure and make it difficult for different grasps to be compared. Moreover, the ability of a grasp to resist external forces and moments depend on the direction of the external load acting on the body. Consider two grasps: an object resting freely on the palm of a hand and an object held by fingers on its lateral sides. If the only external force acting on the object is gravity, then, in the former grasp, the palm supports this load while in the latter case frictional forces at the contacts support the weight. If the coefficient of friction at the contact surface is less than one, the former definitely is a more desirable grasp.

To account for the information lost when representing grasp quality measure as a number, a graphical representation is introduced. The graphs detail the variation of the finger forces and the objective functions when the direction of the external force/moment change. An example of a planar three-fingered grasp is explained in detail in Section 5.1. Graphs for two-fingered and four-fingered planar grasps and four fingered $3D$ grasps are illustrated [17].

The grasping equations developed in Sections 2 and 3 are summarized below:

$$\begin{array}{ll} (a) & \underline{\mathbf{F}} = \underline{\mathbf{G}} \cdot \underline{\mathbf{f}} \\ (b) & \|\underline{\mathbf{f}}_{a1} + \underline{\mathbf{f}}_{\mu 1}\| \leq (\mu_s + 1)\|\underline{\mathbf{f}}_{11}\| \\ (c) & \underline{\mathbf{f}}_{ai} \cdot \underline{\mathbf{n}}_i \leq 0 \\ (d) & \mathbf{Min}(\mathbf{S}(\underline{\mathbf{f}})) \end{array} \quad (6)$$

Equation (6a) is the grasp matrix representation of the force and moment balances. Equation (6b) satisfies the frictional constraints at the contact surfaces. The normal finger forces can assume positive and negative values if they are left as free variables during optimization. Since suction by fingers is not allowed, Equation (6c) is introduced to ensure that the forces are directed into the body. n_i in Equation (6c) is the normal at the point of contact. Equation (6d) denote the objective functions used to obtain a unique solution.

5.1 Three-Fingered Planar Grasp

Figure 3 shows the schematic diagram of a three-fingered grasp. Dimensions of the object and the positions of the fingers are shown in Figure 3. The value of static coefficient of friction is chosen to be 0.4. Since the applied finger forces ($\mathbf{f_{i1}}$) are perpendicular to the sides of the rectangle, Equations (6 b and c) can be recast as:

$$\mathbf{f_{i2}} \leq \mu_s\, \mathbf{f_{i1}} \qquad \text{for i=1,2, and 3.}$$

$$\mathbf{f_{i1}} \geq 0.0 \qquad \text{for i=1,2, and 3.}$$

Finger forces are calculated using the three objective func-

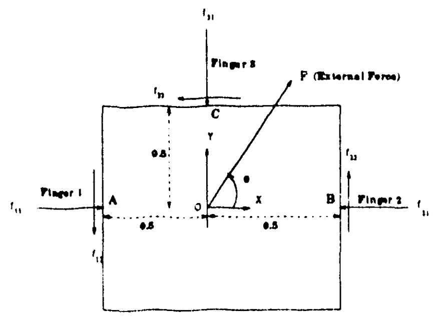

Figure 3: A Three Fingered Grasp of a Square Planar Object

tions developed in Section 3. To obtain a unique solution of the finger forces, numerical optimization of the six variables in nine constraint equations and an objective function is performed. For the Minimax objective function, one additional variable and three additional equations are required, since the representation of the objective function requires an additional variable. Figures 4 and 5 show the plots obtained using the Norm and Entropy objective functions, respectively.

As expected from symmetry, for variations of the external force, fingers 1 and 2 have symmetrical force distribution. Although the magnitude of the finger forces differ, irrespective of the criteria used, the maximum is obtained when θ is 230° and 310° for Finger 1 and 2 respectively. When external force lie in Quadrants III and IV, the finger forces are maximum.

since friction alone has to resist the external force. The value of the force at finger 3 when θ is 90^0 is less than 1.0 since frictional forces at fingers 1 and 2 resist part of the load. This is not true when the external load is completely supported by fingers 1 and 2 when θ is 180° and 0°, respectively.

The finger forces assume zero values when Norm function is used as the objective criterion while it never reache zero value when the Entropy criterion is used.

The Norm of the finger forces is calculated for the entropy objective function to compare the grasps. The magnitude of the norm for the Entropy criterion is larger than those obtained using the Norm criteria. Norm of the finger forces for the two objective functions assume minimum values when θ is 45° and 135° and maximum values when θ is 245° and 295°. The discontinuities in the norm plot at these values of θ are due to the geometry of the object and the symmetrical placement and direction of the finger contact forces.

The value of the entropy function give a measure of the unevenness of the distribution of the finger forces. From Figure 5 it can be seen that the relative difference in magnitude of the finger forces is small when angle θ is near 90°. The value of the entropy function is large in III and IV quadrants when the finger 3 apply smaller force compared to fingers 1 and 2.

Examples of two and four finger grasp and torque as the external disturbing force are given in [17]. With the graphical representation one is able to quickly infer the effect of an additional finger contact on the grasp. Depending on the direction of the external load (or a range of external load directions) the method is able to compare different grasps of the same object.

Friction coefficients using Equation (3) are plotted for each objective function. Figures 6 and 7 give the normalized values of the friction at the contact points. Observation of the discontinuities of the normalized friction coefficients in Figure 6 suggests that there are six modes of motion for this case. The modes of motion are the translations opposite to the direction of finger forces 1, 2, and 3; rotations about finger contacts **A** and **B**; and motion along the direction of application of the external force. Although there are seven possible modes of impending motion, rotation about **C** did not yield a minimum norm for any direction of the external force. When entropy objective function is used, no motion of the object occur when the external force direction θ lie between 1.13 and 2.01 radians. During this interval of θ the object is in static equilibrium. The range over which different modes are applicable differ with the objective criterion used. As an example, the object has impending rotation about finger contact **A** in the III Quadrant when angle θ ranges from 3.22 to 4.709 radians while using the Norm as the objective function and between 3.57 to 4.45 radians when Entropy is used (Figures 6

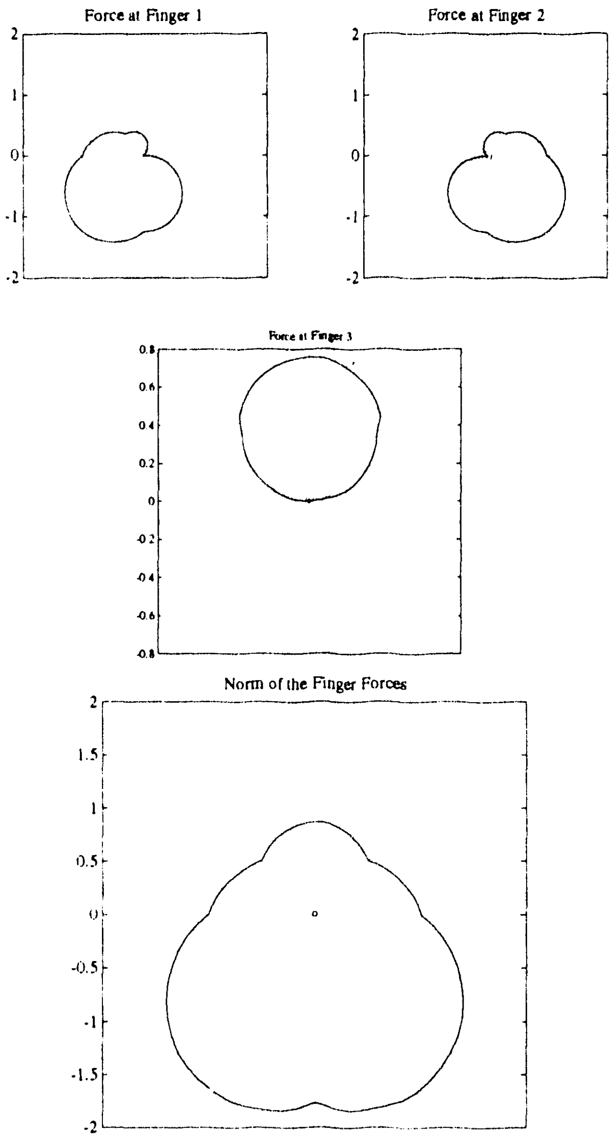

Figure 4: Plots for Three-Fingered Grasp Using Norm

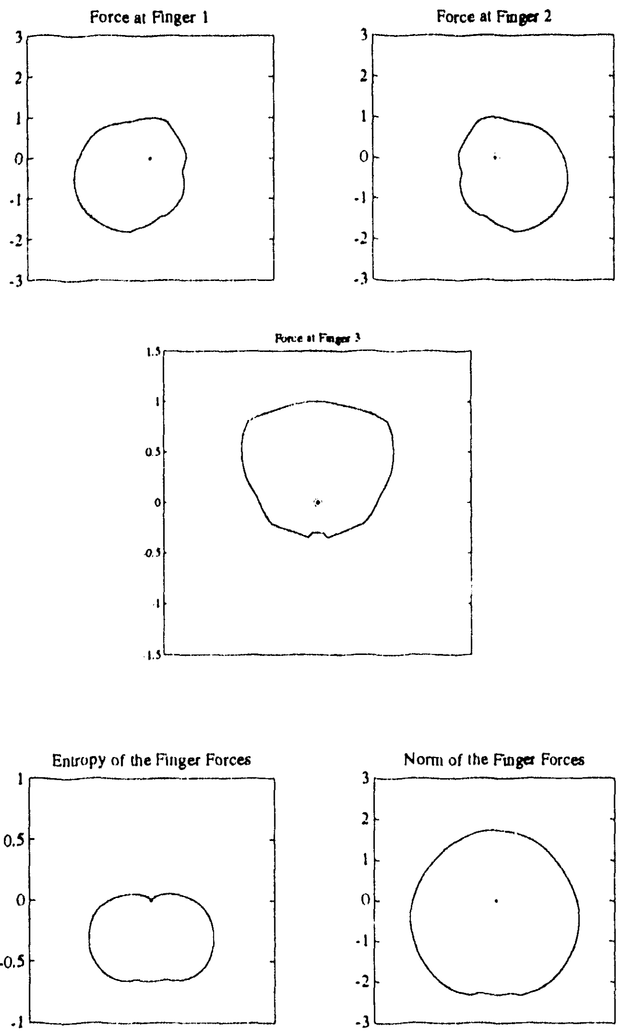

Figure 5: Plots for Three-Fingered Grasp Using Entropy

and 7). This suggests that, depending on the modes of motion desired, different criteria can be chosen and the required finger forces calculated for each contact locations.

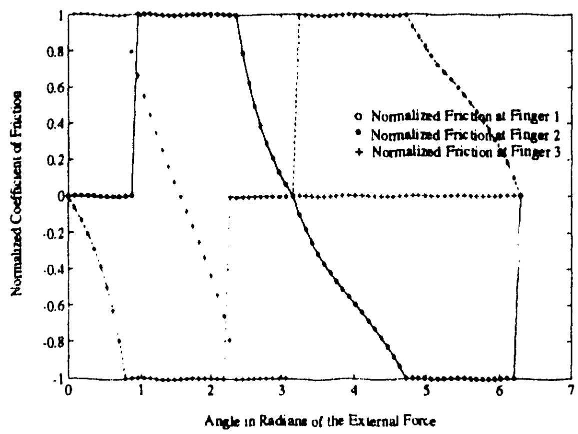

Figure 6: Friction Coefficient Variation for Three-Fingered Grasp Using the Norm as Objective Function.

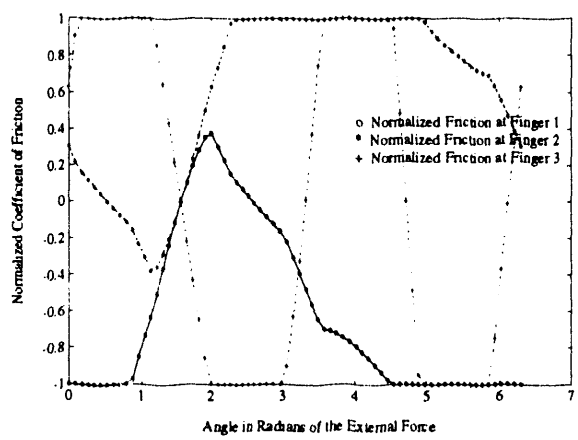

Figure 7: Friction Coefficient Variation for Three-Fingered Grasp Using the Entropy as Objective Function.

6 Discussions and Conclusions

This paper describes a graphical representation of the quality of a grasp. Variations of the optimization functions used to obtain a unique solution and the resulting finger forces are plotted with respect to the direction of the external force.

Knowing the external load direction, a grasp choice is made based on the value of the objective function and the resulting finger force distribution. It was observed that the magnitude of the finger forces calculated from the entropy criterion did not achieve a value of zero for any of the grasping cases. The above result cannot be obtained by introducing a lower bound for the variables while using either the norm or the minimax objective functions. The entropy function is smooth and continuous at the minimum and, hence, will permit values of the forces both below and above the minimum value. Introducing a lower or upper bound on the allowable finger force values yield solutions that lie only on one side of the bound.

Choice of the objective function is also made according to the preferred impending modes of motion of the grasped object under the influence of an external load. Compared to earlier grasp measures that have been suggested by other researchers, this measure describes the influence of the external force on the grasp.

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