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Title: A Bayesian Experimental Design Approach to Structural Health Monitoring

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COVER SHEET

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ABSTRACT

Optimal system design for SHM involves two primarily challenges. The first is the derivation of a proper performance function for a given system design. The second is the development of an efficient optimization algorithm for choosing a design that maximizes, or nearly maximizes the performance function. In this paper we will outline how an SHM practitioner can construct the proper performance function by casting the entire design problem into a framework of Bayesian experimental design. The approach demonstrates how the design problem necessarily ties together all steps of the SHM process.

INTRODUCTION

Structural health monitoring (SHM) is a process of data acquisition, feature extraction, and feature classification for effectively managing structural systems with respect to the potential occurrence of damage. We will show how the design procedure of an SHM system cannot be compartmentalized into any one of these steps, but rather must simultaneously consider the full process. We will cast the SHM design problem in terms of Bayesian experimental design [1] and describe the five steps of the design process: 1) Evaluate risk and cost, 2) Choose feature extraction process, 3) Calculate feature statistics as a function of design parameters, 4) Derive a detector, and 5) Calculate detector performance.

EVALUATE RISK AND COST

The first step in the design process for structural health monitoring systems is to answer four operation questions about the structural and SHM systems:

What are the relevant damage states $\theta \in \Theta$ and their probability of becoming a reality $p(\theta)$?

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These are the damage modes that the SHM system is intended to respond to. They may be a single binary global state (undamaged or damaged), or various levels and/or type of damage (none, minor, moderate, severe, crack, hole, delamination, or loose bolt) distributed over the geometry of the structure.

What actions, $d \in \mathrm{D}$, is the SHM intended to direct in response to the damage modes?

These decisions are usually along the lines of "continue operation", "reduce operation", or "stop operation and manually inspect and repair". One may alternatively assign states such as "decide damage is present" or "decide damage is not present" which carry implied actions. This is where SHM differs from an estimation problem. In estimation, one is traditionally interested in an unbiased prediction of the state of the structure θ. In SHM however, certain damage states may be more probable or more costly if missed, requiring weighted consideration during SHM. The goal of an SHM system isn't necessarily to make the most accurate decisions, but rather to make the safest and most economical ones.

What are the costs, $\mathrm{L}^{(\mathrm{d})}(d,\theta)$, of taking each of those actions?

This is the set of costs for directing each action d when the structure is in each state θ. These are generally associated with the cost of inspecting the structure when it is not damaged and the cost of further damage to or failure of the structure as a result of continuing operation without inspection. One could alternatively cast the problem in terms of maximizing utility; however, we believe the minimization of cost better reflects the SHM intent to mitigate loss due to structural failure.

What are the costs, $\mathrm{L}^{(\mathrm{e})}(e)$, associated with a particular SHM system?

Every SHM design has some associated cost as a function of the design parameters that drives the necessity of optimization (otherwise one could implement an arbitrarily large system). This cost includes both the cost of the SHM hardware as well as a potential increase in operation costs of the structure (such as the extra weight in an aerospace system). In order for an SHM system to be feasible, its ability to decrease risk must be enough to at least offset the cost of the system. To construct a total cost function, the cost of action $\mathrm{L}^{(\mathrm{d})}$ and the cost of design $\mathrm{L}^{(\mathrm{e})}$ need to be in terms of the same units (usually monetary).

Bayes Risk

Once one has developed a cost and damage model for the structure, a practical form of the expected cost, or Bayes Risk, for SHM is

$$E(L)=\sum_{\theta,d}\mathrm{L}^{(\mathrm{d})}(d,\theta)\mathrm{P}(d|\theta,e)\mathrm{P}(\theta)+\mathrm{L}^{(\mathrm{e})}(e) \tag{1}$$

This is the sum of all the possible costs, multiplied by the probability of the combination of events required to incur those costs, plus the cost of the SHM system. In order to make this calculation, one must calculate the conditional probability $\mathrm{P}(d|\theta,e)$. This requires choosing a feature extraction process, calculating feature statistics in terms of the design parameters, determining an appropriate detector $d = \delta(\mathbf{v})$ for transforming acquired features into directed actions, and finally determining the detector performance. These are the next four steps in the SHM design process.

CHOOSE FEATURE EXTRACTION PROCESS

The feature, $\mathbf{v}$, is the final form of the data that is used for statistical detection. We define feature extraction as the physical and computational process that generates $\mathbf{v}$. Features are often interpreted as necessarily being some digitally processed form of the raw data acquired from sensors. In truth, appropriate detectors can just as well be derived with raw acquired waveforms as features, bypassing the step often referred to as "feature extraction" [2]. However, modeling these raw, high dimensional features can sometimes be impractical. Feature extraction provides a means for the SHM practitioner to use past experience and engineering judgment to eliminate irrelevant data.

Unfortunately, it has become common for SHM researchers to simply guess at forms for low dimensional features, presenting the "success" of their feature extraction process on a very small experimental sample. Without any physical or statistical foundation, one runs the risk of 1) Unknowingly throwing away valuable information, 2) Producing highly volatile, noisy features without much broad application, and 3) Reinventing statistically equivalent forms of features developed in previous studies. A more effective approach is for the practitioner to reduce data only to the extent that he or she is certain no relevant information is being thrown away, and then further process the data using statistical-model based methods for further feature reduction and detection.

CALCULATE FEATURE STATISTICS AS A FUNCTION OF DESIGN

In order to design an appropriate detector, one must determine a stochastic model of the features in terms of the chosen design parameters. The model must express the statistical distribution of the features under each of the states identified in risk evaluation stage. In other words determine $\mathrm{p}(\mathbf{v}|\theta, e)$ for all $\theta \in \Theta$.

The model should be as informative as possible; however, appropriate detectors can be derived even if the model includes unknown parameters. For example, one may know the amplitude and arrival time of a wave reflecting from damage, but have no knowledge of the phase. In this scenario, one could derive a detector in a classical sense by using the maximum likelihood estimate of the phase, or in a Bayesian sense by presuming an *a priori* uniform distribution on the phase. Both approaches lead to a detector which compares the envelope of the waveform to some threshold [3].

DESIGN DETECTOR

Given a set of feature statistics, expressed in terms of the feature extraction design parameters, one must in turn design a detector that optimally or near optimally directs some action in response to the realization of some set of features:

$d = \delta(\nu)$. The general form of the optimal detector comes directly from the Bayes Risk formulation [3] as "$\delta(\nu) = \partial$ if

$$\mathrm{E}\left(\mathrm{L}^{(d)} \middle| \partial, \nu\right) < \mathrm{E}\left(\mathrm{L}^{(d)} \middle| \alpha, \nu\right) \quad \alpha \neq \partial \text{,"} \tag{2}$$

$$\text{where } \mathrm{E}\left(\mathrm{L}^{(d)} \middle| \alpha, \nu\right) = \sum_{\theta} \mathrm{L}^{(d)}(\alpha, \theta) \mathrm{P}\left(\nu \middle| \theta, e\right) \mathrm{P}(\theta). \tag{3}$$

In other words, the optimal detector is that which makes the decision that minimizes the Bayes Risk conditioned on the observed feature vector.

With real world application of SHM still in its infancy, the most common task for the detector is to direct whether or not the structural system should be manually inspected, which is a binary detection process. More complicated, continuous task functions, such as assigning a safe operation lifespan or maximum safe operating loads are ideal, but still far from maturity.

CALCULATE DETECTOR STATISTICS AND PERFORMANCE

Calculating the detector statistics is a matter of transforming the feature statistics according to the detector function. This is generally straight forward for linear detectors acting on normally distributed features. In non-linear situations, the exact statistics may not have closed form, and must be calculated numerically. For some optimization search techniques, solutions cannot be found in a reasonable time frame if evaluation of the performance function requires complex numerical algorithms. Therefore, if possible, the exact form of the detector statistics should be calculated in an approximate form for the optimization process, with the exact form being reserved for the in-service detection process. This sometimes may be accomplished by invoking central limit approximations as a result of the detector being formed from the combination of multiple sensor measurements. For a list of approximation techniques, refer to [1].

EXAMPLE DESIGN PROBLEM

Consider the simplified aerospace wing component shown in Figure 1. This component is subject to bending and torsional fatigue loading which tends to propagate defects initiated by impacts on the leading edges and orientated primarily at $\pm 45°$ and $0°$. Dividing the structure into 44,000 uniformly spaced regions, we determine that there is on average a $\mathrm{P}(\theta_k = 1) = 2.3 \times 10^{-9}$ probability of cracks with critical length of 1 cm forming in any given region between tests. With 44,000 regions and three crack orientation, there are a total of $K = 132000$ damage modes we are trying to detect. Figure 1 additionally shows a map of the probability of damage

We wish to design an SHM system that directs whether each region of the structure should or should not be manually inspected for damage, $d_k \in \{1,0\}$. We determine the cost per region inspected, $\mathrm{L}_k^{(d)}(d_k = 1, \theta = 0)$, is \$1, while the resulting cost of leaving a crack unchecked, $\mathrm{L}_k^{(d)}(d = 0, \theta = 1)$, is \$1,000,000. We choose to detect the cracks using a sparse array of piezoceramic discs to actuate and sense ultrasonic guided waves (UGW). Our design parameters are the number, N, and arrangement, $\boldsymbol{X}$, of these actuating and sensing transducers. By distributing the initial and recurring cost of the SHM system over the life of the structure, we determine that the SHM hardware cost is $2 per sensor per test. Our Bayes Risk can then be expressed as

$$E(L)=\sum_{k=1}^{K} 1\,\mathrm{P}\left(d_k=1,\theta_k=0\middle|\boldsymbol{X}\right)+10^6\,\mathrm{P}\left(d_k=0,\theta_k=1\middle|\boldsymbol{X}\right)+2N\,. \tag{4}$$

To obtain features, the system will actuate a narrowband pulse from each of the transducers, one at a time, and sense the response on all receiving transducers. We recognize that the scattering response is linear and that the phase of the scattered waveforms is unpredictable. With this information, we initially process the raw waveforms by band-pass filtering and taking the envelope. Through a generalized likelihood ratio test (GRLT) [3], we discover that each feature can be further reduced without much loss of information by extracting only the value of the envelope corresponding to the time of flight from actuator to each inspection region to the sensor.

If we make use of every possible unique actuator-sensor pair, in one direction, we will have a feature vector, $\boldsymbol{v}_k$ of length $P=N\left(N-1\right)/2$ for each damage mode. The GRLT finally tells us that the near optimal detector is to "Inspect for damage mode k if

$$T_k=\sum_{n=1}^{P}\left(v_k\left[n\right]a_k\left[n\right]/\sigma_k^2\left[n\right]\right)^2>\gamma_k\text{,"} \tag{5}$$

where $a_k[n]$ is the expected amplitude of the scattered waveform as actuated and sensed by transducer pair n and $\sigma_k^2[n]$ is the RMS of the interfering noise.

We can then complete the Bayes Risk calculation by recognizing that the conditional probabilities can be expressed in terms of right tail probabilities:

$$\begin{aligned}\mathrm{P}\left(d_k=1\middle|\theta_k=0,\boldsymbol{X}\right)&=\mathrm{P}\left(T_k>\gamma_k\middle|\theta_k=0,\boldsymbol{X}\right)\\ \mathrm{P}\left(d_k=0\middle|\theta_k=1,\boldsymbol{X}\right)&=1-\mathrm{P}\left(T_k>\gamma_k\middle|\theta_k=1,\boldsymbol{X}\right)\end{aligned}\,. \tag{6}$$

We note that for a sufficient number of actuator sensor pairs, we can consider the detector value T_k to be approximately Gaussian. The right tail probabilities can then be readily calculated using error functions. We determine the scatter amplitude, $a_k[n]$ for the detector value statistics using a ray tracing approach [4] with pre-computed scatter matrices of the three damage orientations and assume a constant noise parameter $\rho_k[n]$. The use of a low order model with pre-computed scatter matrices allows us to efficiently search the optimization space.

We search the space of potential transducer counts and arrangements using a modified genetic algorithm (GA) [5]. We start with a generously large set of transducers, and represent each transducer with a single precision x and y coordinate and a Boolean value that specifies whether that transducer is to be used. The GA then mutates and exchanges combinations of sensors, computing the Bayes Risk of each potential arrangement, and moving the best members forward in the evolution. Through this process, transducers become effectively added and removed through the exchange and mutation of its Boolean value until the algorithm settles on an optimal count and arrangement.

In Figure 1 we show the resulting arrangements for cost functions of $2, $4, and $8 per sensor. The optimal transducer counts were 17, 12, and 7 and the resulting total Bayes Risks were $43, $72, and $111, respectively.

Figure 1. Optimal arrangements for sensor costs of 2, 4, and 8 on map of local prob. of damage

SUMMARY

Casting the SHM design problem in terms of Bayesian experimental design pulls together all aspects of the SHM process. Making use of proper stochastic processing techniques, the SHM practitioner can not only improve their SHM system's capabilities, but they can evaluate and present performance in a way that is meaningful to the owner or operator of the structural system being monitored. This is critical to achieving significant industry use of SHM technology.

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