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NEUTRON COINCIDENCE MEASUREMENTS WHEN NUCLEAR PARAMETERS VARY DURING THE MULTIPLICATION PROCESS*

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INTRODUCTION

In a recent paper, (Reference [1]), a physical/mathematical model was developed for neutron coincidence counting, taking explicit account of neutron absorption and leakage, and using a dual probability generating function to derive explicit formulae for the single and multiple count-rates in terms of the physical parameters of the system.

The results of this modeling proved very successful in a number of cases in which the system parameters (neutron reaction cross-sections, detection probabilities, etc.) remained the same at the various stages of the process (i.e. from collision to collision) (References [2] - [6]). However, there are practical circumstances in which such system parameters change from collision to collision, and it is necessary to accommodate these, too, in a general theory, applicable to such situations. For instance, in the case of the neutron coincidence collar (NCC), which is described in detail in References [8] and [9], the parameters for the initial, spontaneous fission neutrons, are not the same as those for the succeeding induced fission neutrons, and similar situations can be envisaged for certain other experimental configurations. The present note shows how the previous considerations can be elaborated to embrace these more general requirements.

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THEORETICAL MODEL AND RESULTS

The underlying considerations are almost identical to those described in Reference[1], and will not be repeated here. Instead we begin in the middle of Section 3 of the cited reference with the probability generating function for a neutron in the first generation (cf. equation (17) of Reference [1]):

$$G_1(d,g) = p_{d1}d + p_{a1} + p_{f1}\mathbf{J}(g) \quad , \qquad (1)$$

with

$$\mathbf{J}(g) = \sum_{k=0}^{\infty} \mathbf{J}_k g^k .$$

In these expressions the presence of a d denotes the detection of a neutron, while g^i (in the power series expansion of **J**(g)) denotes that i neutrons are generated. p_{d1} is the probability of detection of a neutron, p_{a1} is the probability of neutron leakage and absorption without inducing a fission, and p_{f1} the probability of a neutron inducing fission. **J** is the generating function for the number of induced neutrons where the coefficient $\mathbf{J}_k$ denotes the probability of k induced neutrons in the reaction.

A more detailed discussion of the physical significance of the various terms which can occur in this expression is given in Section 2 of Reference [1].

For the next generation

$$G_2(d,g) = p_{d1}d + p_{a1} + p_{f1}\mathbf{J}(p_{d2}d + p_{a2} + p_{f2}\mathbf{J}(g)) \qquad (2)$$

$$= p_{d1}d + p_{a1} + p_{f1}\mathbf{J}(H_2(d,g)) \qquad (3)$$

where

$$H_2(d,g) = (p_{d2}d + p_{a2} + p_{f2}\mathbf{J}(g)) \qquad (4)$$

Writing $\quad q_i = p_{di}d + p_{ai}$ and $p_i = p_{fi} \qquad (5)$

one finds in general that

$$G_n(d,g) = q_1 + p_1\mathbf{J}\Big(q_2 + p_2\mathbf{J}\Big(q_3 + p_3\mathbf{J}\Big(q_4 \ldots + p_{n-1}\mathbf{J}\big(q_n + p_n\mathbf{J}(g)\big)\ldots\Big)\Big)\Big) \qquad (6)$$

DISCLAIMER

This is an extremely complex expression, but even in the general case it is possible to derive (formal) explicit expressions for certain quantities because of the underlying regularities.

In particular, for the first derivative one finds the pseudo-umbral expression (see Reference [7], p.20ff)

$$G \Rightarrow \frac{q'}{1-(p\mathrm{J}\,')q'} \tag{7}$$

in which, when expanded, powers denote correspondingly indexed variables, i.e.

$$(q')^{n} \rightarrow q_{n}' \tag{8}$$

and

$$(p\mathrm{J}\,')^{n} \Rightarrow p_1 p_2 \dots p_n \mathrm{J}\,'\big(q_2 + p_2\mathrm{J}\,'(\dots)\big)\mathrm{J}\,'\big(q_3 + p_3\mathrm{J}\,'(\dots)\big)\dots\mathrm{J}\,'\big(q_n + p_n\mathrm{J}\,'(\dots)\big) \tag{9}$$

Naturally such a formal expression only becomes useful when there are substantial regularities in the p's and q's.

For the application to the NCC one may note that after the first generation all the p's remain approximately the same. Assuming this to be the case, one has

$$G_n(d,g) = p_{d1}d + p_{a1} + p_{f1}\mathrm{J}\,(H_n(d,g)) \tag{10}$$

$$H_n(d,g) = p_d d + p_a + p_f\mathrm{J}\,(H_{n-1}(d,g)) \tag{11}$$

In the limit, assuming convergence, and dropping the suffix ∞, one finds that

$$G(d,g) = p_{d1}d + p_{a1} + p_{f1}\mathrm{J}\,(H(d,g)) \tag{12}$$

$$H(d,g) = p_d d + p_a + p_f\mathrm{J}\,(H(d,g)) \tag{13}$$

Noting that H(d, g) satisfies the same equations as the G(d, g) in Reference [1], Section 3, we then find , for example, that

$$<n> = p_{d1} + p_d\left(\frac{p_{f1}\overline{\nu}}{(1-p_f\overline{\nu})}\right) \quad (14)$$

and similarly

$$<n(n-1)> = p_{f1}\frac{p_d^2\overline{\nu}}{(1-p_f\overline{\nu})^3} <\nu(\nu-1)> \quad (15)$$

These last two expressions reduce to the usual ones (equations (21) and (22) in Reference [1]) when the probabilities for the source neutrons, e.g. external Am-Li neutrons, and the induced fission neutrons are the same.

Following the discussion in References [1] and [4], when the source neutrons originate in spontaneous fission events with a probability generating function with first and second moments

$$\overline{\nu}_s \ and <\nu_s(\nu_s-1)>$$

respectively, the expressions (14) and (15) above are modified to become

$$<n> = \left(p_{d1} + p_d\left(\frac{p_{f1}\overline{\nu}}{(1-p_f\overline{\nu})}\right)\right)\overline{\nu}_s \quad (16)$$

and

$$<n(n-1)> = p_{f1}\frac{p_d^2}{(1-p_f\overline{\nu})^3} <\nu(\nu-1)> \overline{\nu}_s + \left(p_{d1} + p_d\left(\frac{p_{f1}\overline{\nu}}{(1-p_f\overline{\nu})}\right)\right)^2 <\nu_s(\nu_s-1)> \quad (17)$$

These expressions, too, reduce to the simpler ones derived in Reference [1] when the source and induced neutron reaction probabilities are the same. It should be noted that the second term in the right hand side of equation (17) is now

$$\left(p_{d1} + p_d\frac{p_{f1}\overline{\nu}}{(1-p_f\overline{\nu})}\right)^2 <\nu_s(\nu_s-1)> \text{ rather than } \left(p_d + p_d\frac{p_f\overline{\nu}}{(1-p_f\overline{\nu})}\right)^2 <\nu_s(\nu_s-1)>$$

which would be the case when the nuclear parameters are unchanged throughout the process.

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